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ANALYSIS OF THE DIFFUSION
EQUATION IN ITS SEMI-DISCRETE FORM

by

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ABSTRACT

The partial differential equation describing transient conduction (or diffusion) in non-homogeneous media may be approximated by a set of first order ordinary differential equations if the space-derivatives are replaced by finite-difference expressions. The independent variable in this set of ordinary differential equations is time; the dependent variables are temperatures (or concentrations) at a number of discrete points. If the properties of the media are functions of position only, but not of temperature (or concentration) or time, then the set of differential equations is linear. Further, if the media are isotropic, the coefficient matrix associated with this set of linear differential equations is symmetric. A general method of obtaining a closed form solution of these equations was developed. The solution makes use of certain operational methods of linear algebra, and is in terms of a matrix, which describes the spatial distribution of physical properties in the media, and vectors describing the initial and boundary conditions. Example problems of varying complexity in one- and two-dimensional non-homogeneous media are presented. The solutions were found to agree favourably with solutions obtained by other methods.

The linear transformation of the dependent variables used in obtaining a general solution of the set of ordinary

differential equations decouples these variables. These modified variables are shown to decay logarithmically at rates proportional to the eigenvalues of the coefficient matrix. This property of the modified variables was used in developing a method by which the transport properties of a medium can be determined with good sensitivity. Unsteady-state data required to test this model were produced from experiments using the technique of microcell interferometry. Using this method, the molecular diffusivity of copper sulfate in water and thermal diffusivity of water were determined with acceptable accuracies. The conventional "least-squares" analysis of these data was found to be inadequate.

A mathematical model for estimation of the "conductivity matrix" of a non-homogeneous medium was developed. When tested on some data generated from a semi-analytical solution, the model produced very accurate results. Some predictions about the design of experiments, made by the model were thus verified.

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I. INTRODUCTION

"Et Ignem Regunt Numeri - Plato"

The importance of heat has been known to man since the time he discovered fire. But the nature of laws governing transfer of heat were not known till the time of the great French physicist Jean Baptiste Joseph Fourier (1768 - 1830 A.D.). Fourier deduced certain laws governing heat transfer through "intense observation" and his monumental treatise, "Théorie Analytique de la Chaleur" was a result of these observations. In his treatise, Fourier developed the differential equations for steady-state and transient flow of heat, and developed the famous Fourier series solutions to some simple cases. Since that time, scores of volumes have been written on the subject and some are still being written.

The concept of conductivity of a body was first introduced by Fourier (he called it conducibility) and was mathematically described by him in his famous law of conduction. He mentioned in his treatise that "it could be determined by measuring the times of cooling in vessels of different thicknesses". Evidence exists that he was the first person to make any such measurements and he determined the "conducibility" of cast iron in this manner.

H.S. Carslaw and J.C. Jaeger, in their classical

work (6)*, considered the process of conduction of heat in solids systematically and developed numerous analytical solutions to the partial differential equation describing the process of transient heat conduction for a number of initial and boundary conditions. Practically none of these solutions are capable of treating a non-homogeneous body: that is, a body in which the properties of heat capacity and thermal conductivity are not uniform but are functions of its space coordinates and its temperature. Also, no general method of arriving at an analytical solution was presented for multi-dimensional bodies.

The development of computers in the recent past has made practicable an alternate way of handling the heat conduction equation, namely, the technique of numerical integration. Since the introduction of the explicit finite-difference formulation by Binder (4) and Schmidt (26) for the one-dimensional heat conduction equation, scores of numerical methods have been introduced, and significant progress has been made in this field. The numerical methods are capable of handling the non-homogenieties of a body: this seems to be one of their main advantages over analytical solutions. Nevertheless, these methods suffer from the fact that they lack generality; also, the solutions need to

* Numbers within brackets refer to the list of references at the end of the thesis.

be repeated in their entirety for any change of initial or boundary condition.

The science of estimation of transport properties of a medium has also been the subject of several investigators. Recently, Beck (3) reviewed many of these methods for determination of thermal conductivity of a medium. Numerous techniques for estimation of the diffusivities of a solute in liquids have also been developed and are reviewed at length in Hirschfelder et al (11). A few characteristics of all these methods are striking. The experimental times of most of these methods have been high, so that elaborate arrangements had to be made to eliminate extraneous effects. In many cases, the data could be collected at only a few points of the medium, which restricted the available information. Further, the mathematical techniques used in the treatment of unsteady-state data were rather tedious and insensitive.

The present work takes a different look at the Fourier conduction equation. The technique has been named as semi-analytical since the time derivative of the Fourier equation is treated as a continuous derivative and only the space derivatives are discretised. This treatment transforms the original partial differential equation into a set of ordinary differential equations. The whole development in the present work leans heavily on the fact that there is a first order coupling of flux between adjacent

elements of a body in which the properties are independent of the dependent variable. Also, if the body is isotropic, the interactions between two adjacent elements are equal and opposite. These properties permit setting up of a matrix differential equation in which the coefficient matrix has certain special properties. The theory of linear algebra and matrices can be used to great advantage on this particular mathematical model of the physical system. The fact that the matrix is symmetric allows one to make a few generalisations. Firstly, there is an orthogonal system of eigenvectors associated with it. By defining new dependent variables, which are certain linear combinations of the original variables, the system can be completely uncoupled. Further development shows that a few "straight line paths" can be established, which leads one to a rather simple method of estimation of the properties of a medium, homogeneous or non-homogeneous. The properties of orthogonality and symmetry of the mathematical model of the physical system also lead one to a closed form solution to the Fourier conduction equation.

In the remaining chapters of this work, the statements made above are developed at length. Chapter II deals with the general theory of the system. Chapters III and IV deal with some simple and complex problems involving the solution of the Fourier equation with varying degrees of complexity in properties. In Chapter V, a general method

of estimation of homogeneous thermal and molecular diffusivities of liquids is developed. Chapter VI considers the problem of estimation of properties of a non-homogeneous medium and some interesting conclusions leading to certain improvements in design of experiments are brought out.

II. GENERAL THEORY

In this chapter, the general problem of solving the diffusion equation is discussed. A semi-analytical solution, using some operational methods of linear algebra is developed. The advantages and restrictions of this solution are indicated.

The Fourier equation, which describes the process of transient heat conduction in a body, is derived in great detail in various treatises on heat transfer (6,9,14). In the case where

1. the temperature field extends over a single non-homogeneous, isotropic body; and
 2. no phase changes take place within the field,
- the Fourier equation, written in standard vector notation is

$$c \frac{\partial u}{\partial t} = \nabla \cdot (k \nabla u) + f(\underline{r}, t) \quad (\text{II.1})$$

where u = temperature at any point in the body;
 \underline{r} = "position vector" of the point under consideration, with respect to an arbitrarily chosen origin (A bar _ under a letter is used to denote a vector or a matrix. Small letters are used for vectors and capital letters for matrices);
 t = time
 k = thermal conductivity, a function of \underline{r} and u in general;

c = heat capacity, a function of \underline{r} and u in general;

f = the "generation term". For the purposes of this equation, the form of energy from which heat is obtained is entirely immaterial. However, the entire system of heat sources within the field must be known.

Equation (II.1) is a parabolic partial differential equation in four independent variables (time t and three space variables) and one dependent variable u . If k and c are independent of u , this equation becomes linear, since it contains only the first powers of temperature and its derivatives. Also it is of second order since it contains second partial space derivatives of the temperature.

The general problem of heat conduction, then, is to determine the temperature distribution in the body, given:

1. the temperature distribution at an earlier instant (usually at $t = 0$). This is known as the initial condition.
2. The influences of the surroundings on the boundaries of the body. These are known as the boundary conditions.

The solution to the general conduction problem has been very extensively attempted in the past. Three approaches have been taken, namely:

1. analytical methods
2. semi-analytical methods
3. numerical methods

The excellent treatise by Carslaw and Jaeger (6) deals mainly with the analytical solutions of the conduction equation for a homogeneous one-dimensional body under various initial and boundary conditions. A few problems of heat flow in non-homogeneous bodies are also considered and solutions for these are given. However, practically no multidimensional problems in a non-homogeneous body are treated in this book.

Numerical methods of solving the Fourier equation have been extensively developed since the advent of high speed digital computers. The various numerical methods may be classified into two broad categories, namely the explicit and implicit methods. Stable procedures have been developed using both of these techniques (1,13,23). Recently, Varga (30) generalised these various methods and developed a few general criteria for stability of these methods.

One main disadvantage of numerical methods is that they lack generality; a change in initial or boundary condition requires that the complete solution be repeated. Another disadvantage is that the integration procedures are invariably stepwise in time, and as such require that the solution has to be evaluated at the previous time intervals

to get the solution at the time required - obviously a time - consuming procedure.

The semi-analytical method proposed here for the solution of the general conduction equation combines many of the advantages of analytical and numerical methods and, hopefully, minimises their disadvantages. This method uses the concept of lumped parameters, so commonly used in numerical analysis, along with some operational methods of linear algebra and matrix theory.

Semi-analytical methods are not altogether new: However, the application of semi-analytical methods to problems of interest to the chemical engineer has been very limited. Varga (29) suggested a solution to the general diffusion equation by integrating the set of ordinary differential equations obtained when the partial derivatives on the space side are put in finite difference form. He also established that the set of ordinary differential equations is stable from a consideration of the properties of the associated coefficient matrix.

The most important work of interest to the chemical engineer in which the semi-analytical method is made use of is that of Wei and Prater (32). Paradoxically enough, there is practically no mention of a possible extension of their theory - essentially developed for a system of complex first order reactions - to the solution of the diffusion equation or to the determination of transport properties, where generally the rate equation is indeed

first order.

By an appropriate method of spatial discretisation, equation (II.1) can be brought into the form

$$\underline{D} \frac{d\underline{u}(t)}{dt} = \underline{B}' \underline{u}(t) + \underline{s}'(t) \quad (\text{II.2})$$

where \underline{D} = a diagonal matrix of constants;

$\underline{u}(t)$ = temperature vector;

\underline{B}' = coefficient matrix of constants;

$\underline{s}'(t)$ = boundary condition vector.

By the nature of the problem, \underline{D} is a positive real diagonal matrix and hence has an inverse \underline{D}^{-1} , whose elements are simply the reciprocals of the elements of \underline{D} . The notation $\underline{D}^{1/2}$ and $\underline{D}^{-1/2}$ is used here to denote the diagonal matrices whose elements are the square roots of the elements of \underline{D} and \underline{D}^{-1} , respectively.

The derivation of equation (II.2) from equation (II.1) is deferred to a later stage in the thesis.

A new variable $\underline{w}(t)$ is defined by the equation

$$\underline{w}(t) = \underline{D}^{1/2} \underline{u}(t) \quad (\text{II.3})^*$$

which gives

$$\underline{u}(t) = \underline{D}^{-1/2} \underline{w}(t) \quad (\text{II.4})$$

Substitution of equation (II.4) into equation (II.2) and pre-multiplication by $\underline{D}^{-1/2}$ gives

* As will be seen in the presented examples, $\underline{D} \equiv \underline{I}$, the identity matrix in many cases, which makes $\underline{w}(t) \equiv \underline{u}(t)$.

Let $f(x) = \sum_{n=0}^{\infty} a_n x^n$ be a power series with radius of convergence $R > 0$. Then

$$f'(x) = \sum_{n=1}^{\infty} n a_n x^{n-1} \quad (1)$$

Proof. Let $x \in (-R, R)$. Then $|x| < R$ and so $|x| < R/2$. For each $n \in \mathbb{N}$, we have

$$|n a_n x^{n-1}| \leq n |a_n| |x|^{n-1}$$

and so by (1) we have $|f'(x)| \leq \sum_{n=1}^{\infty} n |a_n| |x|^{n-1}$. Since $|x| < R/2$, we have

$$\sum_{n=1}^{\infty} n |a_n| |x|^{n-1} < \sum_{n=1}^{\infty} n |a_n| (R/2)^{n-1} < \sum_{n=1}^{\infty} n |a_n| R^{n-1} < \infty$$

and so the series $\sum_{n=1}^{\infty} n |a_n| |x|^{n-1}$ converges. By (1) we have

$$f'(x) = \sum_{n=1}^{\infty} n a_n x^{n-1} \quad (2)$$

and so $f'(x)$ is a power series with radius of convergence R . Let $x \in (-R, R)$. Then

$$f'(x) = \sum_{n=1}^{\infty} n a_n x^{n-1} = \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n$$

$$= \sum_{n=0}^{\infty} a_{n+1} (n+1) x^n = \sum_{n=0}^{\infty} a_{n+1} x^{n+1} = f(x) - a_0$$

$$= f(x) - a_0$$

$$f'(x) = f(x) - a_0$$

and so $f'(x) = f(x) - a_0$. Let $x \in (-R, R)$. Then $|x| < R$ and so $|x| < R/2$. For each $n \in \mathbb{N}$, we have

$$|a_{n+1} (n+1) x^n| \leq |a_{n+1}| (n+1) |x|^n \leq |a_{n+1}| (n+1) (R/2)^n < |a_{n+1}| R^n < \infty$$

and so the series $\sum_{n=0}^{\infty} |a_{n+1}| R^n$ converges. By (2) we have

$$\sum_{n=0}^{\infty} |a_{n+1}| R^n < \infty$$

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$$\frac{d \underline{w}(t)}{dt} = \underline{B} \underline{w}(t) + \underline{s}(t)$$

where $\underline{s}(t) = \underline{D}^{-\frac{1}{2}} \underline{s}'(t)$ and $\underline{B} = \underline{D}^{-\frac{1}{2}} \underline{B}' \underline{D}^{-\frac{1}{2}}$.

Based on equation (II.5), it is possible to pose two problems:

1. Given the coefficient matrix \underline{B} , an initial temperature vector $\underline{u}(0)$ and the boundary condition vector $\underline{s}'(t)$, find the temperature profiles as a function of time. This is known as the "Forward Problem".
2. Given the temperature profiles $\underline{u}(t)$ for various elapsed times and the boundary condition vector $\underline{s}'(t)$, find the coefficient matrix \underline{B} . This is known as the "Inverse Problem".

The classical work of Wei and Prater (32) deals with the solution of the inverse problem. However, there are a number of marked differences between the problem of Wei and Prater and the problem considered in this work. These are:

1. The system studied by Wei and Prater is a complex first order reaction system. This will be referred to as a "chemical system" from here on.
2. It is possible to study a chemical system experimentally under a wide variety of initial conditions. This may not be possible in a physical system.

3. In a chemical system, one more equation, the conservation of mass is available. As may be noticed from Wei and Prater's work, their development leans heavily on this fact.
4. In a chemical system, no boundary conditions are encountered, again changing the formulation of the model quite significantly.
5. The coefficient matrix is, in general, unsymmetric in the case of a chemical system, contrary to the physical systems*. The fact that \underline{B} is symmetric in a physical system makes the size of the physical problems that can be handled fairly large, simply because the techniques of handling symmetric matrices are developed better.

* This can be proved as follows:

Restricting the present discussion, for the moment, to the two-dimensional case, consider a plane region R of unit thickness (Fig. II.1). This region is arbitrarily divided into n elemental areas. In accordance with the lumped parameter concept, u , k , c are considered constant throughout each element. The properties are in fact chosen to be those of a central point for each of these elements. Expansion of the space derivative $\nabla \cdot (k \nabla u)$ for a two-dimensional case results in

$\frac{\partial}{\partial x} (k \frac{\partial u}{\partial x}) + \frac{\partial}{\partial y} (k \frac{\partial u}{\partial y})$. This expression, written in finite difference form for each one of the mesh points results in the symmetric matrix \underline{B}' , which when pre- and post-multiplied by the same diagonal matrix $\underline{D}^{1/2}$ still remains symmetric: (This is true of any two symmetric matrices \underline{X} and \underline{Y} , for

$$(\underline{X} \underline{Y} \underline{X})^T = \underline{X}^T \underline{Y}^T \underline{X}^T = \underline{X} \underline{Y} \underline{X})$$

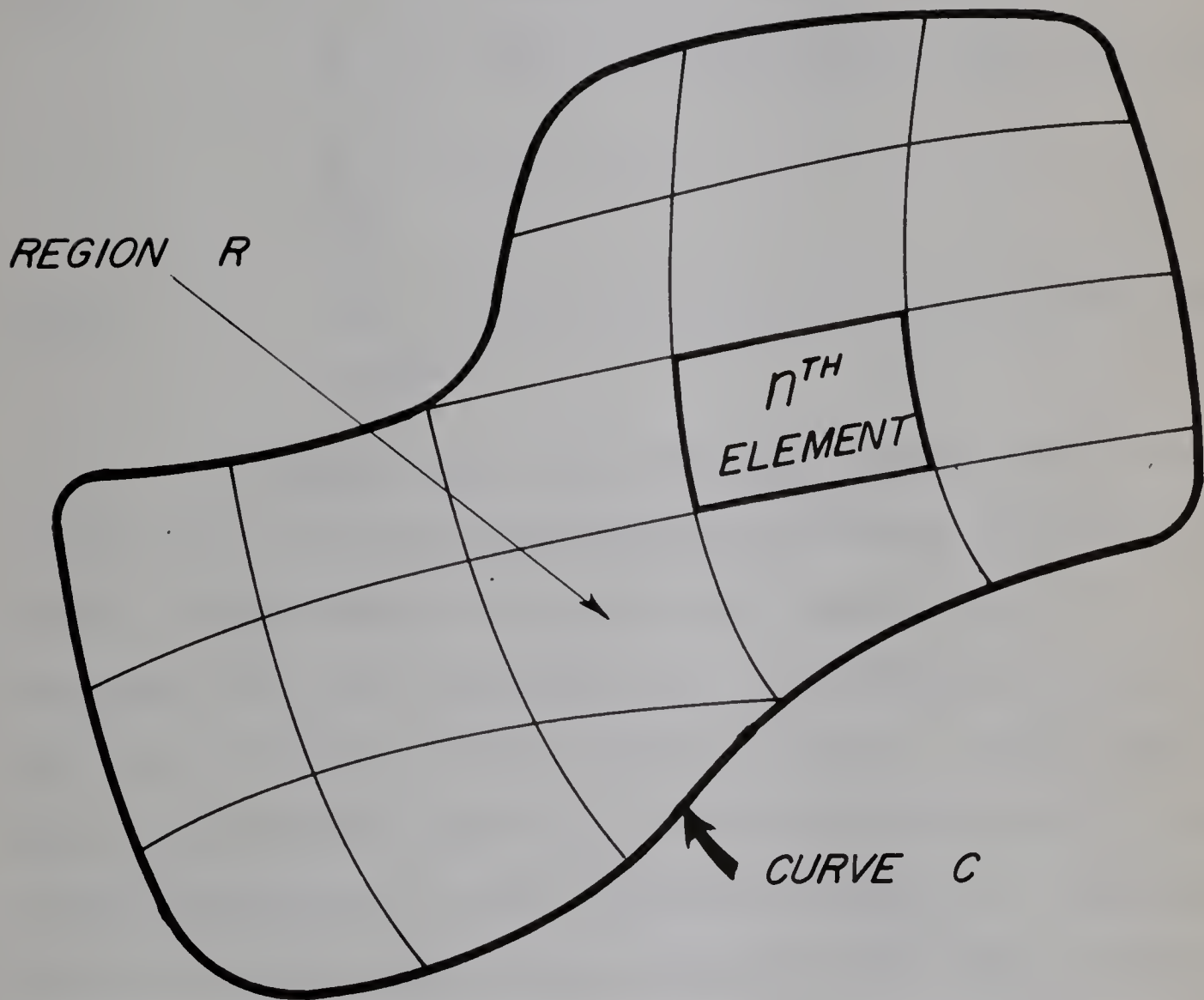


FIGURE II.1 A TWO-DIMENSIONAL PLANE

In the work of Wei and Prater, the matrix differential equation (II.5) simplifies to

$$\frac{d\alpha}{dt} = \underline{K} \alpha \quad (\text{II.6})$$

where α = composition vector; and
 \underline{K} = rate constant matrix, with the constraints

$$\sum_{i=1}^n \alpha_i = 1.0 \quad (\text{II.7})$$

where α_i = mole fraction of i th component; and
 $\alpha_i \geq 0$ (II.8).

In their work, Wei and Prater prove that for every system which can be represented by equations (II.6) through (II.8), there exists a hypothetical species β_i ($i = 1$ to n), for which the rate equations are completely uncoupled, unlike the system represented by equation (II.6). They also develop this hypothetical species β , which they prove to be a certain linear combination of the original species α . The weighting factors used in the linear combination are shown to be the elements of the eigenvectors of the rate constant matrix \underline{K} , or in matrix notation

$$\alpha = \underline{X} \beta \text{ and } \beta = \underline{X}^{-1} \alpha \quad (\text{II.9})$$

where \underline{X} is the matrix of eigenvectors of \underline{K} , in which the eigenvectors are arranged column wise. The lengths of each of these eigenvectors is dictated by other conditions in the

problem, and will not be discussed here. Equation (II.6) in terms of the hypothetical species becomes

$$\frac{d\beta}{dt} = \underline{\Delta} \beta \quad (\text{II.10})$$

where $\underline{\Delta}$ = a diagonal matrix whose diagonal elements are the eigenvalues of \underline{K} .

Geometrically, the β species may be looked upon as the α - species in a different coordinate system, namely the coordinate system of the eigenvectors of \underline{K} . Each "component" of the β -species is a projection onto one of these eigenvectors. One of the very important properties of an eigenvector of a matrix is that when premultiplied by the matrix, the eigenvector undergoes a change in length only, but not in its direction. In other words, these eigenvectors are the "straight line paths" in an n-dimensional space, and Wei and Prater developed a very elegant method of determining these paths for a complex first order reaction system; when once these paths are determined, the determination of the rate constant matrix \underline{K} is easily accomplished through a similarity transformation. After the mathematical analysis is started with any arbitrary initial conditions, the method is capable of dictating the next initial condition for the experiments so that the straight line paths can be arrived at with a minimum of experimental effort.

The method proposed here, which is based on the

semi-discrete form (II.5) of the diffusion equation, is considered from the two points of view, the forward problem and the inverse problem. The mathematics developed for the forward problem are rigorous and the method is general: it can account for any type of spatial variation of properties, provided the discretisation using the lumped parameter concept does not introduce large errors; any time-dependent, but linear boundary conditions can be handled. However, the condition of radiation at a boundary, proportional to the fourth power of temperature cannot be handled.

Because of the nature of the inverse problem, the theory can be divided into two clear categories:

1. theory for a homogeneous medium; and
2. theory for a non-homogeneous medium.

As may be readily seen, the coefficient matrix for a homogeneous medium is known (except for a constant multiplicative factor, which is the homogeneous diffusivity), contrary to the case of a non-homogeneous medium. However, certain known properties of the coefficient matrix for a non-homogeneous medium are made use of in developing the theory for this case. This is discussed in Ch. VI. The theory of the inverse problem for a homogeneous medium is given here.

A. Forward Problem.

The solution to the forward problem is a general solution of equation (II.5):

$$\frac{d \underline{w}(t)}{dt} = \underline{B} \underline{w}(t) + \underline{s}(t) \quad (\text{II.5})$$

with the initial conditions $\underline{w}(0) = \underline{w}_0$.

Since \underline{B} is a real, symmetric and negative-definite matrix, it is evident that*:

1. \underline{B} has n real, non-positive eigenvalues (n is the order of the matrix); and
2. \underline{B} has n real and mutually orthogonal eigenvectors \underline{q}_j ($j=1$ to n) satisfying the relation

$$\underline{q}_j \cdot \underline{q}_i = 0 \text{ if } i \neq j \text{ and } \underline{q}_i \cdot \underline{q}_i = \text{constant} \quad (\text{II.11})$$

Furthermore, if the vectors \underline{q}_j are so chosen that they are of unit length, then they are orthonormal, which makes

$$\underline{Q}^{-1} = \underline{Q}^T \quad (\text{II.12})$$

\underline{Q} = matrix of eigenvectors of \underline{B} , whose columns are the normalised eigenvectors \underline{q}_j . Superscript T refers to the transpose.

This particular way of choosing the individual eigenvectors make \underline{Q} an orthogonal matrix.

A new variable \underline{v} is defined by the equation

$$\underline{v}(t) = \underline{Q}^T \underline{w}(t) \text{ and } \underline{w}(t) = \underline{Q} \underline{v}(t) \quad (\text{II.13})$$

* The proofs of various theorems of matrix algebra are given in Appendix A.

Replacement of $\underline{w}(t)$ by $\underline{v}(t)$ in equations (II.5) yields

$$\underline{Q} \frac{d\underline{v}(t)}{dt} = \underline{B} \underline{Q} \underline{v}(t) + \underline{s}(t) \quad (\text{II.14})$$

Premultiplication by \underline{Q}^T gives

$$\frac{d \underline{v}(t)}{dt} = \underline{Q}^T \underline{B} \underline{Q} \underline{v}(t) + \underline{Q}^T \underline{s}(t) \quad (\text{II.15})$$

Now, the operation $\underline{Q}^T \underline{B} \underline{Q}$ of equation (II.15) on the symmetric matrix \underline{B} is a similarity transformation, which serves to diagonalise the matrix \underline{B} resulting in the diagonal matrix \underline{P} , whose diagonal elements are the eigenvalues of \underline{B} . In matrix notation,

$$\underline{Q}^T \underline{B} \underline{Q} = \underline{P} = \begin{bmatrix} p_1 & & & & \\ & p_2 & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & p_n \end{bmatrix} \quad (\text{II.16})$$

where p_i ($i = 1$ to n) are the eigenvalues of \underline{B} .

Also,

$$\underline{Q} = \begin{bmatrix} q_{11} & q_{12} & \cdot & \cdot & \cdot & \cdot & q_{1n} \\ q_{21} & & & & & & \cdot \\ \cdot & & & & & & \cdot \\ \cdot & & & & & & \cdot \\ \cdot & & & & & & \cdot \\ q_{n1} & \cdot & \cdot & \cdot & \cdot & \cdot & q_{nn} \end{bmatrix} \quad (\text{II.17})$$

$$\underline{Q}^T = \begin{bmatrix} q_{11} & q_{21} & \cdot & \cdot & \cdot & \cdot & \cdot & q_{n1} \\ q_{12} & & & & & & & \cdot \\ \cdot & & & & & & & \cdot \\ \cdot & & & & & & & \cdot \\ \cdot & & & & & & & \cdot \\ q_{1n} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & q_{nn} \end{bmatrix} \quad (\text{II.18})$$

$$\underline{Q}^T \underline{s}(t) = \left[\underline{q}_1^T \underline{s}(t), \underline{q}_2^T \underline{s}(t), \cdot \cdot \cdot, \underline{q}_n^T \underline{s}(t) \right]^T \quad (\text{II.19})$$

Equation (II.15) now becomes

$$\frac{d\underline{v}(t)}{dt} = \underline{P} \underline{v}(t) + \underline{Q}^T \underline{s}(t) \quad (\text{II.20})$$

Each individual equation of the matrix equation (II.20) may be written as

$$\frac{d v_j(t)}{dt} = p_j v_j(t) + \underline{q}_j^T \underline{s}(t) \quad (\text{II.21})$$

(j = 1 to n)

This is a first order linear ordinary differential equation in one variable $v_j(t)$ with the initial condition

$$v_j(0) = v_{j,0} \quad (= \underline{q}_j^T \underline{w}(0)) \quad (\text{II.22})$$

and is effectively decoupled from the other variables v_i ($i \neq j$). Equation (II.21) may be integrated to give

$$v_j(t) = e^{p_j t} \left[v_{j,0} + \int_0^t e^{-p_j \sigma} \left[\underline{q}_j^T \underline{s}(\sigma) \right] d\sigma \right] \quad (\text{II.23})$$

$$(j = 1 \text{ to } n)$$

Equation (II.23) is the general solution of equation (II.5), but can be put into a compact matrix form through some rearrangement.

In long form, equation (II.23) may be written as

$$\begin{bmatrix} v_1(t) \\ v_2(t) \\ \cdot \\ \cdot \\ \cdot \\ v_n(t) \end{bmatrix} = \begin{bmatrix} e^{p_1 t} & & & & \\ & e^{p_2 t} & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \cdot \\ & & & & e^{p_n t} \end{bmatrix} \begin{bmatrix} v_{1,o} \\ v_{2,o} \\ \cdot \\ \cdot \\ \cdot \\ v_{n,o} \end{bmatrix} + \\
 \begin{bmatrix} e^{p_1 t} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ e^{p_n t} \end{bmatrix} \begin{bmatrix} \int_0^t e^{-p_1 \sigma} \left[\underline{q}_1^T \underline{s}(\sigma) \right] d\sigma \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \int_0^t e^{-p_n \sigma} \left[\underline{q}_n^T \underline{s}(\sigma) \right] d\sigma \end{bmatrix} \quad (II.24)$$

Premultiplication of equation (II.24) with \underline{Q} gives

$$\begin{bmatrix} w_1(t) \\ w_2(t) \\ \cdot \\ \cdot \\ \cdot \\ w_n(t) \end{bmatrix} = \begin{bmatrix} q_{11} & q_{12} & \cdot & \cdot & \cdot & q_{1n} \\ q_{21} & & & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ q_{n1} & \cdot & \cdot & \cdot & \cdot & q_{nn} \end{bmatrix} \begin{bmatrix} e^{p_1 t} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ e^{p_n t} \end{bmatrix} \begin{bmatrix} v_{1,o} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ v_{n,o} \end{bmatrix} +$$

$$\begin{bmatrix} q_{11} & q_{12} & \cdot & \cdot & \cdot & q_{1n} \\ q_{21} & & & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ q_{n1} & \cdot & \cdot & \cdot & \cdot & q_{nn} \end{bmatrix} \begin{bmatrix} e^{p_1 t} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ e^{p_n t} \end{bmatrix} \begin{bmatrix} \int_0^t e^{-p_1 \sigma} \underline{q}_1^T \underline{s}(\sigma) d\sigma \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \int_0^h e^{-p_n \sigma} \underline{q}_n^T \underline{s}(\sigma) d\sigma \end{bmatrix} \quad (II.25)$$

$$\text{But } \underline{v}_0 = \underline{Q}^T \underline{w}_0 \quad (II.26)$$

$$\text{and } \underline{Q}^T \underline{Q} = \underline{I} \quad (II.27)$$

So, substitution of equation (II.26) into the first term of equation (II.25) and of equation (II.27) into the integral of the second term of equation (II.25) gives

$$\begin{bmatrix} w_1(t) \\ \cdot \\ \cdot \\ \cdot \\ w_n(t) \end{bmatrix} = \begin{bmatrix} q_{11} & \cdot & \cdot & \cdot & \cdot & q_{1n} \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ q_{n1} & \cdot & \cdot & \cdot & \cdot & q_{nn} \end{bmatrix} \begin{bmatrix} e^{p_1 t} \\ \cdot \\ \cdot \\ \cdot \\ e^{p_n t} \end{bmatrix} \times$$

$$\begin{bmatrix} q_{11} & \cdot & \cdot & \cdot & \cdot & q_{n1} \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ q_{1n} & \cdot & \cdot & \cdot & \cdot & q_{nn} \end{bmatrix} \begin{bmatrix} w_{1,0} \\ \cdot \\ \cdot \\ \cdot \\ w_{n,0} \end{bmatrix} + \begin{bmatrix} q_{11} & \cdot & \cdot & \cdot & \cdot & q_{1n} \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ q_{n1} & \cdot & \cdot & \cdot & \cdot & q_{nn} \end{bmatrix} \times$$

$$\begin{bmatrix} e^{p_1 t} & & & \\ & \cdot & & \\ & & \cdot & \\ & & & \cdot \\ & & & & e^{p_n t} \end{bmatrix} \begin{bmatrix} q_{11} & \cdot & \cdot & \cdot & q_{n1} \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ q_{1n} & \cdot & \cdot & \cdot & q_{nn} \end{bmatrix} \int_0^t \begin{bmatrix} q_{11} & \cdot & \cdot & \cdot & q_{1n} \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ q_{n1} & \cdot & \cdot & \cdot & q_{nn} \end{bmatrix} x$$

$$\begin{bmatrix} e^{-p_1 \sigma} \left[\underline{q}_1^T \underline{s}(\sigma) \right] d\sigma \\ \cdot \\ \cdot \\ \cdot \\ e^{-p_n \sigma} \left[\underline{q}_n^T \underline{s}(\sigma) \right] d\sigma \end{bmatrix} \quad (II.28)$$

The following notation is employed:

$$\underline{E}(t) = \begin{bmatrix} e^{p_1 t} & & & \\ & \cdot & & \\ & & \cdot & \\ & & & \cdot \\ & & & & e^{p_n t} \end{bmatrix} \quad (II.29a)$$

$$\underline{E}^{-1}(t) = \begin{bmatrix} e^{-p_1 t} & & & \\ & \cdot & & \\ & & \cdot & \\ & & & \cdot \\ & & & & e^{-p_n t} \end{bmatrix} \quad (II.29b)$$

$$\underline{h}(t) = \begin{bmatrix} \int_0^t e^{-p_1 \sigma} \underline{q}_1^T \underline{s}(\sigma) d\sigma \\ \cdot \\ \cdot \\ \int_0^t e^{-p_n \sigma} \underline{q}_n^T \underline{s}(\sigma) d\sigma \end{bmatrix} \quad (\text{II.29c})$$

$$\text{Now, } \underline{h}(t) = \int_0^t \begin{bmatrix} e^{-p_1 \sigma} \\ \cdot \\ \cdot \\ e^{-p_n \sigma} \end{bmatrix} \begin{bmatrix} q_{11} & \cdot & \cdot & \cdot & q_{n1} \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ q_{1n} & \cdot & \cdot & \cdot & q_{nn} \end{bmatrix} \underline{x} \quad (\text{II.29d})$$

$$\begin{bmatrix} s_1(\sigma) \\ \cdot \\ \cdot \\ \cdot \\ s_n(\sigma) \end{bmatrix} d\sigma \quad (\text{II.30})$$

So, equation (II.28) may be written as

$$\underline{w}(t) = \underline{Q} \underline{E}(t) \underline{Q}^T \left[\underline{w}_0 + \int_0^t \underline{Q} \underline{E}^{-1}(\sigma) \underline{Q}^T \underline{s}(\sigma) d\sigma \right] \quad (\text{II.31})$$

Finally, $\underline{u}(t) = \underline{D}^{-1/2} \underline{w}(t)$ from equation (II.4).

Also, $\underline{w}_0 = \underline{D}^{1/2} \underline{u}(0)$ and $\underline{s}(\sigma) = \underline{D}^{-1/2} \underline{s}'(\sigma)$.

So, premultiplication by $\underline{D}^{-\frac{1}{2}}$ and substitution for the other values gives

$$\begin{aligned} \underline{u}(t) &= \underline{D}^{-\frac{1}{2}} \underline{Q} \underline{E}(t) \underline{Q}^T \left[\underline{D}^{\frac{1}{2}} \underline{u}(0) + \right. \\ &\quad \left. \int_0^t \underline{Q} \underline{E}^{-1}(\sigma) \underline{Q}^T \underline{D}^{-\frac{1}{2}} \underline{s}(\sigma) d\sigma \right] \\ \text{or, } \underline{u}(t) &= \underline{D}^{-\frac{1}{2}} \underline{Q} \underline{E}(t) \underline{Q}^T \underline{D}^{\frac{1}{2}} \left[\underline{u}_0 + \int_0^t \underline{D}^{-\frac{1}{2}} \underline{Q} \underline{E}^{-1}(\sigma) \right. \\ &\quad \left. \underline{Q}^T \underline{D}^{-\frac{1}{2}} \underline{s}'(\sigma) d\sigma \right] \end{aligned} \quad (\text{II.32})$$

Equation (II.32) represents the general solution, rigorous after the approximation of discretisation of space derivatives, to the general diffusion equation (II.1). If the boundary condition vector $\underline{s}'(t)$ is simple, the integral term of equation (II.32) may be evaluated analytically; otherwise, recourse to numerical integration methods has to be sought.

The solution, of course, involves the evaluation of the eigenvalues and eigenvectors of the matrix \underline{B} .

B. Inverse Problem For a Homogeneous Medium.

The general theory is developed by considering the process of unsteady-state heat transfer as an example. For the case of a one-dimensional, homogeneous body, the starting partial differential equation is

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2} \quad (\text{II.33})$$

where α = diffusivity of the medium and
 x = distance.

Typical initial and boundary conditions are:

$$\left. \begin{array}{ll} \text{I.C:} & \text{at } t = 0, \quad u = f(x) \\ \text{B.C:} & \text{at } x = 0, \quad u = \text{constant}, c_1 \\ & \text{at } x = 1, \quad \frac{\partial u}{\partial x} = \text{constant}, c_2 \end{array} \right\} \quad (\text{II.34})$$

(The boundary conditions do not have to be exactly as they are in equation (II.34); they can be any combination of these two at the two ends. Fig. II.2 illustrates the typical grid spacing for the above boundary conditions and may have to be altered a little to suit the particular boundary conditions).

The distance $0 \leq x \leq 1$ of the body is divided into a finite number of distance increments (n) of equal width h , with an increment of width $\frac{h}{2}$ next to the boundary where flux is specified, as shown in Fig. II.2. The temperatures at each of these grid points is specified by u_1, u_2, \dots, u_n .

A set of ordinary differential equations can now be written to describe the unsteady-state process in the body, as follows:

$$\frac{du_1}{dt} = \frac{\alpha}{h^2} (c_1 - 2u_1 + u_2) \quad (\text{II.35.1})$$

$$\frac{du_2}{dt} = \frac{\alpha}{h^2} (u_1 - 2u_2 + u_3) \quad (\text{II.35.2})$$

$$\frac{du_{n-1}}{dt} = \frac{\alpha}{h^2} (u_{n-2} - 2u_{n-1} + u_n) \quad (\text{II.35.}(n-1))$$

$$\frac{du_n}{dt} = \frac{\alpha}{h^2} (u_{n-1} - 2u_n + u'_n) \quad (\text{II.35.}n')$$

Using the flux condition of equation (II.34) an expression for temperature at the hypothetical point u'_n in terms of u_n may be obtained as follows:

At $n + 1$: the boundary condition specifies that

$$\frac{u_n - u_{n+1}}{(h/2)} = c_2, \text{ or } u_{n+1} = u_n - c_2 \frac{h}{2} \quad (\text{II.35.}(n+1))$$

Also, from conservation of energy, the following expression holds:

$$\frac{u_{n+1} - u'_n}{(h/2)} = c_2 \text{ or } u_{n+1} = u'_n + c_2 \frac{h}{2} \quad (\text{II.35.}(n+2))$$

Combining the last two equations gives

$$u'_n = u_n - c_2 h \quad (\text{II.35.}(n+3))$$

Substitution of the above expression into equation (II.35. n') gives

$$\frac{du_n}{dt} = \frac{\alpha}{h^2} (u_{n-1} - u_n - c_2 h) \quad (\text{II.35.}n)$$

Equations (II.35.1) through (II.35. n) may be

written as a single matrix equation as follows:

$$\frac{h^2}{\alpha} \frac{d}{dt} \begin{bmatrix} u_1 \\ . \\ . \\ . \\ . \\ u_n \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 & . & . & . & 0 \\ 1 & -2 & 1 & . & . & . & 0 \\ 0 & . & . & . & . & . & 0 \\ . & . & . & . & . & . & 0 \\ . & . & . & 1 & -2 & 1 & . \\ 0 & . & . & 0 & 1 & -1 & . \end{bmatrix} \begin{bmatrix} u_1 \\ . \\ . \\ . \\ . \\ u_n \end{bmatrix} + \begin{bmatrix} c_1 \\ 0 \\ . \\ . \\ 0 \\ -c_2 h \end{bmatrix} \quad (\text{II.35})$$

In matrix notation, this equation may be written as

$$\frac{h^2}{\alpha} \frac{d}{dt} (\underline{u}(t)) = \underline{B} \underline{u}(t) + \underline{s} \quad (\text{II.36})$$

with the initial conditions $\underline{u}(0) = \underline{u}_0$.

A new variable $\underline{v}(t)$ is defined by the equation

$$\underline{v}(t) = \underline{Q}^T \underline{u}(t) \text{ and } \underline{u}(t) = \underline{Q} \underline{v}(t) \quad (\text{II.37})$$

where matrices \underline{Q} and \underline{Q}^T are as defined before. Substitution in equation (II.36) gives

$$\frac{h^2}{\alpha} \underline{Q} \frac{d\underline{v}(t)}{dt} = \underline{B} \underline{Q} \underline{v}(t) + \underline{s} \quad (\text{II.38})$$

Premultiplication by \underline{Q}^T gives

$$\frac{h^2}{\alpha} \frac{d\underline{v}(t)}{dt} = \underline{Q}^T \underline{B} \underline{Q} \underline{v}(t) + \underline{Q}^T \underline{s} \quad (\text{II.39})$$

$$\text{Or } \frac{h^2}{\alpha} \frac{d\underline{v}(t)}{dt} = \underline{P} \underline{v}(t) + \underline{Q}^T \underline{s} \quad (\text{II.40})$$

where the matrix \underline{P} is as defined before.

Each individual equation is written as

$$\frac{h^2}{\alpha} \frac{dv_j(t)}{dt} = p_j v_j(t) + r_j \quad (\text{II.41})$$

$$(j = 1 \text{ to } n)$$

where $r_j = \underline{q}_j^T \underline{s}$ and $\underline{r} = \underline{Q}^T \underline{s}$

The initial conditions for these ordinary differential equations are $v_j(0) = v_{j,0} (= \underline{q}_j^T \underline{u}(0))$ (II.42)

Integration of equation (II.41) gives

$$\ln z_j(t) = \ln \left[\frac{p_j v_j(t) + r_j}{p_j v_{j,0} + r_j} \right] = \frac{\alpha p_j}{h^2} t \quad (\text{II.43})$$

$$(j = 1 \text{ to } n).$$

Equation (II.43) says that a plot of $\ln z_j(t)$ as ordinate with t as abscissa should result in a straight line, whose slope equals $\frac{\alpha p_j}{h^2}$. Therefore, if data of temperature profiles as a function of time are available, the left hand side of equation (II.43) can be easily calculated (knowing, of course, the eigenvalues and eigenvectors of \underline{B}) and these values can be plotted and the slope evaluated. From the known values of slope, p_j and h , α can be readily calculated.

Although this derivation is presented for the case of transient thermal conduction, the model is just as valid for the process of diffusion of a substance due to concentration gradients, or the molecular diffusion of a substance.

III. FORWARD PROBLEM

In this chapter, solutions to some of the transient conduction problems are presented. The coefficient matrices are developed for each one of these cases, and the techniques used for the determination of eigenvalues and eigenvectors of the coefficient matrix are discussed in some detail.

A. Homogeneous Media

1. One-Dimensional Media: Firstly, it is pertinent at this stage to establish a check on the accuracy of the semi-analytical method. For this purpose, a very simple problem involving unsteady-state heat transfer was solved. The problem may be stated as follows:

A one-dimensional slab of thickness $2l$, and uniform thermal diffusivity α is initially at a uniform temperature of unity. At time zero, both its boundaries are suddenly brought to a temperature zero and maintained at that temperature. It is required to evaluate the temperature profiles at various elapsed times.

The partial differential equation describing the process of heat transfer in the above problem is

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2} \quad (\text{III.1})$$

$$\begin{array}{l}
 \text{I.C: at } t = 0, \quad u = 1.0, \quad 0 \leq x \leq 2l \\
 \text{B.C: at } x = 0 = 2l, \quad u = 0.0 \\
 \text{Also at } x = l, \quad \frac{\partial u}{\partial x} = 0 \text{ by symmetry}
 \end{array} \quad \left. \vphantom{\begin{array}{l} \text{I.C:} \\ \text{B.C:} \\ \text{Also at } x = l \end{array}} \right\} \quad (\text{III.2})$$

An analytical solution to equation (III.1) with the boundary conditions (III.2) is given by McAdams (14), and is as follows:

$$\begin{aligned}
 u = \frac{4}{\pi} \left[\exp(-a_1 x) \sin\left(\frac{\pi x}{2l}\right) + \frac{1}{3} \exp(-9a_1 x) \sin\left(\frac{3\pi x}{2l}\right) \right. \\
 \left. + \frac{1}{5} \exp(-25a_1 x) \sin\left(\frac{5\pi x}{2l}\right) + \dots \right] \quad (\text{III.3})
 \end{aligned}$$

where $a_1 = \left(\frac{\pi}{2}\right)^2$; and $x = \frac{\alpha t}{l^2}$.

Equation (III.3) is used to check the accuracy of the solutions obtained by the semi-analytical method.

Since the body is symmetrical about the centerline, only one-half ($0 \leq x \leq l$) is considered in the semi-analytical method. This distance is divided into a five-point grid of equal distances h , with a half-slab $\frac{h}{2}$ near the insulated boundary (Fig. III.1). When the space derivative at these grid points is replaced by the appropriate finite-difference formulae, the following matrix ordinary differential equation is obtained:

$$\frac{d}{dt} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix} = \frac{\alpha}{h^2} \begin{bmatrix} -2 & 1 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix} \quad (\text{III.4})$$

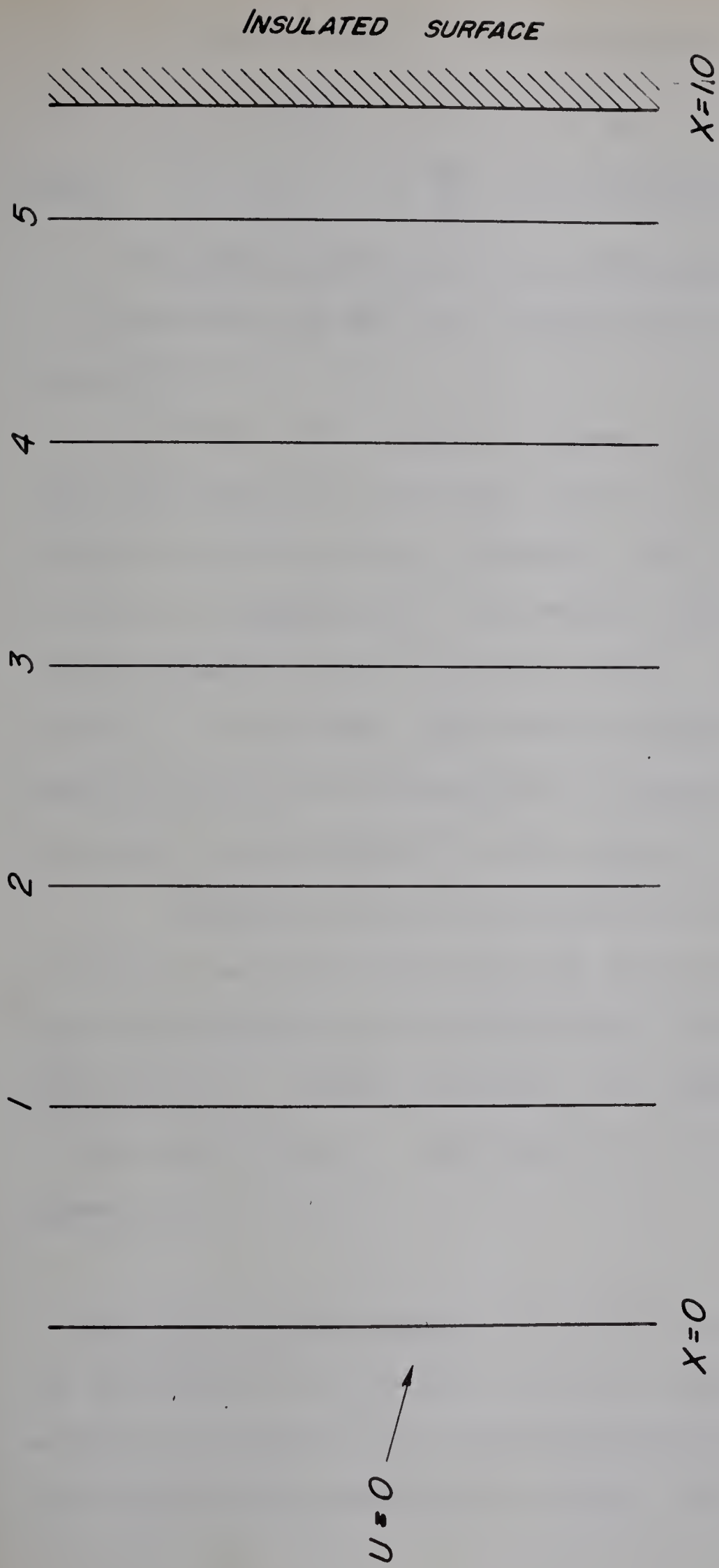


FIGURE III . 1
ILLUSTRATION OF A TYPICAL FIVE POINT GRID
ONE-DIMENSIONAL HOMOGENEOUS MEDIA

The solution is straightforward, and is given by

$$\underline{u}(\theta) = \underline{Q} \underline{E}(\theta) \underline{Q}^T \underline{u}_0 \quad (\text{III.5})$$

where $\theta = \frac{\alpha t}{h^2}$, the dimensionless time.

For this particular case, the coefficient matrix \underline{B} , the matrix of eigenvectors \underline{Q} and the diagonal matrix $\underline{E}(\theta)$ are given in Appendix B.

Table III.1 gives a summary of results, together with the analytical solution and the associated errors. The complete set of results, together with the computer program is given in Appendix B. As may be noted from Table III.1, the associated errors are very small. This verifies the validity of the model and confirms that the numerical errors associated with the evaluation of eigenvalues and eigenvectors are quite insignificant in this case.

Appendix B also gives \underline{B} , \underline{Q} and $\underline{E}(\theta)$ obtained by employing a ten-point grid for the same problem. The results and comparisons from the analytical solution are given in Table III.2. As may be noted, the comparisons are an order of magnitude better. The complete results are given in Appendix B.

2. Two-Dimensional Media: Very few analytical solutions are available for problems in multi-dimensional media. The methods of solution that have been successfully used to date are essentially numerical in nature. Both explicit and

TABLE III.1

One-Dimensional Homogeneous Media

Summary of Results* - Five-point Grid.

Grid Point Dimensionless time θ	1	3	5
4.0	0.2769 0.2759 -0.0010	0.7046 0.7065 0.0019	0.8825 0.8890 0.0065
12.0	0.1348 0.1348 -0.0000	0.3616 0.3616 0.0000	0.4734 0.4735 0.0001
20.0	0.0705 0.0702 -0.0003	0.1891 0.1883 -0.0008	0.2476 0.2466 -0.0010

* The first entry in each block refers to the value of u , calculated by the semi-analytical technique; the second entry refers to the value of u calculated analytically; the third entry represents the error - the value calculated analytically minus the value calculated by the proposed method.

A complete set of results in tabular form is available in Appendix B.

TABLE III.2One-Dimensional Homogeneous MediaSummary of Results* - Ten-point Grid

Grid Point Dimensionless time θ	1	5	10
4	0.2775 0.2763 -0.0012	0.9195 0.9229 0.0034	0.9990 0.9995 0.0005
20	-0.1246 0.1246 0.0000	0.5592 0.5594 0.0002	0.8030 0.8042 0.0012
70	0.0397 0.0396 -0.0001	0.1810 0.1808 -0.0002	0.2653 0.2650 -0.0003

* The first entry in each block refers to the value of u , calculated by the semi-analytical technique; the second entry refers to the value of u calculated analytically; the third entry represents the error - the value calculated analytically minus the value calculated by the proposed method.

A complete set of results in tabular form is available in Appendix B.

implicit methods have been used (12,13,23) successfully.

In the semi-analytical solution of transient heat conduction problem in a two-dimensional medium, the simplest initial and boundary conditions were chosen. The reason for this, again is that an analytical solution is available for this simple case (15), so that comparison of results was possible. This analytical solution was in fact used for the purposes of comparison with the results of the semi-analytical solution.

The solution to the following problem was considered:

$$\frac{\partial u}{\partial t} = \alpha \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (\text{III.6})$$

$$- a \leq x \leq a$$

$$- b \leq y \leq b$$

with the initial and boundary conditions

$$\left. \begin{array}{l} \text{I.C: at } t = 0, \quad u = 1.0 \\ \text{B.C: at } x = a = -a, \quad u = 0 \\ \quad \quad \quad \text{at } y = b = -b, \quad u = 0 \end{array} \right\} \quad (\text{III.7})$$

These boundary conditions imply that

$$\left. \begin{array}{l} \text{at } x = 0, \quad \frac{\partial u}{\partial x} = 0 \\ \text{at } y = 0, \quad \frac{\partial u}{\partial y} = 0 \end{array} \right\} \quad (\text{III.7a})$$

This problem was solved for two cases:

- i) $a = b$, or a square body.
- ii) $a \neq b$, or a rectangular body.

In each of the above two cases, the solution for one quarter of the body was determined, since the solution for the rest of the body can be determined from this solution.

The derivation of the coefficient matrix B is illustrated for the case of a rectangular body with the following values of the parameters:

$$\left. \begin{array}{l} a = 6.5, \quad \Delta x = 1.0 \\ b = 3.5, \quad \Delta y = 1.0 \end{array} \right\} \quad (\text{III.8})$$

It is not necessary to have $\Delta x = \Delta y$ so that the matrix B is symmetric, but in the example, this was convenient.

The finite-difference grid is shown in Fig. III.2. The set of ordinary differential equations for this problem may be written as:

$$\frac{du_1}{dt} = \alpha(-4u_1 + u_2 + u_7) \quad (\text{III.9.1})$$

$$\frac{du_2}{dt} = \alpha(u_1 - 4u_2 + u_3 + u_8) \quad (\text{III.9.2})$$

$$\frac{du_3}{dt} = \alpha(u_2 - 4u_3 + u_4 + u_9) \quad (\text{III.9.3})$$

$$\frac{du_4}{dt} = \alpha(u_3 - 4u_4 + u_5 + u_{10}) \quad (\text{III.9.4})$$

$$\frac{du_5}{dt} = \alpha(u_4 - 4u_5 + u_6 + u_{11}) \quad (\text{III.9.5})$$

$$\frac{du_6}{dt} = \alpha(u_5 - 3u_6 + u_{12}) \quad (\text{III.9.6})$$

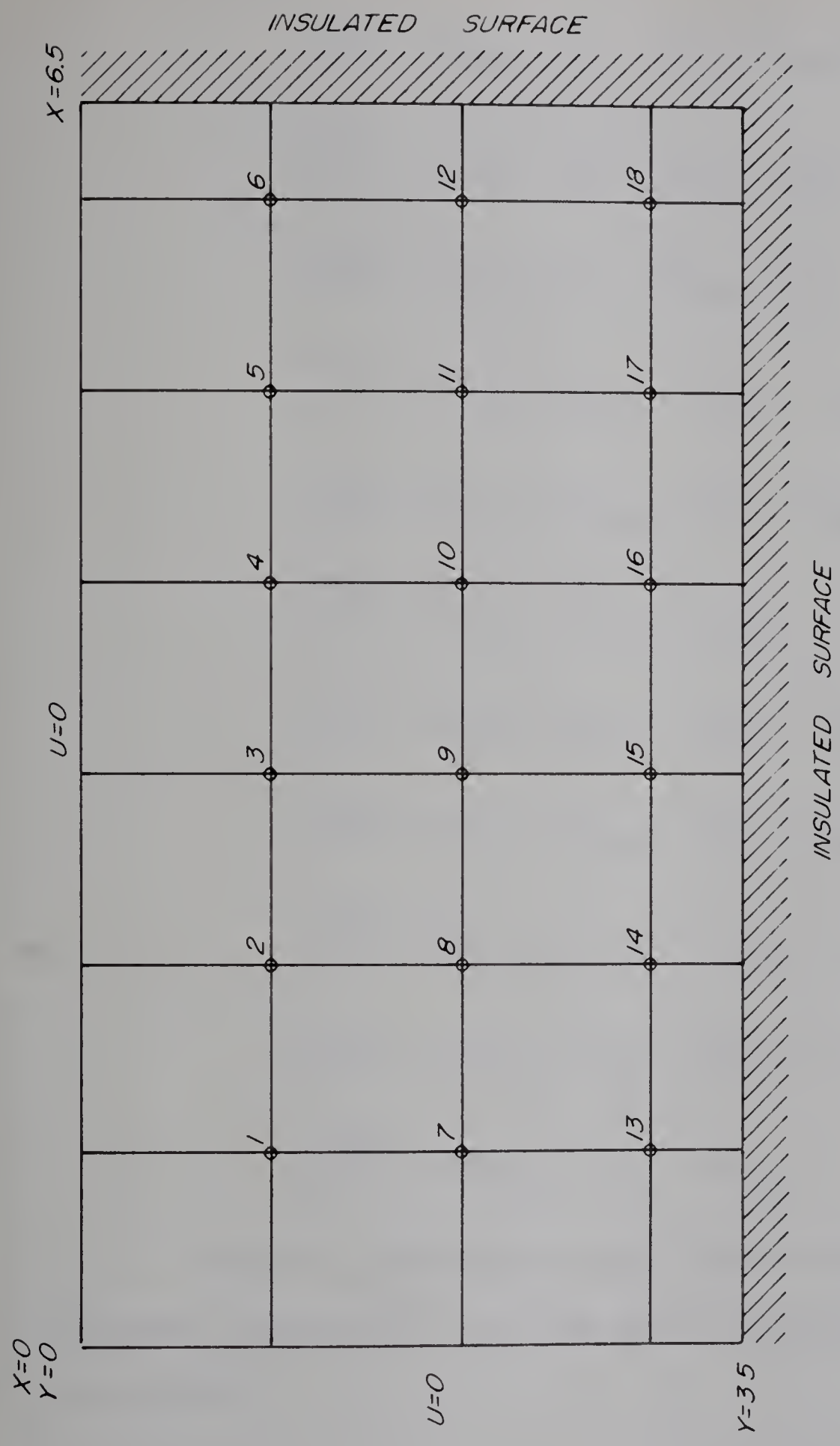


FIGURE III. 2

TWO DIMENSIONAL MEDIA
RECTANGULAR BODY

$$\frac{du_7}{dt} = \alpha(u_1 - 4u_7 + u_8 + u_{13}) \quad (\text{III.9.7})$$

$$\frac{du_8}{dt} = \alpha(u_2 + u_7 - 4u_8 + u_9 + u_{14}) \quad (\text{III.9.8})$$

$$\frac{du_9}{dt} = \alpha(u_3 + u_8 - 4u_9 + u_{10} + u_{15}) \quad (\text{III.9.9})$$

$$\frac{du_{10}}{dt} = \alpha(u_4 + u_9 - 4u_{10} + u_{11} + u_{16}) \quad (\text{III.9.10})$$

$$\frac{du_{11}}{dt} = \alpha(u_5 + u_{10} - 4u_{11} + u_{12} + u_{17}) \quad (\text{III.9.11})$$

$$\frac{du_{12}}{dt} = \alpha(u_6 + u_{11} - 3u_{12} + u_{18}) \quad (\text{III.9.12})$$

$$\frac{du_{13}}{dt} = \alpha(u_7 - 3u_{13} + u_{14}) \quad (\text{III.9.13})$$

$$\frac{du_{14}}{dt} = \alpha(u_8 + u_{13} - 3u_{14} + u_{15}) \quad (\text{III.9.14})$$

$$\frac{du_{15}}{dt} = \alpha(u_9 + u_{14} - 3u_{15} + u_{16}) \quad (\text{III.9.15})$$

$$\frac{du_{16}}{dt} = \alpha(u_{10} + u_{15} - 3u_{16} + u_{17}) \quad (\text{III.9.16})$$

$$\frac{du_{17}}{dt} = \alpha(u_{11} + u_{16} - 3u_{17} + u_{18}) \quad (\text{III.9.17})$$

$$\frac{du_{18}}{dt} = \alpha(u_{12} + u_{17} - 2u_{18}) \quad (\text{III.9.18})$$

These equations are obtained by substitution of the following expressions for the space derivatives at these grid points:

$$\left. \frac{\partial^2 u}{\partial x^2} \right|_{x,y} = \frac{u_{x-\Delta x} - 2u_x + u_{x+\Delta x}}{\Delta x^2} \Big|_y; \quad \text{and} \quad (III.10)$$

$$\left. \frac{\partial^2 u}{\partial y^2} \right|_{x,y} = \frac{u_{y-\Delta y} - 2u_y + u_{y+\Delta y}}{\Delta y^2} \Big|_x$$

Equations (III.9.1) through (III.9.18) may be put into one matrix differential equation as

$$\frac{1}{\alpha} \frac{d\mathbf{u}}{dt} = \mathbf{B} \mathbf{u} \quad (III.11)$$

The matrix \mathbf{B} of equation (III.11) appears in Appendix C. The solution of the above equation is straightforward. The eigenvectors and eigenvalues of \mathbf{B} are also given in Appendix C. A summary of results at selected grid points along with the results from the analytical solution at these points and the associated errors are given in Table III.3. The analytical solution for this problem was obtained by multiplying the solutions for the two one-dimensional problems, according to the method of Newman (15).

As may be seen from Table III.3, the results are satisfactory. The relatively poor agreement of results at low values of dimensionless time suggests that the method may be getting insensitive as the size of the problem gets bigger. This is probably due to the accuracy with which the eigenvalues and eigenvectors are determined. This is evident from the check of the similarity transformation to produce the original matrix (Appendix C).

TABLE III.3Two-Dimensional Homogeneous MediaSummary of Results* - Rectangular Body

Grid Point $\frac{\alpha t}{\Delta x^2}$	1	2	9	10	17	18
1.0	0.2742 0.2709 -0.0033	0.4357 0.4386 0.0029	0.7925 0.8137 0.0212	0.8221 0.8383 0.0162	0.9426 0.9610 0.0185	0.9440 0.9614 0.0174
6.0	0.0376 0.0374 -0.0002	0.0720 0.0717 -0.0003	0.1817 0.1812 -0.0005	0.2208 0.2207 -0.0001	0.3074 0.3077 0.0003	0.3232 0.3247 0.0015
10.0	0.0128 0.0126 -0.0002	0.0249 0.0245 -0.0004	0.0637 0.0629 -0.0010	0.0788 0.0777 -0.0011	0.1114 0.1098 -0.0016	0.1180 0.1168 -0.0012

* The first entry in each block refers to the value of u , calculated by the semi-analytical technique; the second entry refers to the value of u calculated analytically; the third entry represents the error - the value calculated analytically minus the value calculated by the proposed method.

A complete set of results in tabular form is available in Appendix C.

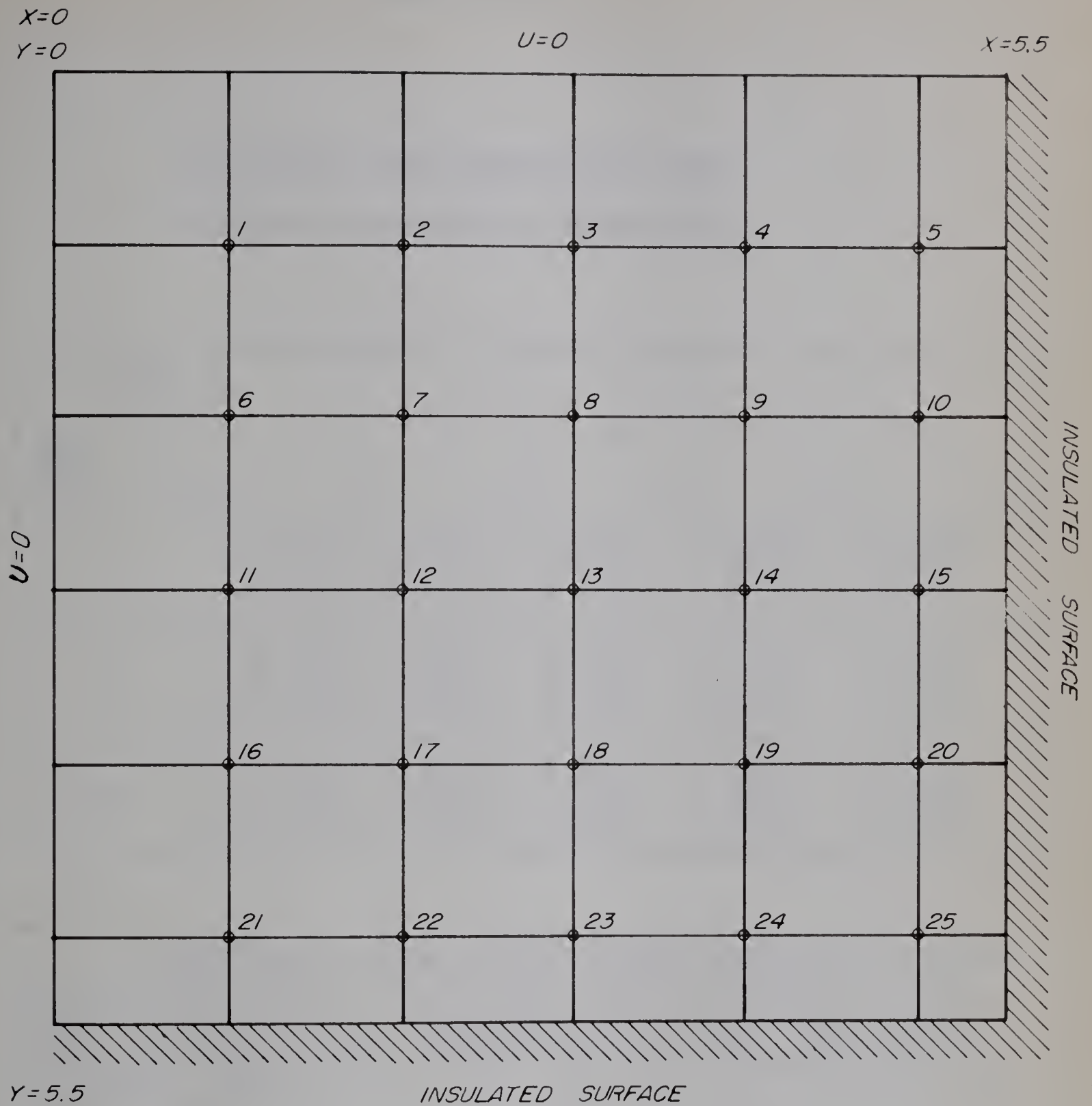


FIGURE 111. 3

TWO DIMENSIONAL MEDIA

SQUARE BODY

TABLE III.4

Two-Dimensional Homogeneous Media
Summary of Results* - Square Body

<div>Grid Point</div> <div>$\frac{\alpha t}{\Delta x^2}$</div>	1	7	13	19	25
1.0	0.2743 0.2709 -0.0034	0.6927 0.7101 0.0174	0.9107 0.9334 0.0226	0.9800 0.9907 0.0107	0.9958 0.9991 0.0033
6.0	0.0501 0.0501 0.0000	0.1821 0.1824 0.0003	0.3495 0.3512 0.0017	0.4980 0.5020 0.0040	0.5838 0.5896 0.0058
12.0	0.0182 0.0182 0.0000	0.0669 0.0669 0.0000	0.1307 0.1308 0.0001	0.1893 0.1894 0.0001	0.2241 0.2242 0.0001

* The first entry in each block refers to the value of u , calculated by the semi-analytical technique; the second entry refers to the value of u calculated analytically; the third entry represents the error - the value calculated analytically minus the value calculated by the proposed method.

A complete set of results in tabular form is available in Appendix C.

The matrix B for the case of a square body (Fig. III.3) is given in Appendix C; a summary of results appears in Table III.4.

The complete sets of results and the relevant programs are given in Appendix C.

B. Non-Homogeneous Media

Analytical solutions to problems involving non-homogeneous media are very scarce (6,10); the few that are available are for special cases of property variations across the medium (6). Most problems of this nature are usually solved using numerical methods.

The semi-analytical solution for the case of a non-homogeneous medium, where the properties are dependent on the three space coordinates but not on time or temperature is presented here. A one-dimensional body with no sources or sinks is considered as an example.

The partial differential equation describing the process is

$$c \frac{\partial u}{\partial t} = \frac{\partial}{\partial x} \left(k \frac{\partial u}{\partial x} \right) \quad (\text{III.12})$$

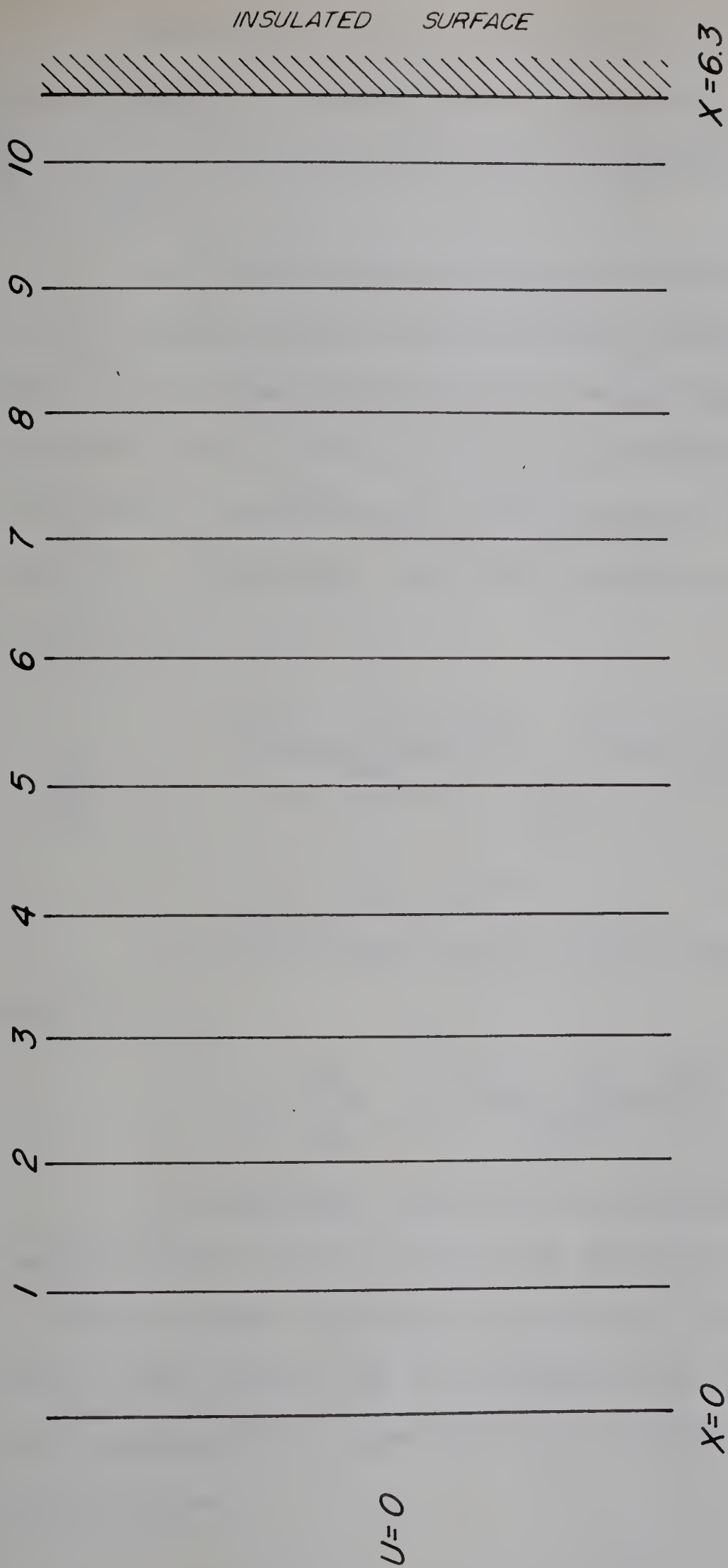
where

$$c = f_1(x)$$

$$k = f_2(x) \text{ and with the initial and boundary}$$

conditions

$$\left. \begin{array}{l} \text{I.C: at } t = 0, \quad u(x) = u_0 \\ \text{B.C: at } x = 0, \quad u = \bar{u} \\ \quad \quad \text{at } x = 1, \quad \frac{\partial u}{\partial x} = 0 \end{array} \right\} \quad (\text{III.13})$$



GRID SPACING USED FOR ONE-DIMENSIONAL
Non-HOMOGENEOUS MEDIA

FIGURE III. 4

The body $0 \leq x \leq 1$ is divided into a finite number of increments (n) of width Δx each, with a slab of width $\frac{\Delta x}{2}$ near the boundary where flux is specified, as shown in Fig. III.4.

If the thermal conductivity and the heat capacity of the medium may be represented by discrete values for each one of these grid points, the partial differential equation (III.12) can be written as a set of ordinary differential equations by discretisation of the space derivative. Typically, the heat balance on the i th element is given by

$$c_i \frac{du_i}{dt} = \frac{\frac{k_{i-\frac{1}{2}} (u_{i-1} - u_i)}{\Delta x} - \frac{k_{i+\frac{1}{2}} (u_i - u_{i+1})}{\Delta x}}{\Delta x} \quad (\text{III.14})$$

($i = 1$ to $(n-1)$)

At the boundary where flux is specified, the equation is

$$c_n \frac{du_n}{dt} = \frac{k_{n-\frac{1}{2}} (u_{n-1} - u_n)}{\Delta x^2} \quad (\text{III.15})$$

In equations (III.14) and (III.15), the effective thermal conductivity between the i th and $(i+1)$ th elements is taken as that value for a point half way between the i th and $(i+1)$ th planes and is designated by $k_{i+\frac{1}{2}}$. This is a valid approximation provided the function $k = f_2(x)$ is not discontinuous.

The equations (III.14) and (III.15) may be written in matrix form as follows:

$$\frac{1}{\Delta x^2} \begin{bmatrix} c_1 & & & & & & \\ & c_2 & & & & & \\ & & \ddots & & & & \\ & & & \ddots & & & \\ & & & & \ddots & & \\ & & & & & \ddots & \\ & & & & & & c_n \end{bmatrix} \frac{d}{dt} \begin{bmatrix} u_1 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ u_n \end{bmatrix} =$$

$$\begin{bmatrix} -(k_{1\frac{1}{2}} + k_{1\frac{1}{2}}) & k_{1\frac{1}{2}} & 0 & \dots & \dots & \dots & 0 \\ k_{1\frac{1}{2}} & -(k_{1\frac{1}{2}} + k_{2\frac{1}{2}}) & k_{2\frac{1}{2}} & 0 & \dots & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ k_{n-1\frac{1}{2}} & -(k_{n-1\frac{1}{2}} + k_{n-\frac{1}{2}}) & k_{n-\frac{1}{2}} & \dots & \dots & \dots & 0 \\ & k_{n-\frac{1}{2}} & -(k_{n-\frac{1}{2}}) & \dots & \dots & \dots & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ u_n \end{bmatrix} + \begin{bmatrix} \bar{u}k_{1\frac{1}{2}} \\ 0 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ 0 \end{bmatrix} \quad (\text{III.16})$$

In accordance with equation (II.2), the various matrices are designated as

$$\underline{D} = \begin{bmatrix} c_1/\Delta x^2 & & & & & & \\ \vdots & \ddots & & & & & \\ & & \ddots & & & & \\ & & & \ddots & & & \\ & & & & \ddots & & \\ & & & & & \ddots & \\ & & & & & & c_n/\Delta x^2 \end{bmatrix}; \quad \underline{B}' = \begin{bmatrix} -(k_{1\frac{1}{2}} + k_{1\frac{1}{2}}) & k_{1\frac{1}{2}} & 0 & \dots & \dots & \dots & 0 \\ k_{1\frac{1}{2}} & -(k_{1\frac{1}{2}} + k_{2\frac{1}{2}}) & k_{2\frac{1}{2}} & 0 & \dots & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ k_{n-1\frac{1}{2}} & -(k_{n-1\frac{1}{2}} + k_{n-\frac{1}{2}}) & k_{n-\frac{1}{2}} & \dots & \dots & \dots & 0 \\ & k_{n-\frac{1}{2}} & -(k_{n-\frac{1}{2}}) & \dots & \dots & \dots & 0 \end{bmatrix};$$

$$\underline{s}'(t) = \begin{bmatrix} \bar{u} \quad k_{\frac{1}{2}} \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \quad (\text{III.17})$$

\underline{D} is a positive diagonal matrix since the heat capacity of each of the elements is positive.

The solution for equation (III.16) is given by equation (II.32), and is

$$\underline{u}(t) = \underline{D}^{-\frac{1}{2}} \underline{Q} \underline{E}(t) \underline{Q}^T \underline{D}^{\frac{1}{2}} \left[\underline{u}_0 + \int_0^t \underline{D}^{-\frac{1}{2}} \underline{Q} \underline{E}^{-1}(\sigma) \underline{Q}^T \underline{D}^{-\frac{1}{2}} \underline{s}'(\sigma) d\sigma \right] \quad (\text{III.18})$$

where σ is a dummy variable;

$$\underline{D}^{\frac{1}{2}} = \begin{bmatrix} (c_1/\bar{\Delta x}^2)^{\frac{1}{2}} \\ \cdot \\ \cdot \\ \cdot \\ (c_n/\bar{\Delta x}^2)^{\frac{1}{2}} \end{bmatrix}; \quad \underline{D}^{-\frac{1}{2}} = \begin{bmatrix} (\bar{\Delta x}^2/c_1)^{-\frac{1}{2}} \\ \cdot \\ \cdot \\ \cdot \\ (\bar{\Delta x}^2/c_n)^{-\frac{1}{2}} \end{bmatrix};$$

$$\underline{B} = \underline{D}^{-\frac{1}{2}} \underline{B}' \underline{D}^{-\frac{1}{2}};$$

$$\underline{Q} = \text{matrix of eigenvectors of } \underline{B};$$

$$p_i = \text{eigenvalues of } \underline{B} (i = 1 \text{ to } n);$$

$$\underline{E}(t) = \begin{bmatrix} e^{p_1 t} & & \\ & \ddots & \\ & & e^{p_n t} \end{bmatrix}; \quad \underline{E}^{-1}(t) = \begin{bmatrix} e^{-p_1 t} & & \\ & \ddots & \\ & & e^{-p_n t} \end{bmatrix}$$

The results presented are for the following values of the various parameters:

$$\left. \begin{aligned} f_1(x) &= 1.0 \text{ (a constant)} \\ f_2(x) &= 1.0 + 0.5x \text{ (a linear variation of } k) \\ \underline{u}_0 &= \underline{0} \text{ (initially at zero temperature)} \\ \bar{u} &= 1.0 \\ l &= 6.3 \\ n &= 10 \\ \Delta x &= 0.6 \end{aligned} \right\} \quad (\text{III.19})$$

The matrix \underline{B} , eigenvalues p_i and the matrix of eigenvectors \underline{Q} are given in Appendix D.

An analytical solution could not be found for this case; hence a numerical solution using the explicit method was obtained for two different values of Δx and Δt . The solutions agreed to 4 significant figures. Hence, this solution was assumed to approach the true solution.

A summary of results from the semi-analytical solution and the numerical solution, along with the associated errors is given in Table III.5. The results, as may

TABLE III.5One-Dimensional Non-Homogeneous MediaSummary of Results* - Ten-Point Grid

Grid Point Time	1	4	7	10
1.0	0.6505 0.6500 -0.0005	0.1466 0.1422 -0.0044	0.0272 0.0242 -0.0030	0.0068 0.0052 -0.0016
8.0	0.8706 0.8698 -0.0008	0.6341 0.6323 -0.0018	0.5267 0.5245 -0.0022	0.4929 0.4905 -0.0024
16.0	0.9450 0.9446 -0.0004	0.8446 0.8435 -0.0011	0.7989 0.7976 -0.0013	0.7846 0.7832 -0.0014

* The first entry in each block is the value of u calculated by the semi-analytical method; the second entry is the value calculated by the numerical solution; the third entry is the error - the value calculated by numerical method minus the value calculated by the proposed method.

A complete set of results in tabular form is available in Appendix D.

be noticed, are highly satisfactory.

The complete set of results and the computer program are given in Appendix D.

C. Eigenvalues and Eigenvectors of \underline{B} .

The various methods of evaluating the eigenvalues and eigenvectors of a symmetric matrix may broadly be divided into two categories:

1. Non-iterative or direct methods. These usually require a large number of multiplications and hence are time-consuming on the computer. Typical of these procedures are due to Leverrier, Hessenberg, and Givens. These are described in some detail by Lapidus (12).
2. Iterative methods. These methods are the most widely used on the digital computers. They have certain advantages over the direct methods, namely:
 - a) all the eigenvalues are determined at the same time.
 - b) all the eigenvectors are determined simultaneously, through a set of orthogonal transformations.Typical of these methods are those due to Jacobi (25).

and Householder (33).

No attempt is made here to develop the algorithms for any of the above methods; complete details are available in various texts on numerical analysis. However, the

central ideas of the two methods used in this thesis, the Jacobi's procedure and Householder's procedure are presented.

Jacobi's method: This method applies to symmetric matrices, and involves the annihilation of the off-diagonal elements of a symmetric matrix using a series of orthogonal matrices. To illustrate the procedure, a matrix \underline{A} of order 2 is considered. A matrix \underline{A}' is formed by the transformation

$$\underline{A}' = \underline{Q}_1^T \underline{A} \underline{Q}_1 \quad (\text{III.20})$$

where

$$\underline{Q}_1 = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}.$$

This type of transformation is known as rotation.

If the off-diagonal elements of \underline{A}' are to be zero, then the following identity has to be satisfied:

$$\tan 2\alpha = \frac{2a_{12}}{a_{11} - a_{22}} \quad (\text{III.21})$$

The diagonal elements of \underline{A}' are λ_1 and λ_2 , the two eigenvalues of \underline{A} (the eigenvalues of \underline{A} and \underline{A}' are the same since \underline{A} and \underline{A}' are similar).

In the case of a matrix of order n , the matrix \underline{Q}_1 is replaced by the matrix

$$\underline{Q}_1 = \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & \ddots & & & \\ & & & (\cos \alpha)_{ii} & \dots & -(\sin \alpha)_{ij} \\ & & & & 1 & \\ & & & & & \ddots \\ & & & & & & 1 \\ & & & (\sin \alpha)_{ji} & \dots & (\cos \alpha)_{jj} \\ & & & & & & 1 \\ & & & & & & & \ddots \\ & & & & & & & & 1 \end{bmatrix} \quad (\text{III.22})$$

This is a modified unit matrix, with its i th and j th diagonal elements replaced by $(\cos \alpha)$, the (ij) th zero by $(-\sin \alpha)$ and the (ji) th zero by $(\sin \alpha)$ ($j > i$). The angle α is determined by the expression

$$\tan 2\alpha = \frac{2a_{ij}}{a_{ii} - a_{jj}} \quad (\text{III.23})$$

The similarity transformation of equation (II.20) annihilates the elements a_{ij} and a_{ji} . In this method, the largest off-diagonal element is first annihilated, but this might show a non-zero value at a later stage. Hence the method has no fixed number of iterations.

When the procedure converges, the following equation is the result:

$$\underline{Q}_r^T \underline{Q}_{r-1}^T \dots \underline{Q}_2^T \underline{Q}_1^T \underline{A} \underline{Q}_1 \underline{Q}_2 \dots \underline{Q}_{r-1} \underline{Q}_r = \underline{\Delta} \quad (\text{III.24})$$

where r refers to the number of iterations and $\underline{\Delta}$ is the

diagonal matrix of eigenvalues of \underline{A} .

If the matrix $\underline{Q}_1 \underline{Q}_2 \dots \underline{Q}_r$ is denoted by \underline{Q} , then

$$\underline{Q}_r^T \underline{Q}_{r-1}^T \dots \underline{Q}_2^T \underline{Q}_1^T = \underline{Q}^T ; \text{ then} \quad (\text{III.25})$$

$$\underline{Q}^T \underline{A} \underline{Q} = \underline{\Lambda}$$

Premultiplication by \underline{Q} gives

$$\underline{A} \underline{Q} = \underline{Q} \underline{\Lambda} \quad (\text{III.26})$$

Or \underline{Q} is the matrix of eigenvectors of \underline{A} .

Householder's Method: The two salient features of Householder's method are:

1. It uses orthogonal matrices which do not produce pure rotation.
2. An element set to zero does not show up again, reducing the number of iterations.

In Householder's procedure, a matrix \underline{P} is defined by the equation

$$\underline{P} = \underline{I} - 2 \underline{w} \underline{w}^T \quad (\text{III.27})$$

where

\underline{I} = unit matrix and

\underline{w} = a vector of unit length, whose elements are chosen according to the method suggested in (33).

Now, \underline{P} is a symmetric matrix, by inspection of equation (III.27). Also, it is orthogonal, since

$$\begin{aligned}
\underline{P}^T \underline{P} &= (\underline{I} - 2 \underline{w} \underline{w}^T) (\underline{I} - 2 \underline{w} \underline{w}^T) \\
&= \underline{I} - 4 \underline{w} \underline{w}^T + 4 \underline{w} (\underline{w}^T \underline{w}) \underline{w}^T \\
&= \underline{I} - 4 \underline{w} \underline{w}^T + 4 \underline{w} \underline{w}^T = \underline{I}
\end{aligned} \tag{III.28}$$

The vector \underline{w}_r is chosen such that its first $(r-1)$ elements are zero, or $\underline{w}_r^T = (0, 0, \dots, 0, x_r, x_{r+1}, \dots, x_n)$

(III.29)

satisfying the relation $x_r^2 + x_{r+1}^2 + \dots + x_n^2 = 1$.

The transformation of \underline{A} to triple diagonal form is affected in $(n-2)$ similarity transformations with $\underline{P}_2, \underline{P}_3, \dots, \underline{P}_{n-1}$, formed with $\underline{w}_2, \underline{w}_3, \dots, \underline{w}_{n-1}$. The elements of the \underline{w} -vectors are found from a series of relations obtained by multiplying the original matrix \underline{A} with \underline{w} , the details of which are available in (33). The resulting tri-diagonal matrix is treated by the method of Givens to obtain the eigenvalues and eigenvectors of the tri-diagonal matrix. The eigenvectors are then premultiplied by the series of matrices $\underline{P}_{n-1}, \dots, \underline{P}_2$ to obtain the eigenvectors of the original matrix \underline{A} .

The storage requirements of Householder's method are shown to be very satisfactory, and are shown to be only $(\frac{1}{2} n (n+1) + n)$ memory locations (33). This indeed is a very significant feature of the method, and is consequently the most commonly used.

The computer time for the determination of eigenvalues and eigenvectors by Householder's method is also found to be less than that required by Jacobi's method for

TABLE III.6Householder's MethodEstimated Time Requirements in Minutes on IBM 7090*

<u>Order of the matrix</u>	<u>Eigenvalues only</u>	<u>Eigenvalues and all the eigenvectors.</u>
10	0.01	0.03
30	0.14	0.24
50	0.43	0.69
100	2.40	4.40

* From a leaflet published by the Department of Educational Psychology, University of Alberta, Edmonton.

the same matrix. Typical time requirements by Householder's method are given in Table III.6.

In this thesis, Householder's method was most extensively used. The only place where Jacobi's method was used was when dual eigenvalues are encountered. In this situation, Householder's method did not produce the proper eigenvectors due to some inherent property of the method, whereas Jacobi's method produced the proper eigenvectors.

IV. A TWO-DIMENSIONAL PETROLEUM RESERVOIR

The petroleum reservoir is a very complex physical system encountered by the engineer. Calhoun (5) gives an excellent summary of the general nature and behaviour of a petroleum reservoir. To quote:

- "
1. Each petroleum reservoir is an individual and must be treated as such in its analysis and operation;
 2. The well system is a part of the reservoir.
Reservoir control and well control must be considered together, not separately;
 3. The natural underground physical state of reservoir rocks and reservoir fluids is different from that observed when they are taken from the reservoir;
 4. Oil, water and gas coexist intimately in most reservoirs;
 5. Petroleum is not produced from the containing rock without some displacing energy, such as encroaching water or expanding gas;
 6. Petroleum reservoirs are generally heterogeneous in rock properties, fluid properties or energy available for displacing oil; and
 7. All reservoir control is accomplished by control of fluid transfer to, from or within the reservoir."

The behaviour of a producing petroleum reservoir is best described by the continuity equation (IV.1).

$$\phi S_o h \frac{\partial \rho}{\partial t} = \frac{\partial}{\partial x} \left(\frac{kh\rho}{\mu} \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{kh\rho}{\mu} \frac{\partial P}{\partial y} \right) - Q(x, y, t) \quad (IV.1)$$

where ϕ = porosity of formation;
 S_o = fraction of oil saturation;
 h = thickness of formation;
 k = permeability of formation;
 μ = viscosity of oil;
 Q = rate of withdrawal of oil (at reservoir conditions) per unit area;
 ρ = density of oil;
 P = pressure of oil;
 x, y = distance coordinates;
 t = time.

If the quantity $\frac{1}{\rho} \frac{\partial \rho}{\partial P} = c$ is essentially a constant, and if the terms $\frac{kh\rho}{\mu} \left(\frac{\partial P}{\partial x} \right)^2$ and $\frac{kh\rho}{\mu} \left(\frac{\partial P}{\partial y} \right)^2$ are small compared to the terms $\frac{\partial}{\partial x} \left(\frac{kh}{\mu} \frac{\partial P}{\partial x} \right)$ and $\frac{\partial}{\partial y} \left(\frac{kh}{\mu} \frac{\partial P}{\partial y} \right)$ respectively, then, equation (IV.1) may be reduced to equation (IV.2).

$$\phi S_o h c \frac{\partial P}{\partial t} = \frac{\partial}{\partial x} \left(\frac{kh}{\mu} \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{kh}{\mu} \frac{\partial P}{\partial y} \right) - Q(x, y, t) \quad (IV.2)$$

If the quantities k and ϕ are functions of position only and the quantities c and μ are essentially constant, then equation (IV.2) is linear.

Fig. IV.1 shows a typical formation. If the closed curve C represents the reservoir, the following boundary conditions are usually applicable:

$$\frac{\partial P}{\partial r} = 0 \text{ on } C, \text{ where } r \text{ is the direction normal to } C.$$

This implies that the formation is impermeable beyond the reservoir.

$P(x,y)$ is a known function at some known t .

$Q(x,y,t)$ is a known function throughout the reservoir.

In general, the permeability and porosity of the formation vary throughout the region and the variations cannot usually be represented by a simple equation. The best approach seems to be that of assigning discrete point values of k and ϕ throughout the medium.

Numerical solutions to equation (IV.2) have been developed in the recent past (1,23). A semi-analytical solution of equation (IV.2) is presented here for the following reasons:

1. to obtain an estimate of the size of the problems that can be handled by this approach without running into numerical difficulties; and
2. to demonstrate the versatility of this approach.

The reservoir bounded by the curve C was divided into a finite-difference grid with 40 grid points, as shown in Fig. IV.1. Each grid point was assigned with a number. Three producing wells were placed arbitrarily in blocks 8, 19 and 25.

If the area around any grid point i is denoted by s_i , then the partial differential equation (IV.2) can be integrated over this elemental area to get

$$\iint_{s_i} \psi_i \frac{\partial P_i}{\partial t} dx dy = \iint_{s_i} \left[\frac{\partial}{\partial x} \left(k'_i \frac{\partial P_i}{\partial x} \right) + \frac{\partial}{\partial y} \left(k'_i \frac{\partial P_i}{\partial y} \right) \right] dx dy - \iint_{s_i} Q(x, y, t) dx dy \quad (IV.3)$$

where $\psi = \phi \cdot 1 - s_0 \cdot h \cdot c$ and $k' = \frac{kh}{\mu}$.

If l_i is the contour enclosing the area s_i , then Green's theorem says that (31):

$$\iint_{s_i} \left[\frac{\partial}{\partial x} \left(k'_i \frac{\partial P_i}{\partial x} \right) + \frac{\partial}{\partial y} \left(k'_i \frac{\partial P_i}{\partial y} \right) \right] dx dy = \int_{l_i} \left(k'_i \frac{\partial P_i}{\partial x} dy - k'_i \frac{\partial P_i}{\partial y} dx \right) \quad (IV.4)$$

The R.H. S. for each of these can be evaluated simply by considering the gradients $\frac{\partial P_i}{\partial x}$ and $\frac{\partial P_i}{\partial y}$ along the contour and employing a consistent system of signs to the derivatives and distance increments. As an example, the interior point 15 is considered:

$$\begin{aligned} \Delta x \Delta y \psi_{15} \frac{d P_{15}}{dt} &= \frac{(k'_{14} + k'_{15})}{2} \cdot \frac{(P_{14} - P_{15})}{\Delta x} \cdot \Delta y \\ &- \frac{(k'_{9} + k'_{15})}{2} \cdot \frac{(P_{9} - P_{15})}{\Delta y} \cdot (-\Delta x) \\ &+ \frac{(k'_{15} + k'_{16})}{2} \cdot \frac{(P_{15} - P_{16})}{\Delta x} \cdot (-\Delta y) - \frac{(k'_{15} + k'_{23})}{2} \cdot \Delta y \end{aligned}$$

$$\frac{(P_{15} - P_{23})}{\Delta y} (\Delta x) - Q_{15}(x, y, t) \Delta x \Delta y \quad (\text{IV.5})$$

Division throughout by $\Delta x \Delta y$ and rearrangement gives

$$\begin{aligned} \psi_{15} \frac{dP_{15}}{dt} = & \frac{(k'_{9} + k'_{15})}{2\Delta y^2} P_9 + \frac{(k'_{14} + k'_{15})}{2\Delta x^2} P_{14} - \left[\frac{(k'_{9} + k'_{15})}{2\Delta y^2} \right. \\ & + \frac{(k'_{14} + k'_{15})}{2\Delta x^2} + \frac{(k'_{15} + k'_{16})}{2\Delta x^2} + \left. \frac{(k'_{15} + k'_{23})}{2\Delta y^2} \right] P_{15} \\ & + \frac{(k'_{15} + k'_{16})}{2\Delta x^2} P_{16} + \frac{(k'_{15} + k'_{23})}{2\Delta y^2} P_{23} - Q_{15}(x, y, t) \end{aligned} \quad (\text{IV.6})$$

Interior point 21 is an example of a grid point around which the contour is odd-shaped. Integration of equation (IV.2) over this area gives

$$\begin{aligned} \iint_{s_{21}} \psi_{21} \frac{\partial P_{21}}{\partial t} dx dy = & \iint_{s_{21}} \left[\frac{\partial}{\partial x} \left(k'_{21} \frac{\partial P_{21}}{\partial x} \right) \right. \\ & + \left. \frac{\partial}{\partial y} \left(k'_{21} \frac{\partial P_{21}}{\partial y} \right) \right] dx dy - \iint_{s_{21}} Q_{21} dx dy \end{aligned} \quad (\text{IV.7})$$

Application of Green's theorem to the R.H.S. of equation (IV.7) gives the expression

$$\oint_{l_{21}} \left(k'_{21} \frac{\partial P_{21}}{\partial x} dy - k'_{21} \frac{\partial P_{21}}{\partial y} dx \right) - Q_{21} A_{21} \quad (\text{IV.8})$$

The line integral of the above expression can be written as a numerical integral as in equation (IV.5) to give the following equation:

$$\begin{aligned} A_{21} \psi_{21} \frac{dP_{21}}{dt} = & \frac{(k'_{13} + k'_{21})}{2} \frac{(P_{13} - P_{21})}{\Delta y} \Delta x - \frac{(k'_{21} + k'_{22})}{2} \frac{(P_{21} - P_{22})}{\Delta x} \Delta y \\ & - \frac{(k'_{21} + k'_{29})}{2} \frac{(P_{21} - P_{29})}{\sqrt{\Delta x^2 + \Delta y^2}} \cdot \frac{\Delta y}{2} \cos 45^\circ - Q_{21} A_{21} \end{aligned} \quad (\text{IV.9})$$

Division throughout by $\Delta x \Delta y$ and rearrangement gives

$$\begin{aligned} \frac{A_{21}}{\Delta x \Delta y} \psi_{21} \frac{dP_{21}}{dt} = & \frac{(k'_{13} + k'_{21})}{2 \Delta y^2} P_{13} - \left[\frac{(k'_{13} + k'_{21})}{2 \Delta y^2} + \frac{(k'_{21} + k'_{22})}{2 \Delta x^2} \right. \\ & \left. + \frac{(k'_{21} + k'_{29})}{4 \Delta x \sqrt{2 \Delta x^2 + 2 \Delta y^2}} \right] P_{21} + \frac{(k'_{21} + k'_{22})}{2 \Delta x^2} P_{22} \\ & + \frac{(k'_{21} + k'_{29})}{4 \Delta x \sqrt{2 \Delta x^2 + 2 \Delta y^2}} P_{29} - Q_{21} \frac{A_{21}}{\Delta x \Delta y} \end{aligned} \quad (\text{IV.10})$$

Formulation of equations of the type of equations (IV.6) and (IV.10) for the grid points 1 through 40 yields the following matrix differential equation:

$$\underline{D} \frac{d\underline{p}}{dt} = \underline{B}' \underline{p} + \underline{s}' \quad (\text{IV.11})$$

It is needless to point out that the matrix \underline{B}' is symmetric. In equation (IV.11), \underline{D} , \underline{p} , \underline{B}' , \underline{s}' and t have the same meaning as \underline{D} , \underline{u} , \underline{B}' , \underline{s}' and t , respectively of equation (II.2).

The solution of equation (IV.11) is straightforward. Table IV.1 gives a summary of results. Appendix E gives the computer program, the "property matrices" of the formation and the complete results.

To obtain a check on the results, the amount of oil produced was calculated by two independent methods:

1. Amount of oil produced at stock tank conditions is given by the equation

$$\text{Total production} = (\text{production of the wells/time}) \times \text{time of production} \quad (\text{IV.12})$$

2. The amount given by equation (IV.12) should be equal to the depletion of oil from the reservoir itself. This quantity may be derived as follows:

$$\text{Pore volume available for oil in block } j \text{ of the field} = (1-s_w)(\phi h)_j A_j \quad (\text{IV.13})$$

$$\text{Equivalent volume at stock tank conditions} = N_j = \frac{(1-s_w)(\phi h)_j A_j}{B} \quad (\text{IV.14})$$

where B = formation volume factor. $1 - s_w = S_o$

If the oil pressure in a block has decreased by ΔP_j in a certain period of time and $N_{P,j}$ is the oil produced in that period, then,

$$\frac{N_{P,j}}{N_j} = c(\Delta P)_j \quad (\text{IV.15})$$

TABLE IV.1A Two-Dimensional Petroleum ReservoirSummary of Results

Initial condition: at time - 0, the formation is at a uniform pressure of 1500 psia.

Size of the field: The complete rectangle measures 2 miles x 1.375 miles.

Number of wells: Three.

Total rate of Production: 400 STB/day.

Elapsed time, days	Pressures at selected grid points, psia					Total production to date, STB x 10 ⁻⁵	Check obtained by material balance STB x 10 ⁻⁵
	1	10	20	30	40		
40	1361.8	1361.3	1358.7	1361.3	1359.2	0.1600	0.1598
120	1082.9	1082.4	1079.8	1082.3	1080.3	0.4800	0.4795
200	803.9	803.4	800.8	803.4	801.3	0.8000	0.7992

or
$$N_{P,j} = N_j c(\Delta P)_j \quad (\text{IV.16})$$

giving

$$\text{Total production} = \sum_{j=1}^n N_j c(\Delta P)_j \quad (\text{IV.17})$$

The barrels of oil at stock tank conditions produced, calculated using equations (IV.12) and (IV.17) at each time appear in Table IV.1 and Appendix E. As may be noted, the agreement is extremely good.

V. INVERSE PROBLEM FOR A HOMOGENEOUS MEDIUM

In this chapter, the technique of estimating the transport property of a homogeneous medium using the semi-analytical method is described in detail. The experimental methods used are also dealt with in some detail.

A. Diffusivity of Copper Sulfate in Water.

The data used in estimating the diffusivity of copper sulfate in water were produced from unsteady-state experiments using the technique of microcell interferometry. These experiments were performed by Dr. R.N. O'Brien of the Chemistry Department, University of Alberta, Edmonton, and are described in complete detail elsewhere (17,18,19, 20). The salient features of the experiments and measuring techniques are presented here.

The main apparatus consists of an interferometer cell, its light source, and the recording instrument, a motion picture camera in this case.

The cell is essentially a wedge type interferometer, which can be set up as an electrochemical cell or a thermal conductivity cell. Fig. V.1 shows the various parts of the principal section of the cell. The cell is thermostated and the wedge angle can be changed. The actual cell itself, which was made of two copper electrodes sandwiched between two glass flats coated with partially reflecting

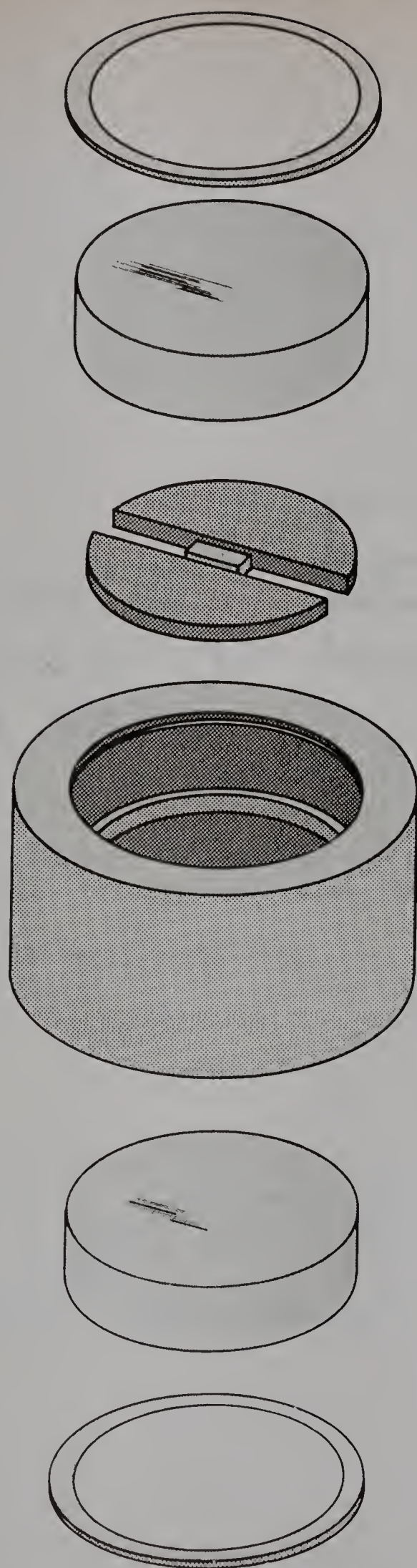


Fig. V.1. Details of the interferometer cell.

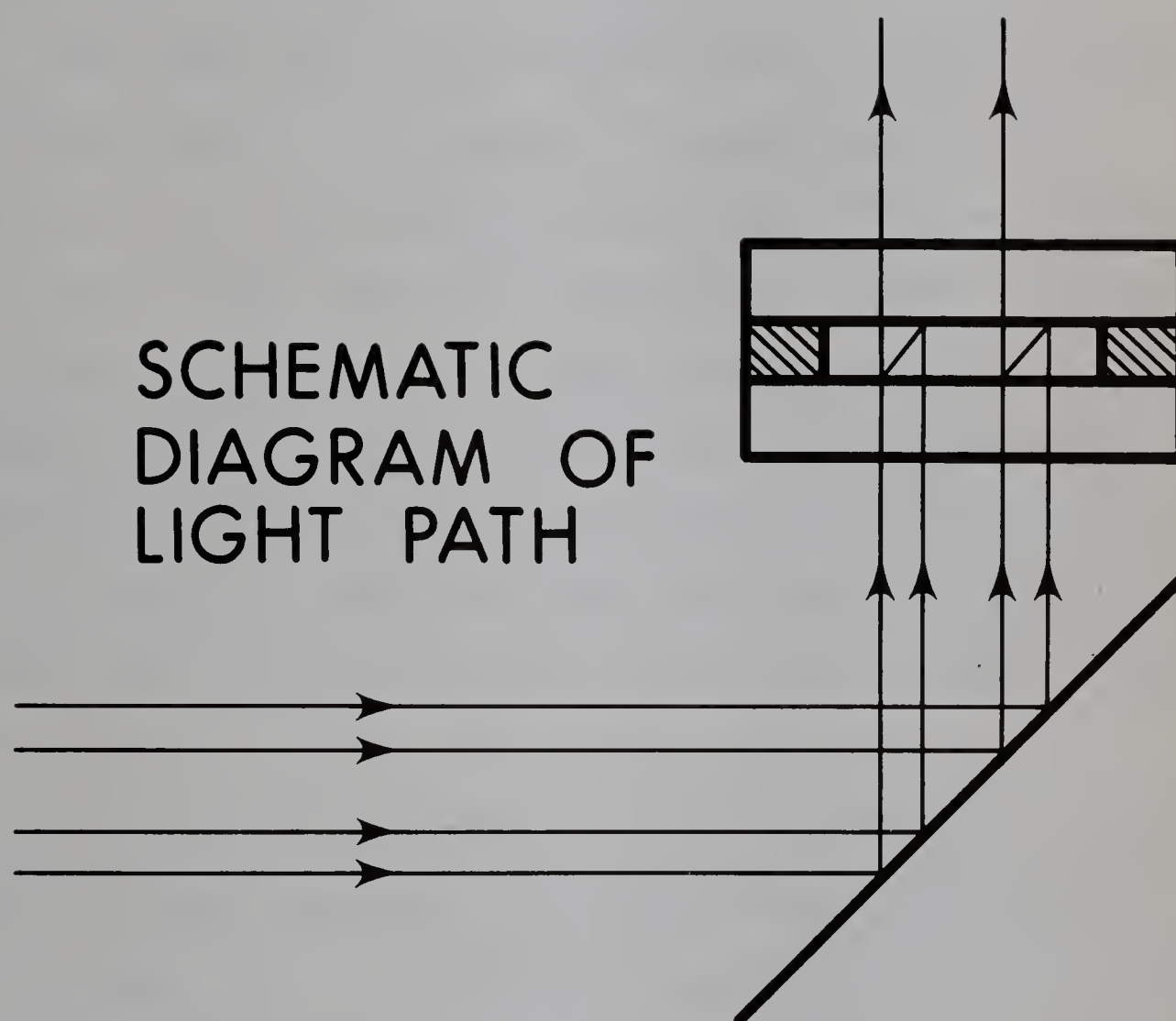


Fig. V.2. Schematic diagram of the light path showing the splitting of the beam amplitude and production of two coherent transmitted beams. The two halves of each of the split beams shown are actually coincident or very close to it. The displacement shown is for convenience of display only.

dichroic coating at the inner surfaces, was held in place by two 3/4-in brass retaining plates. The thermostating fluid was passed through the channels drilled in the retaining plates. The electrodes could be held firmly at a desired separation by teflon spacers.

The cell was operated successfully using a Gates Sodium vapor lamp as the source of monochromatic light. The glass at the first glass-solution interface encountered by the sodium light had a 90% reflecting dichroic coating and thus allowed about 10% of the light to traverse the cell to the other solution - glass interface. This glass had a 70% reflecting coating which split the amplitude of the light falling on it into a transmitted beam (3% of the original) and a reflecting beam (7% of the original). This process repeated itself, (Fig. V.2), and interference occurred between the transmitted beams, producing a system of light and dark fringes. The orientation and spacing of the fringes, which is controlled by the wedge angle, was adjusted to be about three per mm.

In diffusion experiments, the copper blocks, 0.2 cm thick and 4 cm long were set 0.313 cm apart and connected to a potential source as parallel-faced, horizontal electrodes with the cathode above the anode in a plain 0.1 N solution of copper sulfate of pH 4.5. The electrical circuit was arranged to give a constant current density of about 1 ma per sq cm. After about three minutes of

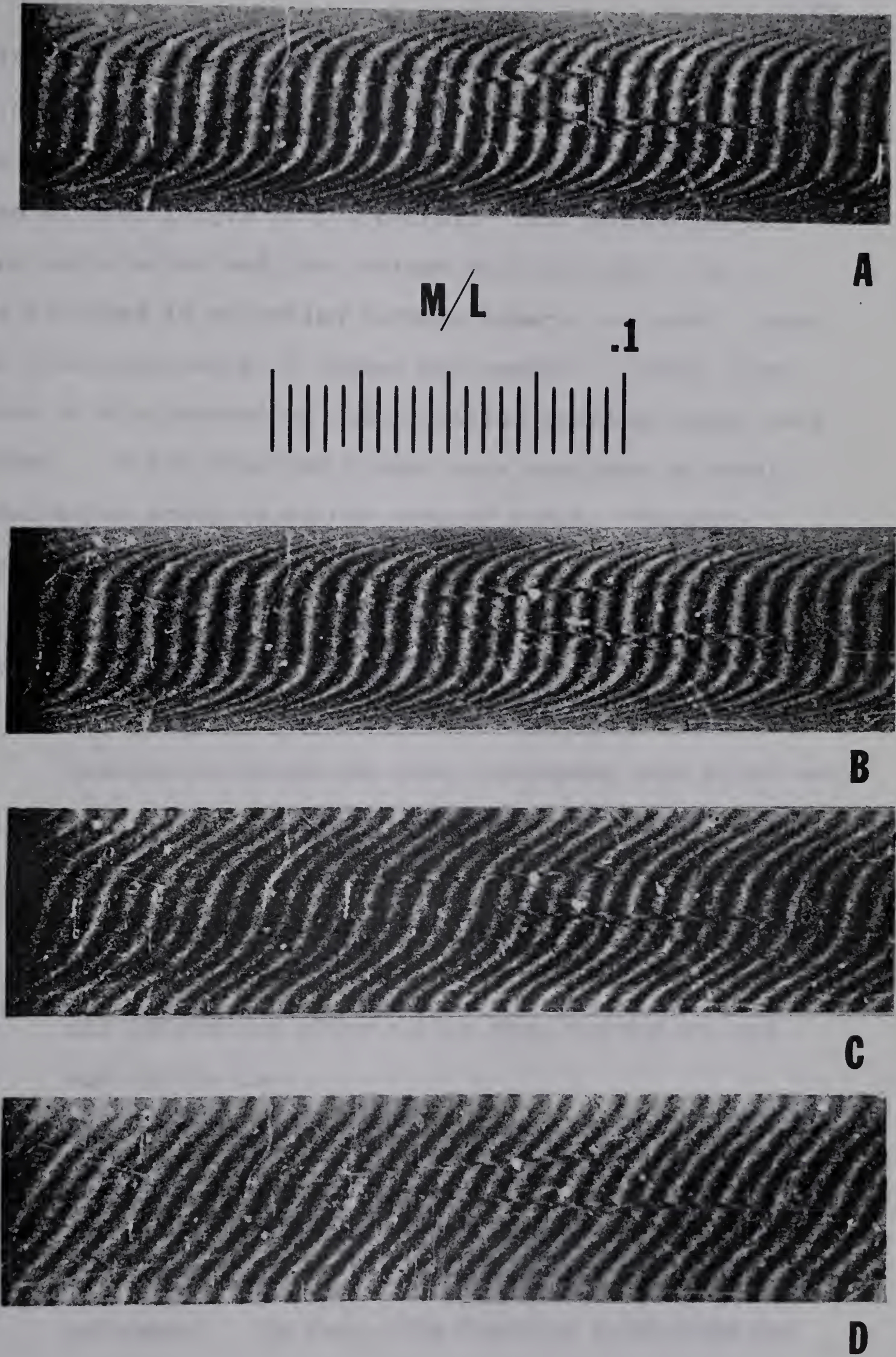


Fig. V.3 Photographs of Typical Fringes

electrolysis, a concentration gradient was established that extended toward the center of the cell from both the anode and the cathode but leaving an essentially unaltered volume of electrolyte in the center. The motion picture camera was started and the current switched off. A Bolex Paillard 16 mm motion picture camera was used; Ansco hypan film was used at 8 frames per second. About five minutes of the process of concentration gradient decay were recorded. A few selected frames were analysed to obtain concentration profiles at the elapsed times. Typical fringes are shown in Fig. V.3.

A few striking advantages of the microcell technique over the conventional techniques are:

1. Since the cell width is very small, the concentration build-up and decay processes take place very fast, the time required being in the order of a few minutes. In the case of thermal experiments, this was in the order of a few seconds. This very short duration of experiments virtually eliminates all extraneous effects like free convection and eddy diffusion.
2. The methods in the past (8) could collect data at only a limited number of points. They also employed very small gradients to avoid the undesirable effects. The present set-up does not suffer from any of these problems. In fact, the complete concentration

profile is available for mathematical analysis, and the concentrations at the required grid points are obtained by employing an appropriate interpolation method. Further, a wide variety of initial concentration gradients can be selected, including very steep ones.

The fringes were measured by two different methods. In some cases, the fringe was projected onto a graph paper at a magnification of about 50, the contour sketched out on the paper and the values at various points obtained from the sketched in curve. In other cases, the film negative was placed in a digitalised measuring microscope and computer punch cards made for various points on the selected fringe.

The fringes obtained in the microcell method are refractive index contours. The relationship between the refractive index and fringe width is derived as follows (17):

The basic formula for the position of fringes on a wedge is $n\lambda = 2\mu l \cos \theta$ (V.1)

where n = order of interference
 μ = refractive index
 λ = wavelength of light
 l = distance between reflecting surfaces,
 (i.e) thickness of the wedge at a given point.

θ = angle of incidence, 0° in this case.

Or, $n\lambda = 2\mu l$ (V.2)

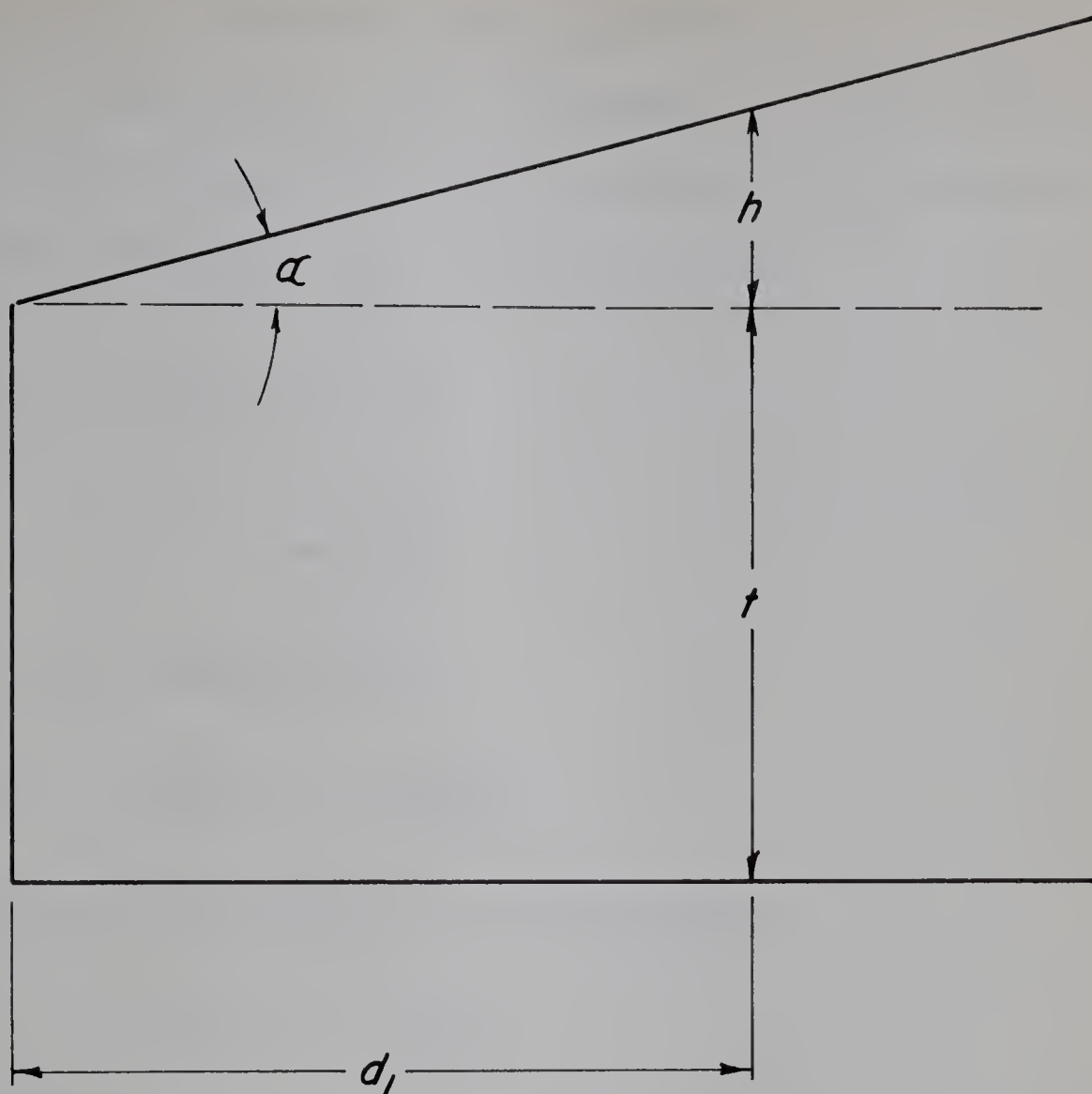


FIGURE V.4

SCHEMATIC DIAGRAM OF
THE INTERFEROMETER WEDGE

If it is assumed that the fringes are generated in a medium of refractive index μ between the reflecting surfaces in such a manner that the fringes are a distance d apart, then from Fig. V.4, it follows that

$$\tan \alpha = \frac{h_1}{d_1} \quad (\text{V.3})$$

and
$$\lambda = \frac{2\mu_1}{h_1} \quad (\text{V.4})$$

where α is the wedge angle.

Therefore,
$$\tan \alpha = \frac{\lambda}{2\mu_1 d_1} \quad (\text{V.5})$$

For a second point, equation (V.5) becomes

$$\tan \alpha = \frac{\lambda}{2\mu_2 d_2} \quad (\text{V.6})$$

Combining equations (V.5) and (V.6), the relationship

$$\frac{\mu_1}{\mu_2} = \frac{d_2}{d_1} \quad (\text{V.7})$$

is obtained.

If AA and A'A' are two different fringes, equation (V.2) may be written as

$$n_{AA} \lambda = 2\mu_1 l \quad (\text{V.8.1})$$

and
$$n_{A'A'} \lambda = 2\mu_2 (l+h) \quad (\text{V.8.2})$$

where h is the change in thickness of the wedge. Since the order of interference is essentially a constant, the above

two equations may be combined to give

$$\mu_1 l = \mu_2 (1+h) \quad (\text{V.9})$$

But $\tan \alpha = \frac{h}{d_2}$ (V.10)

from equation (V.3).

Combining equations (V.10) and (V.5), the relationship

$$h = \frac{\lambda d_2}{2\mu_1 d_1} \quad (\text{V.11})$$

is obtained.

Substitution of equation (V.11) into equation (V.9)

gives $\mu_1 l = \mu_2 \left[1 \pm \frac{\lambda d_2}{2\mu_1 d_1} \right]$ (V.12)

Substitution of equation (V.12) into equation (V.8.1) results in

$$\mu_1 = \mu_2 \pm \frac{\mu_2 d_2}{n_{AA} d_1} \quad (\text{V.13})$$

or $\mu_2 = \frac{\mu_1}{(1 \pm F/n)}$ (V.14)

where $F = d_2/d_1$ and is called the fringe shift. The order of interference is easily calculated for the system using equation (V.2).

To obtain the concentration (or concentration change) at any point on the fringe, a calibration curve of refractive index versus concentration is needed. The curve for copper sulfate at 25°C appears in Fig. V.5. It may be noticed

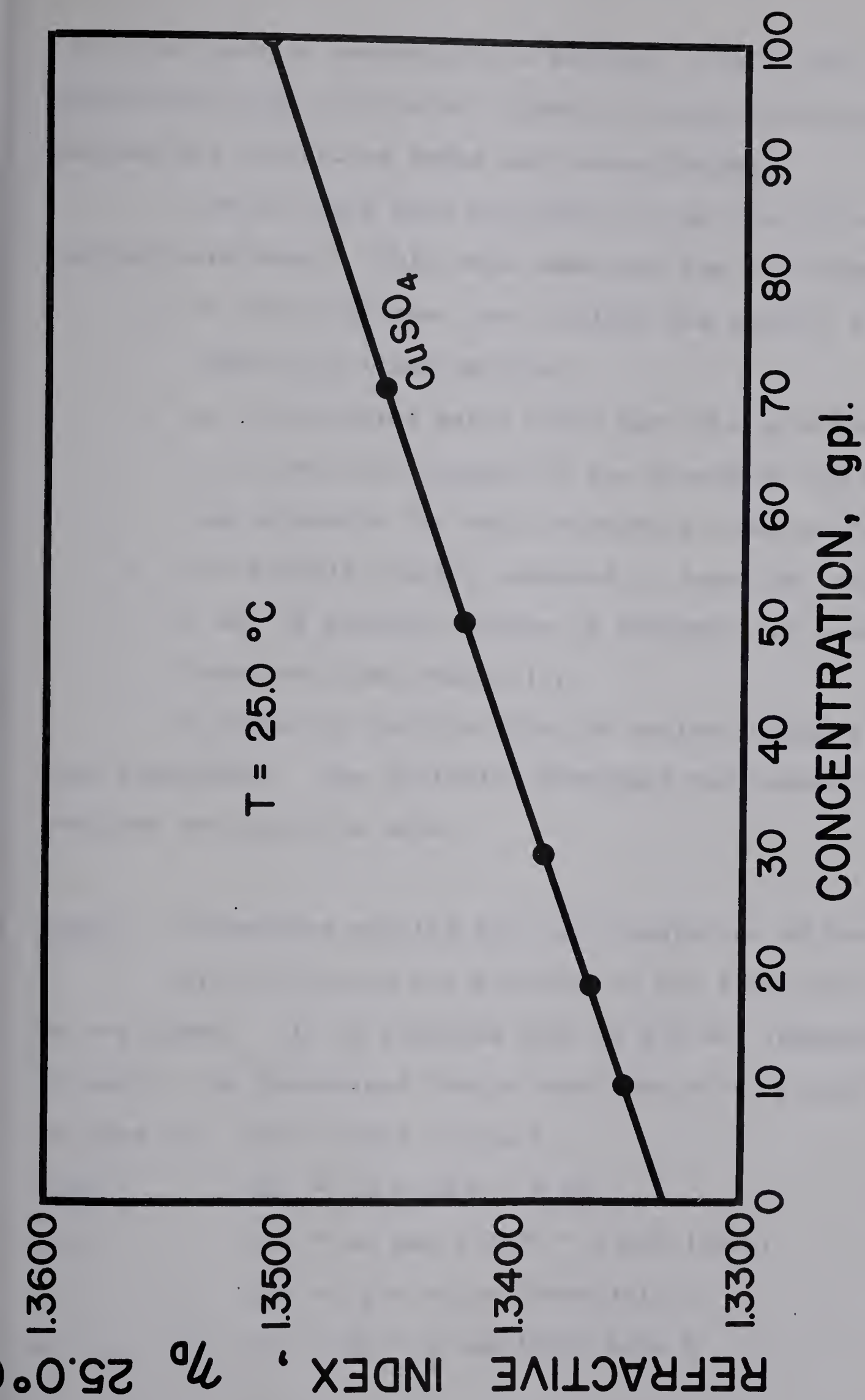


Fig. V.5

that the curve is essentially a straight line in the concentration range of interest, giving a linear relationship between the refractive index and concentration.

The raw data were available in the form of a punched card deck. This deck contained the following:

1. an identification card (called the master) for each refractive index profile;
2. an "orientation card" which gave the orientation of the frame with respect to the y-axis of the measuring apparatus for each refractive index profile; and
3. the profile itself, measured in Angström units, as a set of discrete points of distance (x) versus refractive index change (y).

A number of such profiles at various elapsed times were available. The following procedure was adopted on the computer to treat the data.

Step 1. Correction applied for the orientation of the frame:

Fig. V.6 gives the geometry of the frame with respect to x-y plane. It is required that OE and OC' (denoted by x' and y') be determined from a knowledge of x, y and θ .

To find y' : $OC' = CE = CD \sin \theta$.

Also, $CD = CG + GD = y + GD$.

But $GD = OG \tan (90-\theta) = x \tan (90-\theta)$

$\therefore CE = (y + x \tan (90-\theta)) \sin \theta$.

or $y' = (y + x \tan (90-\theta)) \sin \theta$ (V.15)

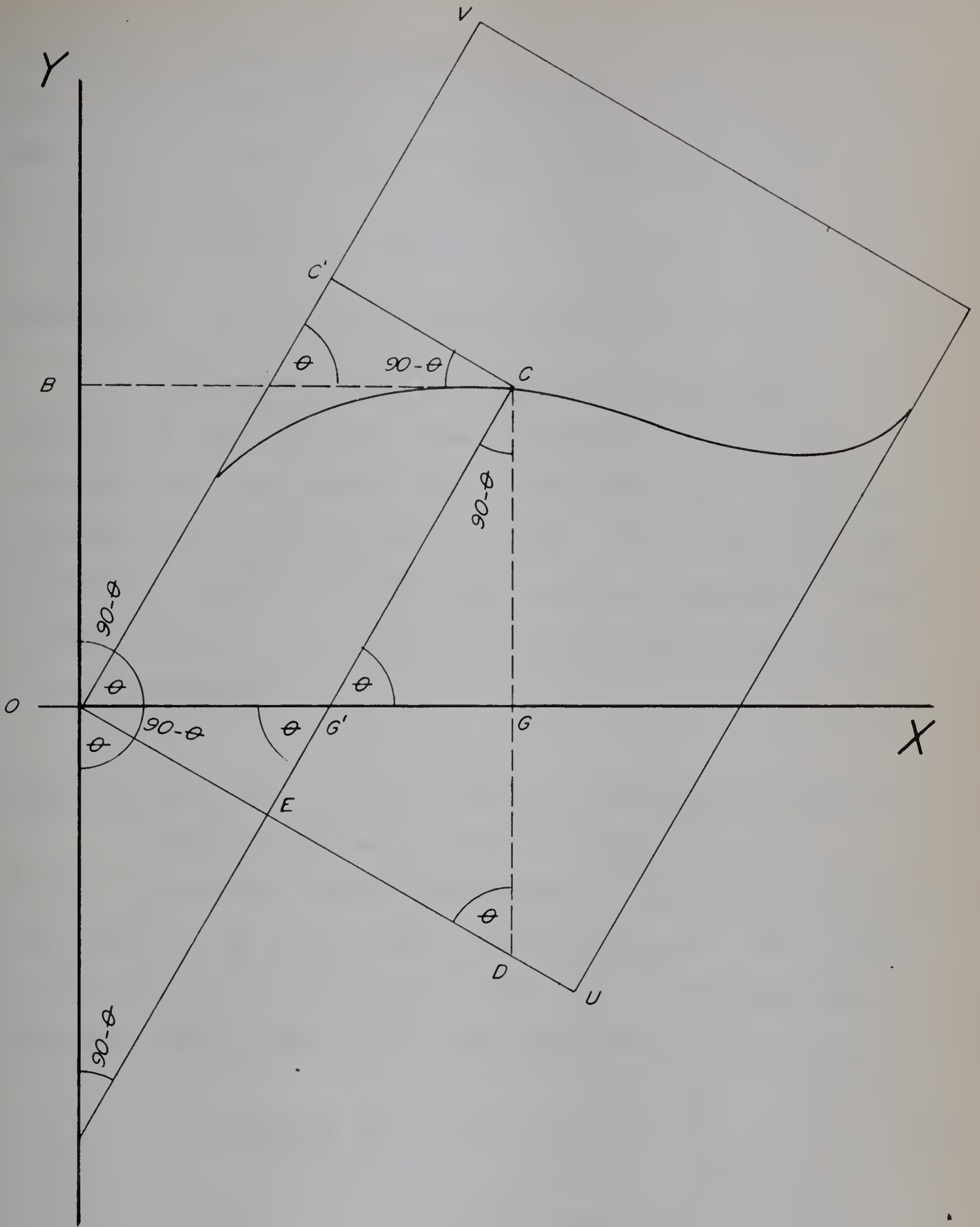


FIGURE V.6 ORIENTATION OF THE FRINGES

To find x' : $OE = OG' \sin \theta$.

But $OG' = OG - GG' = x - GG'$

and $GG' = \frac{GC}{\tan \theta} = \frac{y}{\tan \theta}$, giving

$$OG' = x - \frac{y}{\tan \theta}.$$

Therefore $x' = (x - \frac{y}{\tan \theta}) \sin \theta$ (V.16)

Each orientation card contained 3 points of y versus x on the line OC' . These points were fitted to a straight line using least squares, and the angle θ obtained from the slope of this straight line. After the angle was determined, equations (V.15) and (V.16) were applied to each set of x and y on the profile and the corresponding x' and y' were determined.

Step 2. Conversion of the readings on the x -axis into the actual distances of the microcell:

The first and the last points of the x -axis readings were taken to be the two ends of the microcell. The width of the cell was measured to be 0.313 cm; so all the other distances were normalised on this basis using the equation

$$\text{distance } x_j'' = 0.313 \times \frac{(x_j' - x_1')}{(x_N' - x_1')} \quad (V.17)$$

$$(j = 1 \text{ to } N)$$

where $N = \text{number of points on } x\text{-axis.}$

Step 3. Conversion of readings on y-axis to concentrations:

To do this, the change of concentration at any point should be determined. During both the processes of concentration build-up and decay, no copper sulfate has entered the cell from outside; hence the integral under each of the concentration profiles has to be a constant and is equal to the integral under the initial concentration curve; also, assuming that diffusivity of copper sulfate does not change significantly with concentration in the region of interest, the concentration versus distance curve should be a mirror image, (turned upside down) at the mid point of the x-axis. In other words, if a horizontal line is drawn to pass through the mid point of concentration curve, the area of the curve to the left and right of this point should be the same.

All these above deductions are borne out from the experimental profiles, as may be seen from Fig. V.7, within experimental limits.

Using trapezoidal rule, the integral under the curve using the actual distances x'' of x-axis and measured values of y-axis (the corrected values y') - is evaluated as follows:

$$\int_{x''_1}^{x''_N} y \, dx'' = I \doteq \frac{1}{2} \sum_{j=1}^{N-1} (y'_j + y'_{j+1}) (x''_{j+1} - x''_j) \quad (V.18)$$

Using the mean value theorem, an average ordinate is given by

$$y_{ave} = \frac{I}{(x_N - x_1)} = \frac{I}{0.313} \quad (V.19)$$

Now, the difference between y_{ave} and y' at any point is directly related to concentration change as follows (21):

- 1) Magnification: The image size is smaller than the actual cell; so the factor to apply for correction is

$$\text{Magnification} = \frac{3130}{(x_N - x_1)} \quad (V.20)$$

This factor is different for each frame.

- 2) Conversion factor: From the calibration curve, this was given to be 0.227×10^{-4} (gm mole/lit)/micron (21).

$$\begin{aligned} \text{Or, } c_j &= 0.05 + (y'_j - y_{ave}) \times 0.227 \times 10^{-4} \\ &\times \frac{3130}{(x_N - x_1)} \quad (V.21) \\ (j &= 1 \text{ to } n) \end{aligned}$$

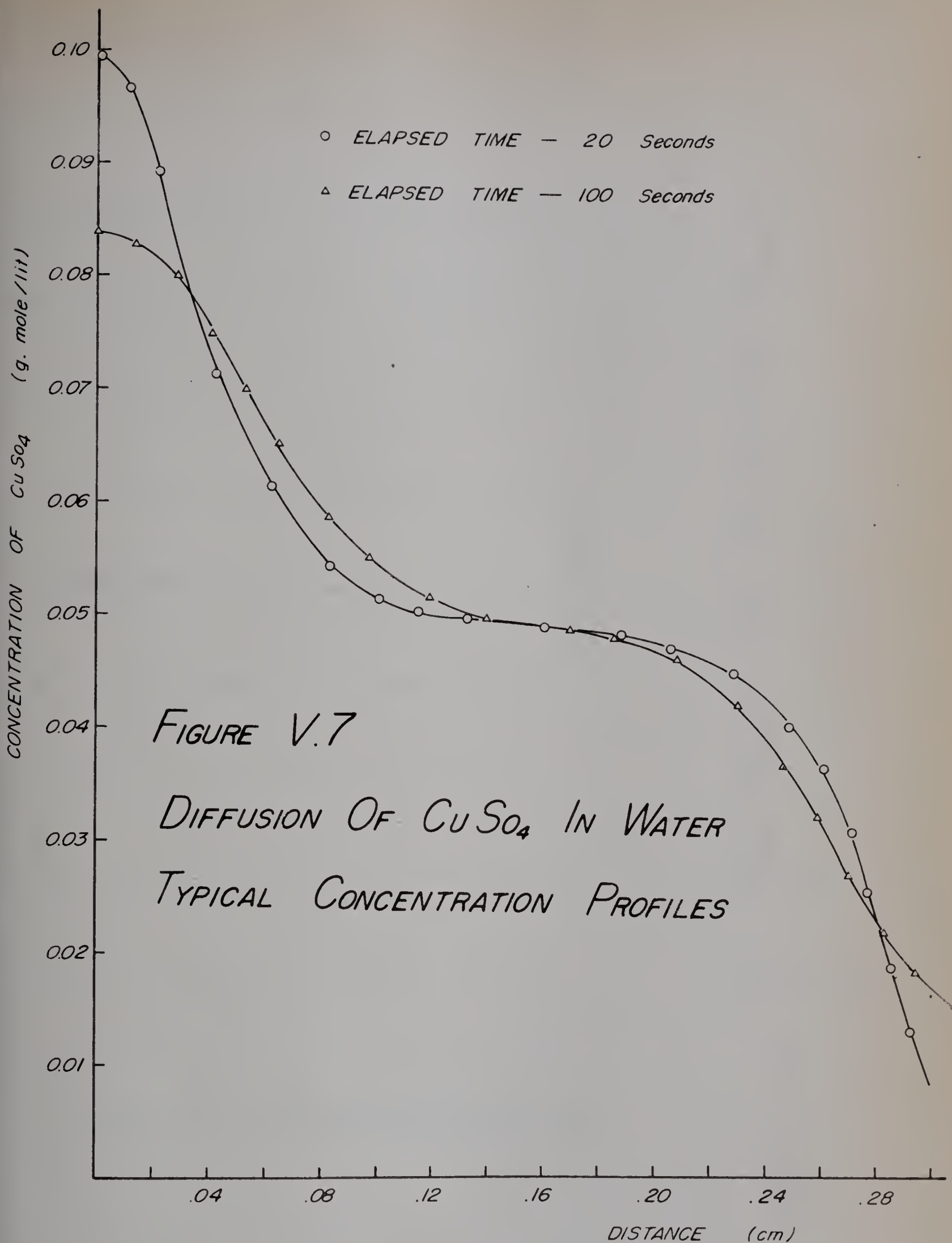
where c_j = concentration of copper sulfate, gm mole/lit at x_j .

Application of equations (V.15) through (V.21), done in a computer program yielded the curve of concentration versus distance at each elapsed time. Typical figures appear in Fig. V.7.

Step 4. Formation of the finite difference grid:

It was observed that the concentration at the mid point of the cell was unchanged within experimental limits, as was predicted from theoretical considerations. So, the following boundary conditions were employed:

$$\left. \begin{aligned} \text{At } x &= 0 = 0.313 \text{ cm, } \frac{\partial c}{\partial x} = 0 \\ \text{at } x &= 0.1565 \text{ cm, } c = \text{constant} \end{aligned} \right\} \quad (V.22)$$



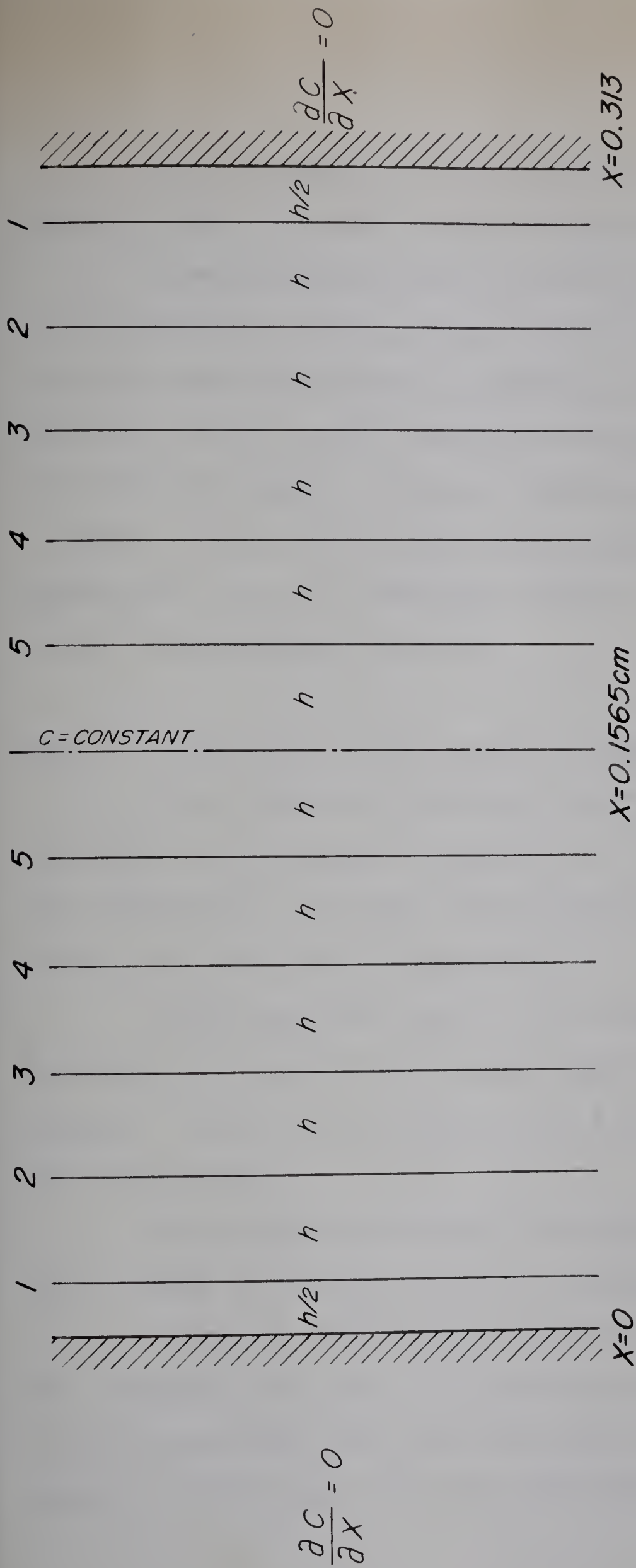


FIGURE V. 8

GRID SPACING USED FOR THE TWO HALVES
OF THE INTERFEROMETER CELL

The two halves of the microcell were treated separately, and a five point grid was employed (Fig. V8).

The concentrations at the grid points were obtained by interpolation using an appropriate interpolation formula. Since the data were unequally spaced, a Lagrangian interpolation formula, using two points on either side of the grid point was used. In some profiles, where the data was a little erratic, the 4-point Lagrangian interpolation formula did not give compatible values; in such profiles, linear interpolation was used.

Step 5. Evaluation of the values of decoupled variable:

The coefficient matrix \underline{B} for the above finite-difference grid appears in Appendix F. The eigenvalues and eigenvectors of \underline{B} were determined using Householder's method, and are given in Appendix F.

Using equation (II.37), the values of $\underline{y}(t)$ were determined at the various elapsed times from the known values of $\underline{c}(t)$. Also, using equation (II.43), the values of $\underline{z}(t)$ were determined.

The complete program, the input data and the complete output of results are given in Appendix F.

From the definition of the uncoupled variables $\underline{y}(t)$ (equation (II.37)), it can be seen that the components $v_i(t)$ of the vector $\underline{y}(t)$ are the scalar products of the potential vector $\underline{u}(t)$ with the individual eigenvectors \underline{q}_i .

($i = 1$ to n). Geometrically, these may be looked upon as the projections of the potential vector $\underline{u}(t)$ onto the individual eigenvectors \underline{q}_i . Another way of looking at these decoupled variables is also of help in interpreting the results. The v_i 's are the variables u_i in an n -dimensional orthogonal space, whose coordinate system is represented by the individual eigenvectors \underline{q}_i . The origin of the original coordinate system and the new coordinate system is, of course, the same.

The variables $z_i(t)$ (equation (II.43)) are functions of v_i , and are normalised values of v_i with respect to the initial value $v_i(0)$ and the modified boundary condition vector component r_i ($i = 1$ to n). It is easily shown (Appendix A) that the eigenvalues of a physical system represent exponential decay constants and hence must be negative numbers. Equation (II.43), then predicts that the values of z_i go to zero exponentially with time. It follows that these z_i 's are largest in numerical value at zero time, and are all equal to unity since all are normalised values. Some of these z_i 's will decay faster than the others depending on the numerical value of the corresponding eigenvalue.

When comparing lengths of vectors, it is desirable and advantageous to use a normalised potential vector $\bar{\underline{u}}(t)$. For example, if the initial condition of the medium is a uniform value of u_0 , one boundary is at a constant value u_b and the other boundary is insulated, the normalised potential

\bar{u}_i is defined by the equation

$$\bar{u}_i = \frac{u_b - u_i}{u_b - u_o} \quad (V.23)$$

On this basis, it can easily be seen that the values of the modified variables \bar{v}_i also decay to zero with time. If the initial potential vector happens to be nearly colinear with one of the eigenvectors (which is the same as having a normalised scalar product close to unity), then the decay process of this particular modified variable is bound to be described significantly better than that of the others by the experimental data. Except under special circumstances, the initial potential vector is not completely colinear with any of the eigenvectors; if this is the case, all the other v_i 's will contribute nothing to the above analysis, since the eigenvectors are orthogonal to each other.

The normalised scalar products of the initial potential vector with the individual eigenvectors, given by the equation

$$p_i = \frac{\underline{u}_o \cdot \underline{q}_i}{|\underline{u}_o| |\underline{q}_i|} \quad (V.24)$$

(i = 1 to n)

appear in Appendix F for the present problem. It may be noticed that p_4 and p_5 are an order of magnitude larger in absolute value than the others. It is also noticed that the eigenvalues corresponding to v_4 and v_5 are an order of magnitude smaller than the others, which makes them decay

slower than the others.

To sum up, although in theory there are n modified variables, each giving rise to a straight line from which the diffusivity of the medium can be calculated, the above considerations show that only a certain number of them are likely to give good results.

The values of z_4 and z_5 only were used therefore for estimation of diffusivity of copper sulfate. These values were curve-fitted to a straight line in a semi-logarithmic manner with the elapsed times. The experimental values of z_4 and z_5 , together with the best fit straight lines through these points appear in Fig. V.9. The estimated values of diffusivity of copper sulfate from z_4 and z_5 appear in Table V.1, along with some reported results (8) and other pertinent information. The estimated values agree favorably with the reported ones, thus confirming the validity of the model.

It was deemed desirable to arrive at a single value of diffusivity. Hence a weighting procedure was adopted: the normalised scalar products of the initial potential vector with the individual eigenvectors were determined, and an initial screening of the values of z_i to be used from the ones deemed undesirable for use was made. The values of diffusivity were determined using the above procedure; the single value of diffusivity then is thought to be best given by the equation

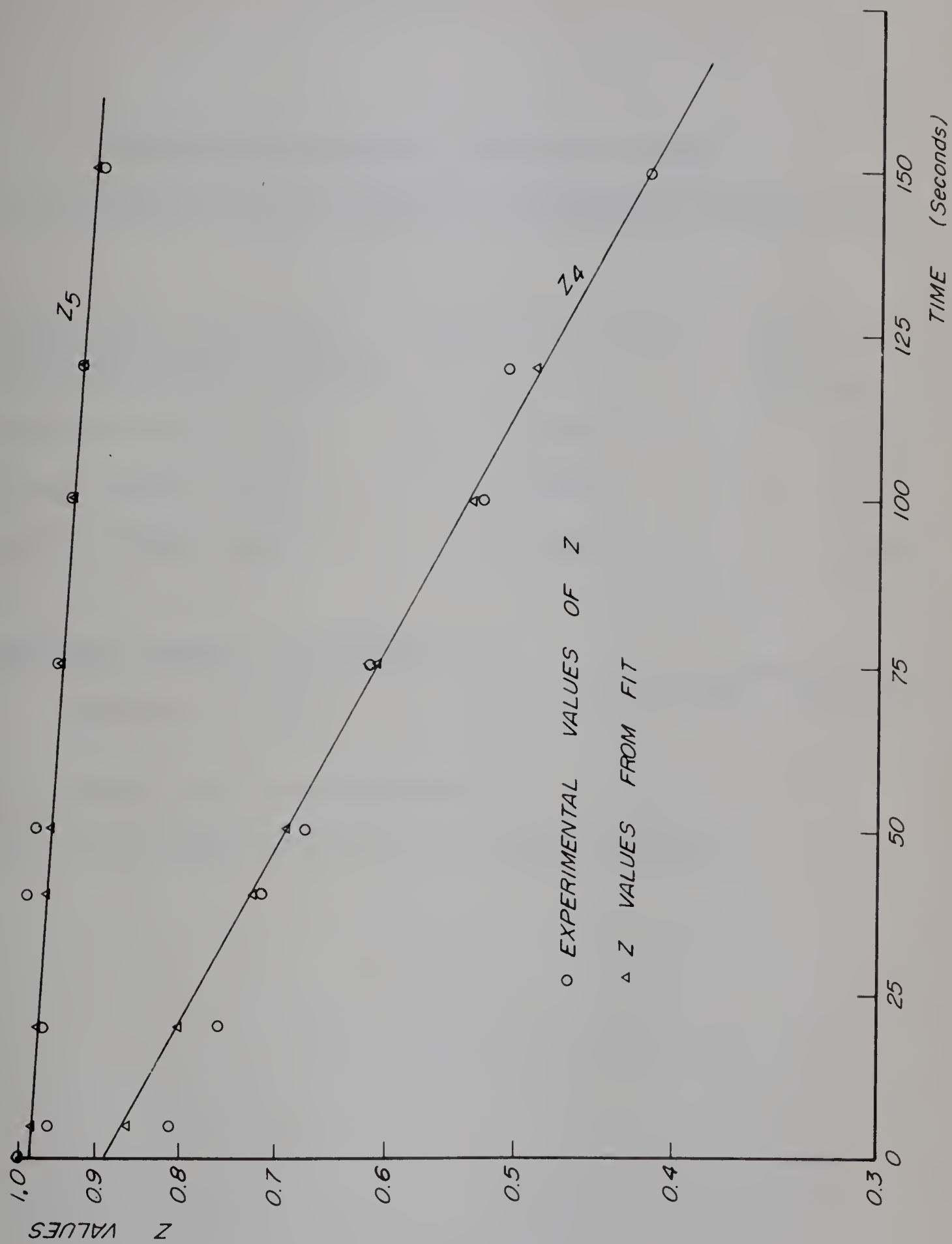


FIGURE V.9 SEMI-LOGARITHMIC LEAST SQUARE FIT OF RESULTS

TABLE V.1Diffusivity of Copper Sulfate in Water(Data from Dr. R.N. O'Brien's experiments. Temp = 25°C)

<u>Column from which the diffusivity was obtained</u>	<u>Weighting factor</u>	<u>Diffusivity x 10⁶ cm²/sec.</u>
z ₅ (High Conc. half)	0.97526	6.2714
z ₄ (Low concn. half)	0.67541	5.8645
z ₅ (Low concn. half)	0.71056	6.4087

$$\left. \begin{array}{l} \text{Weighted average (according to} \\ \text{equation (V.25))} \end{array} \right\} = 6.20 \times 10^{-6} \text{ cm}^2/\text{sec.}$$

Previously reported values (8)

are in the range of $5.5 \text{ to } 8.0 \times 10^{-6} \text{ cm}^2/\text{sec.}$

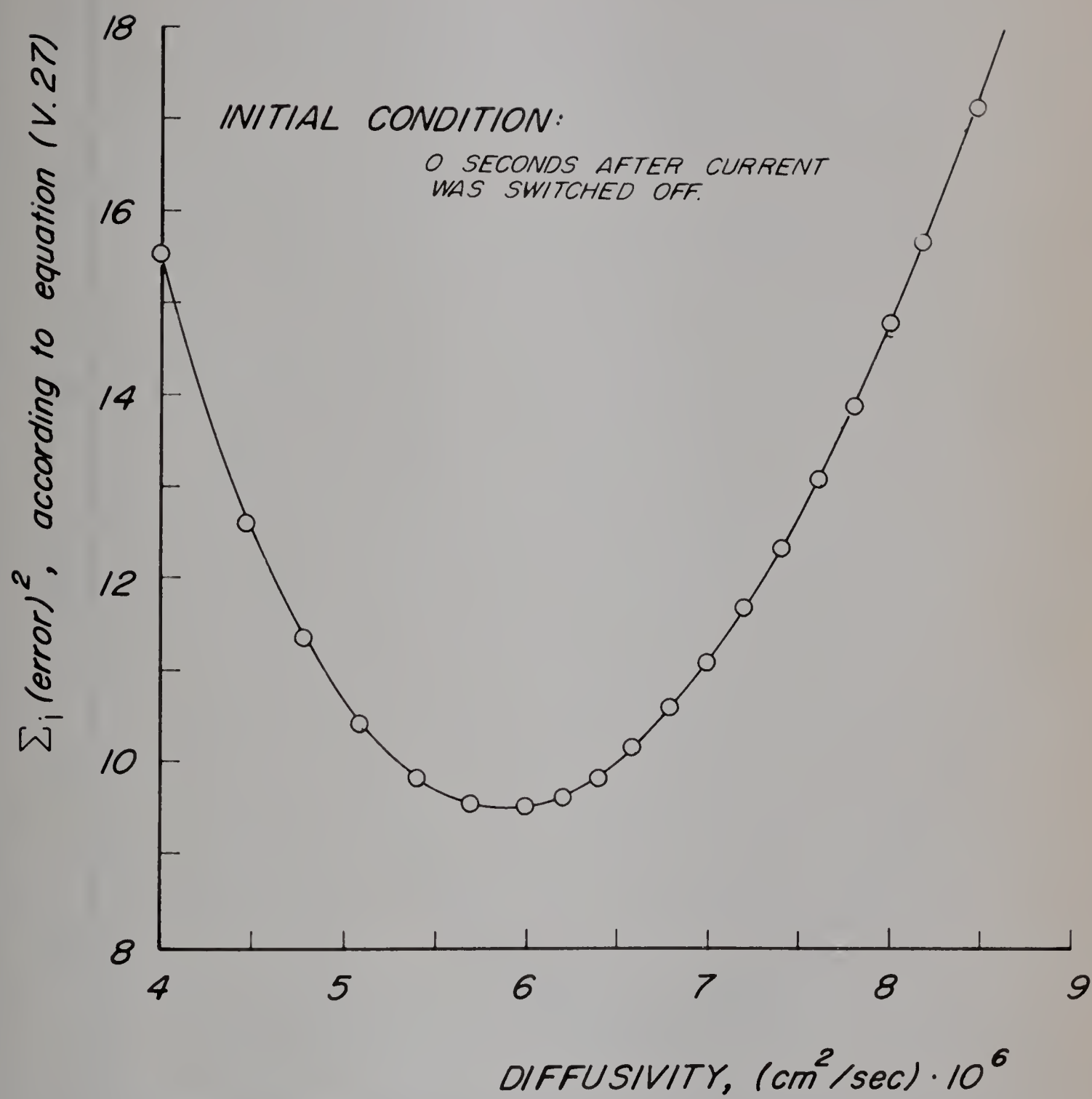
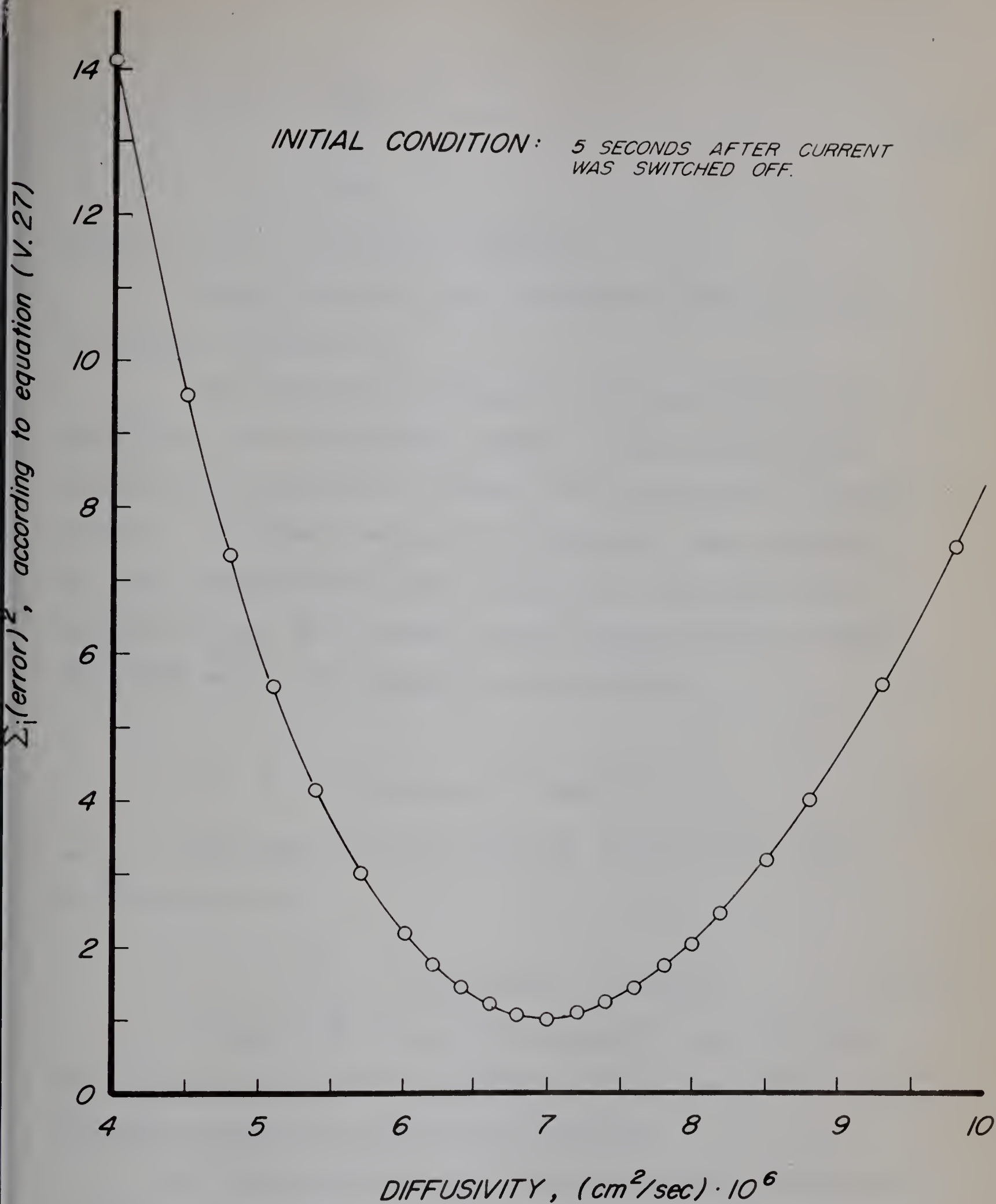


FIG V. 10a

LEAST SQUARE ANALYSIS
OF DIFFUSIVITY DATA



V.10b LEAST SQUARE ANALYSIS OF DIFFUSIVITY DATA

$$\alpha = \frac{\sum_{\text{over } i} p_i \alpha_i}{\sum_{\text{over } i} p_i} \quad (\text{V.25})$$

where p_i = i th scalar product.

These values for the two halves of the microcell are given in Table V.1.

For purposes of comparison, the same data were treated in the conventional manner: that is, analytical solution (7) was used to estimate the diffusivity of copper sulfate. A range of values of diffusivity were selected, and the concentrations were predicted by the analytical solution at the grid points for the various elapsed times. The point error was defined by the equation

$$e_i = (c_{\text{analytical}} - c_{\text{expt}})_i^2 \quad (\text{V.26})$$

The total error for any one value of diffusivity is then given by

$$E = \sum_{\text{over } i} (c_{\text{analytical}} - c_{\text{expt}})_i^2 \quad (\text{V.27})$$

A plot of E versus α was prepared, and the value of α where E was minimum was taken as the true diffusivity of copper sulfate as given by this method.

It was found that the value E was quite insensitive to changes in values of α , in the region where the curve went through a minimum. Further, when the method was repeated for a different initial condition, the

predicted minimum error was at a value of α quite remote from the first one. Hence the method was deemed not suitable.

The results from the analytical solution method are shown in Fig. V.10. The program is given in Appendix F.

The various considerations on the lengths and rate of decay of vectors which led to the present analysis needed a check on their validity. For this purpose extremely accurate data are necessary, and were generated from an analytical solution.

Using these data, the modified variable \underline{y} and the normalised variable \underline{z} were calculated. The corresponding normalised scalar products of the initial condition vector with the individual eigenvectors are also calculated. These values of \underline{y} , \underline{z} and the scalar products appear in Appendix G. Two very important conclusions are borne out from these data, namely:

1. Large eigenvalues tend to decay the values of z_i very fast; and
2. the starting length of a modified variable v_i is very crucial for the accuracy of the method.

Another very important factor is also made clear from the above tables: if the values of z_i are smaller than 0.0001, round off errors and errors in the evaluation of eigenvalues and eigenvectors are likely to play a dominant and detrimental role. Also, appearance of a negative value of z_i (which is theoretically impossible), is a sign

TABLE V.2

Estimation of Thermal Diffusivity of Water
Data Generated From an Analytical Solution

<u>Column from which the diffusivity was obtained</u>	<u>Weighting factor</u>	<u>Diffusivity x 10⁻² cm²/sec.</u>
z ₉	0.30234	0.1425
z ₁₀	0.92083	0.1412

Weighted average (according to equation (V.25)) } = 0.1415 x 10⁻² cm²/sec.

Assumed diffusivity, cm²/sec. = 0.1409 x 10⁻² cm²/sec.

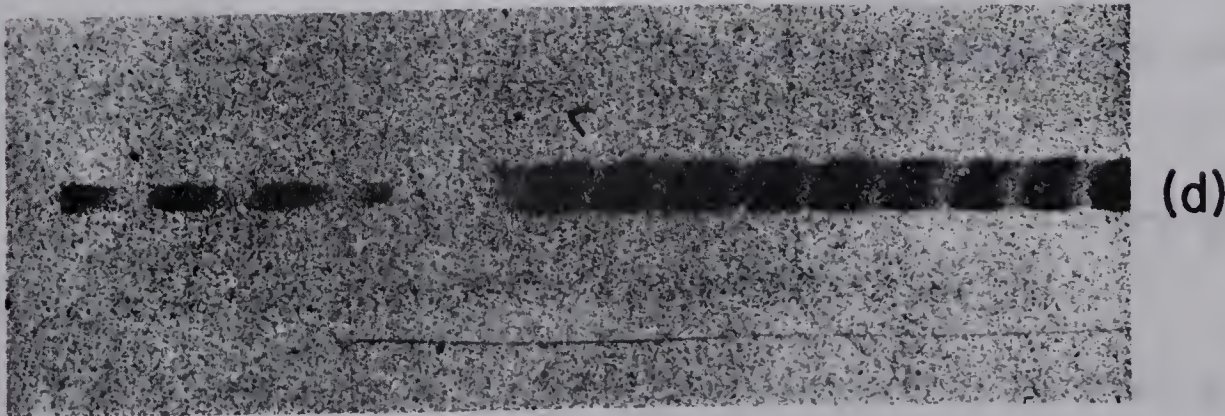
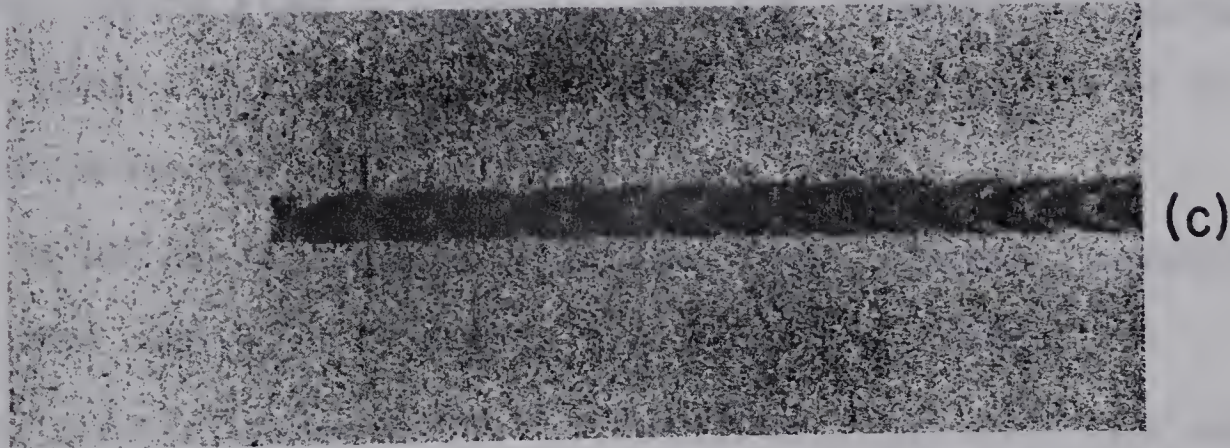
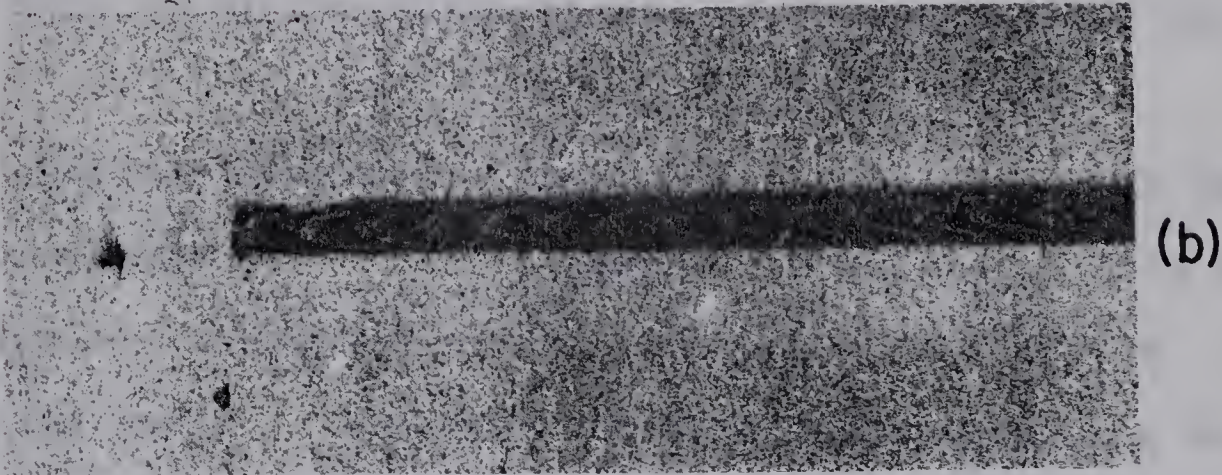
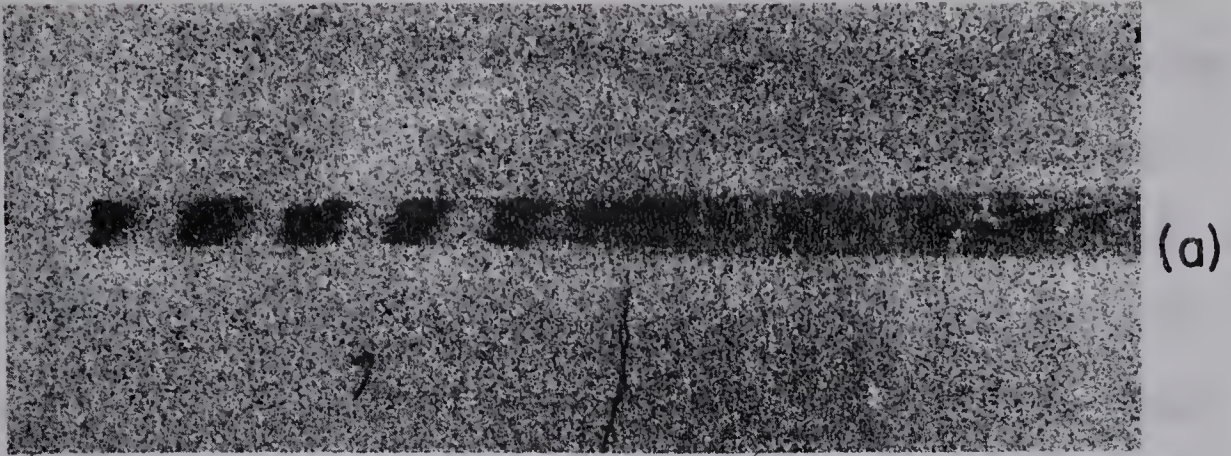
that the data are erratic, and should be handled carefully.

The values of diffusivity calculated from z_9 and z_{10} are given in Table V.2, along with the value used in the generation of data. As may be noticed, these values are extremely accurate.

B. Thermal Diffusivity of Water.

A microcell set up identical to that used in the experiments with copper sulfate solution was used. The 1.95 mm thick copper blocks were set at 0.043 cm apart. The cell was thoroughly thermostated at 25°C, and 0.15 cc of double distilled water at 5.2°C was injected by a hypodermic needle into the cell between the copper blocks. Just before injecting the water, the motion picture camera was started and the fringe development was recorded. The camera was operated at a speed of 24 frames per second. The temperature equalisation was essentially complete in about two seconds. A few typical fringes are shown in Fig. V.11.

A few assumptions made in the analysis of data from thermal experiments need some explanation. In formulating the finite difference grid, the assumption was made that the copper blocks in contact with water remain at the constant temperature of 25°C. This condition has not been met completely, but the error introduced can be shown to be small (22). The thermal conductivities of copper and water



0 10°C

Fig. V.11. Photographs of Typical Fringes

are 0.941 and 0.001409 cal/(cm.sec °C) respectively, or the conductivity of copper is about 660 times that of water. The thermal capacity of the copper blocks used is 15 cal/degree C. To raise 0.15 cc of water from 5.2°C to 25°C, 2.97 calories of heat are required. Therefore, the temperature of copper will decrease less than 0.2°C, or less than 1% of the total change of temperature of the water.

Another assumption made in the model was that the system is essentially one-dimensional in nature. This is not completely satisfied either, but the errors can be shown to be small. Although the heat capacity of the glass flats is about 7 cal/degree C, the thermal conductivity is only about 2% of that of copper, and the conducting surface is about four times smaller, so that the heat supplied by glass should be only about 0.5% of the total.

A third assumption was made that natural convection effects were absent. Although the copper blocks were vertical in this experiment, it is known (16) that the onset of natural convection requires about fifteen seconds, about six times the total experimental time, thus validating the assumption.

Hence, the experiment is expected to be a very good approximation of one-dimensional heat flow.

It was noticed on the motion picture film that the first few frames were quite blurred, making it almost im-

possible to make any accurate measurements. Presumably, the rate of change of temperature in the initial stages of heat transfer was too great for the speed of exposure of the camera to freeze it; so instead of using all the frames, only those frames which seemed to have a natural trend in the temperature profiles were used. The first of these was taken to be at zero time and hence was used as the initial condition vector.

The raw data were available in the form of a punched card deck, and contained the following:

1. An identification card for each refractive index profile;
2. An orientation card: in this deck, there was only one orientation card for all the profiles since they all had the same slanting.
3. The profile itself, measured in Angström units.

Exactly the same steps as in the copper sulfate program were used, with a few changes to meet the particular needs of the deck.

The magnification factor was given to be $0.43/0.345$, and the conversion factor was given to be $0.00597^{\circ}\text{C}/\text{Au}$. In this case, there is no need to integrate the temperature profile; the boundary temperature is known to be 25°C and the temperatures at the other points in the profile were calculated using the formula

$$T_j = 25.0 - \left| y_j' - y_1' \right| \times 0.00597 \times \frac{0.43}{0.345} \quad (\text{V.28})$$

($j = 1$ to N)

TABLE V.3

Estimation of Thermal Diffusivity of Water
Data from Dr. R.N. O'Brien's Experiments

<u>Column from which the diffusivity was obtained</u>	<u>Weighting factor</u>	<u>Diffusivity x 10² cm²/sec.</u>
z ₅ (five-point grid)	0.8996	0.1334
z ₁₀ (ten-point grid)	0.8728	0.1329
Weighted average (according to equation (V.25))		= 0.1332 x 10 ⁻² cm ² /sec.
Reported value (9)		= 0.1409 x 10 ⁻² cm ² /sec.

The boundary conditions employed were:

$$\left. \begin{array}{l} \text{at } x = 0, \quad T = 25.0 \\ \text{at } x = 0.0215 \text{ cm, } \frac{\partial T}{\partial X} = 0 \end{array} \right\} \quad (V.29)$$

A ten-point grid and a five-point grid were used for the half cell width. The coefficient matrix, eigenvalues and eigenvectors for the ten-point grid appear in Appendix G. The corresponding values of the modified variable v, the normalised scalar products and the variable z are also given in Appendix G together with the complete program and output.

It is noticed that in this case only one set of z_i 's, those of z_{10} , are the only ones available for plotting. The reason for this is obvious from the scalar products and the values of z_i themselves. The unavoidable errors in measurements have rendered the values of z_9 unusable.

The estimated values of thermal diffusivity of water from the five-point grid, ten-point grid and the reported value (9) appear in Table V.3; although the results do not agree as well as they do in the case of copper sulfate, they are satisfactory considering the problems encountered. It is hoped that with a better cell design and elimination of recording problems, better results can be obtained.

VI. INVERSE PROBLEM FOR A NON-HOMOGENEOUS MEDIUM

The concept of a homogeneous medium is in fact an approximation, but seems to be satisfactory in some physical situations. However, many situations arise where the approximation of a single value of a transport property for the whole medium is not adequate. A typical example is that of a petroleum reservoir, where the nature of the porous medium cannot be represented by any single value of porosity and permeability. A similar situation is encountered in the flow of heat or mass across the radius of a packed bed, where it has been demonstrated (28) that the effective thermal conductivity and diffusivity vary considerably over the cross-section of any packed bed. A fair amount of research work has been done in this field to establish techniques of estimating the Peclet numbers for heat and mass transfer in packed beds (2, 27).

The inverse problem for a non-homogeneous medium, stated in Chapter II, is as follows:

The temperature (or pressure or concentration) - time history of a non-homogeneous body under the influence of known boundary and initial conditions is available. It is required to predict the transport properties of the body (this is tantamount to predicting the coefficient matrix B for the system).

A solution to the general inverse problem stated

above has been developed using the semi-analytical method. The solution makes use of the following properties of the system:

1. If the body is adequately represented by an n -point grid, the values of potential at these grid points u_1, u_2, \dots, u_n will form a linearly independent system of coordinate axes for the system. Associated with the n -dimensional space of the actual coordinate axes, there exists an orthogonal n -dimensional eigenvector space. These are the eigenvectors associated with the coefficient matrix \underline{B} .
2. Along any of these eigenvectors, a potential vector undergoes only a change in length but not of direction.
3. The rates of decay of the components of the potential vector in the various eigendirections are logarithmically proportional to time. The slopes of these straight lines are proportional to the eigenvalues of the system.

The technique employed in developing a solution to the inverse problem for a non-homogeneous medium (to the physical system) is analogous to the technique employed by Wei and Prater (32) for estimating the rate constant matrix of a complex first order reaction system (the chemical system). There is one chief difference, however. The constraints (II.7) and (II.8) imposed on the chemical system

are not in general applicable to the physical system. For an n -component chemical system, an $(n-1)$ -dimensional phase plane (the reaction "simplex") is immediately defined. On this simplex, $(n-1)$ straight line reaction paths can be established. It must be borne in mind that all of these $(n-1)$ paths are in the real composition space, and are related to the eigendirections of the system, which are not in the real composition space. In the physical system, however, there is no simplex: this implies that there is no $(n-1)$ -dimensional surface on which all the potential vectors will lie. Consequently the straight line paths in the real potential space cannot be established

In the chemical system, the origin of the reaction simplex is the equilibrium composition and is unique for the n -component system under consideration. However, in a physical system approximated by an n -point grid, the boundary conditions determine the steady-state potential vector. This aspect further complicates the generalisation of the physical system.

The following boundary conditions were imposed on the non-homogeneous body considered in this thesis:

$$\text{at } x = 0, \quad u = 0.$$

$$\text{at } x = 1, \quad \frac{\partial u}{\partial x} = 0.$$

The theoretical development which follows is applicable to these boundary conditions.

The steady-state potential vector for this case is obviously the zero vector. This, then, will be the origin for the n -dimensional potential space under consideration. For this particular choice of boundary conditions, any potential vector will obviously lie in the positive orthant of the n -dimensional space. In general, it will have components in the various eigendirections. These components will decay to their origin at rates which are directly related to their eigenvalues. After a certain time, some of these components will become insignificant and the other components will predominate. Since the potential vector always stays in the positive orthant, it follows that the eigendirection in the positive orthant will be the dominant one. This means that the components in this direction will have the slowest rate of decay. Also, since the eigendirections are orthogonal, there is only one eigendirection in the positive orthant. All the above reasoning will then lead to the conclusion that any potential vector will decay to the origin; this decay process, after sufficiently long elapsed time will make the potential vector approach this dominant eigendirection and the rest of the decay will be along this unique direction, which is the only one "straight line path" in the real space associated with the physical system.

The above conclusions are substantiated by the results of the various solutions to the forward problem. There, it was indeed observed that associated with the coefficient matrix \underline{B} , there is one eigendirection which lies entirely in the positive orthant of the actual coordinate system. It is also observed that this eigendirection has the smallest rate of decay. The rate of decay of the length of the component of the potential vector in this direction will be logarithmically proportional to the eigenvalue corresponding to this eigenvector.

After determining this eigenvector, it is possible to eliminate the component of the potential vector in this direction. The resulting "potential vector", then, should converge to the eigendirection corresponding to the next smallest eigenvalue, and so on.

The process of "elimination" of successive eigenvectors to determine new eigenvectors is feasible only if

the original potential vector has reasonable components in the various eigendirections. For determining the first eigenvector, however, this requirement is not critical since any potential vector, which is always in the positive orthant, has a reasonable component in this eigendirection which is also in the positive orthant. No elimination is done while this eigendirection is being determined. Since other components decay faster than the component under consideration, this will eventually predominate.

In planning experiments for the purposes of determining the coefficient matrix \underline{B} , it is desirable to choose the successive sets of initial conditions as closely orthogonal to the determined eigenvectors as possible. Thus, the components of the potential vector in these directions can be suppressed and the components in other eigendirections will be more significant. During the process of elimination of the known components, this type of choice of the initial conditions minimises the loss of information about the unknown components. Since the components in the unknown directions will decay much faster, it is also desirable to have data at shorter elapsed times.

The line of reasoning mentioned above can be put in the form of equations. As an example, a system containing three dependent variables is considered.

A potential vector is represented by \underline{u} . If the vector \underline{u} has components σ_1 , σ_2 and σ_3 along the individual

coordinate axes \underline{u}_1 , \underline{u}_2 , and \underline{u}_3 , then

$$\underline{u} = \sigma_1 \underline{u}_1 + \sigma_2 \underline{u}_2 + \sigma_3 \underline{u}_3 \quad (\text{VI.1})$$

The same vector \underline{u} can also be represented in the orthogonal eigenvector system. If \underline{q}_1 , \underline{q}_2 and \underline{q}_3 are the three unit eigenvectors and β_1 , β_2 and β_3 are the components of \underline{u} in the directions of \underline{q}_1 , \underline{q}_2 and \underline{q}_3 , then

$$\underline{u} = \beta_1 \underline{q}_1 + \beta_2 \underline{q}_2 + \beta_3 \underline{q}_3 \quad (\text{VI.2})$$

It must be remembered, however, that \underline{q}_1 , \underline{q}_2 and \underline{q}_3 are not known. The problem is to find them in the u - system of coordinates.

If one of the eigenvectors, say \underline{q}_1 has been determined by the above process of normalisation of a set of potential vectors, then the projection of any potential vector \underline{u} onto this eigenvector is given by their inner product.

Or
$$\beta_1 = \underline{u}^T \underline{q}_1 \quad (\text{VI.3})$$

β_1 is a scalar.

The "potential vector" that will not have a component in the direction of \underline{q}_1 is given by

$$\underline{u}' = \underline{u} - (\underline{u}^T \underline{q}_1) \underline{q}_1 \quad (\text{VI.4})$$

According to the theory, then, the set of vectors \underline{u}' should converge to the eigendirection \underline{q}_2 . In the three-dependent variable case, determination of \underline{q}_3 is simply accomplished by use of the orthogonality relations

$$\left. \begin{aligned} \underline{q}_1^T \underline{q}_3 &= 0 \quad \text{and} \\ \underline{q}_2^T \underline{q}_3 &= 0 \end{aligned} \right\} \quad (\text{VI.5})$$

and remembering the fact that one of the elements of \underline{q}_3 can be arbitrarily chosen. To determine the third eigenvalue, however, a plot of the components in the \underline{q}_3 direction versus time has to be made for a set of potential vectors.

The above technique can be extended to an n-dimensional case. To determine n eigenvectors, the method requires that (n-1) eliminations should be performed. Some of these eliminations could probably be done with the same data, but it is likely that (n-1) different initial conditions, chosen according to the principle of orthogonality, will give better results.

The technique proposed above was applied to some data generated by a theoretical equation to obtain a check on its validity. Figure VI.1 gives a physical picture of the non-homogeneous medium considered and the boundary conditions employed. The diffusivity of the medium varies linearly with length along the body, and the variation is assumed to be known. When the problem is set up in the form required for a semi-analytical solution using a five-point grid, the following matrix differential equation will result.

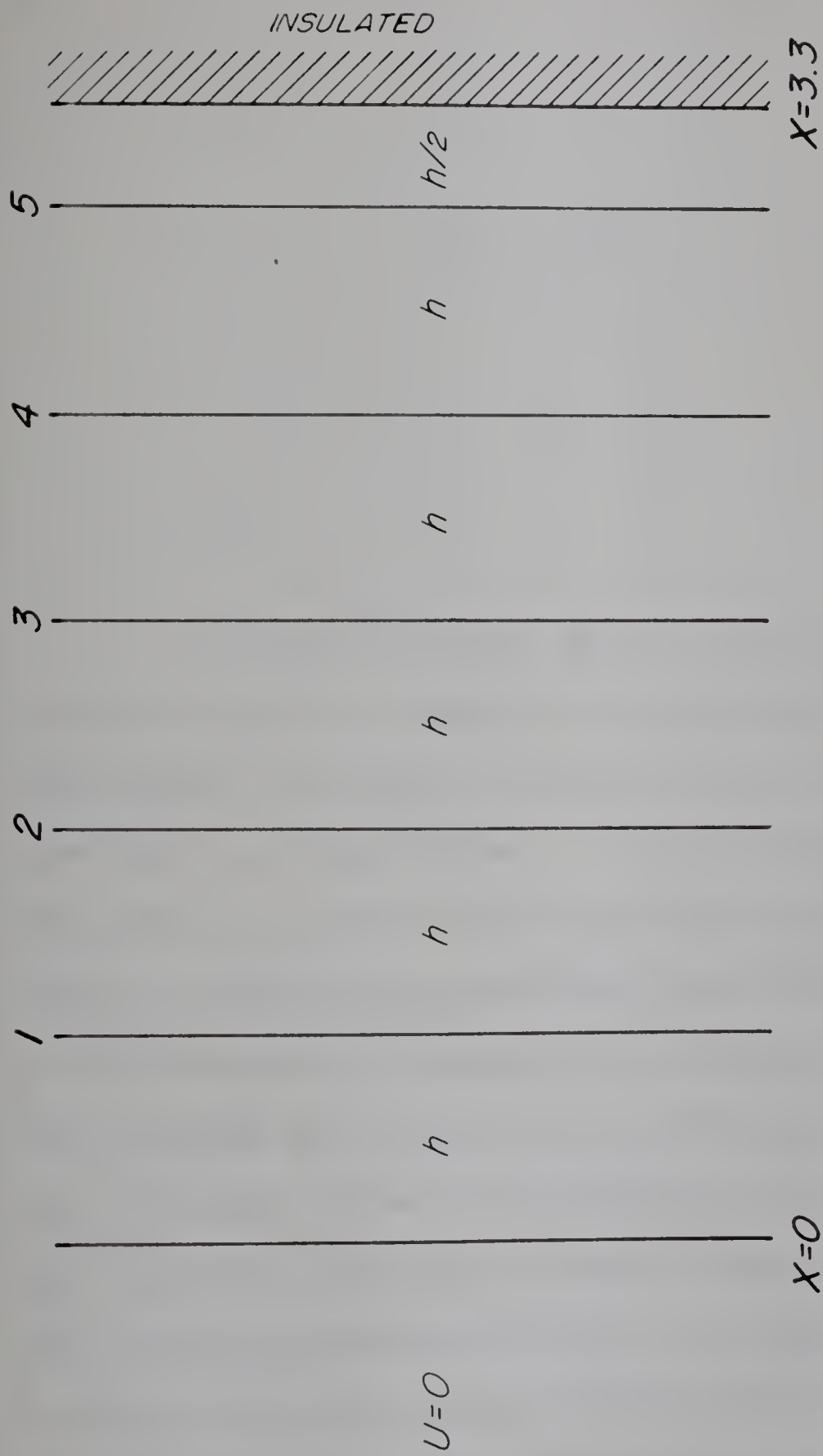


FIGURE VI.1

GRID SPACING USED FOR THE SOLUTION OF
INVERSE PROBLEM — NON-HOMOGENEOUS MEDIUM

$$\frac{d}{dt} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix} = \frac{1}{\Delta x^2} \begin{bmatrix} -(\alpha_{1\frac{1}{2}} + \alpha_{1\frac{1}{2}}) & \alpha_{1\frac{1}{2}} & 0 & 0 & 0 \\ \alpha_{1\frac{1}{2}} & -(\alpha_{1\frac{1}{2}} + \alpha_{2\frac{1}{2}}) & \alpha_{2\frac{1}{2}} & 0 & 0 \\ 0 & \alpha_{2\frac{1}{2}} & -(\alpha_{2\frac{1}{2}} + \alpha_{3\frac{1}{2}}) & \alpha_{3\frac{1}{2}} & 0 \\ 0 & 0 & \alpha_{3\frac{1}{2}} & -(\alpha_{3\frac{1}{2}} + \alpha_{4\frac{1}{2}}) & \alpha_{4\frac{1}{2}} \\ 0 & 0 & 0 & \alpha_{4\frac{1}{2}} & -\alpha_{4\frac{1}{2}} \end{bmatrix} \mathbf{x}$$

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix} + \begin{bmatrix} \alpha_{1\frac{1}{2}} u_0 / \Delta x^2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (\text{VI.6})$$

The solution for equation (VI.6) when the coefficient matrix is known is extensively discussed in Chapters II and III. This method was used to generate data for testing the technique developed in this chapter. The aim of this example is to generate the coefficient matrix of equation (VI.6) and thus obtain a check on the applicability of the technique. As a starting point, a uniform potential of unity across the whole medium was chosen and the boundary u_0 was kept at zero. The potential vector was determined for various elapsed times. The length of each one of these vectors was determined and the normalised potential vector was observed to converge to a unique direction, which checked very closely with one of the eigenvectors. The logarithms of vector lengths versus elapsed times yielded a straight line, whose slope agreed

very closely with the smallest eigenvalue for the system.

A second "experiment" was performed with an initial condition which had a much smaller component in this known eigendirection. The potential vector was determined for various elapsed times. The elapsed times in this second "experiment" were chosen to be an order of magnitude lower than the elapsed times of the first "experiment". The known component was eliminated from these potential vectors using equations (VI.3) and (VI.4) and the resulting vectors were normalised in the same manner. As was predicted, these vectors converged to a unique direction which was the same as the eigenvector corresponding to the second lowest eigenvalue. The rate of decay of the length was also determined and the slope was found to be very close to the second lowest eigenvalue.

The above process was successfully repeated to generate the other eigenvalues and eigenvectors. A similarity transformation using the generated eigenvalues and eigenvectors was made. The resulting matrix agreed very closely with the original coefficient matrix, thus providing an adequate check on the validity of the method.

A mathematical statement of the problem solved, the computer program used and the results obtained are given in Appendix H. A summary of results appears in Table VI.1.

TABLE VI.1INVERSE PROBLEM FOR A NON-HOMOGENEOUS MEDIUMSummary of Results

<u>Expt. No.</u>	<u>Elapse time for convergence to an eigenvector</u>	<u>Estimated eigenvalue</u>	<u>Actual Eigenvalue</u>
1	1.90	-0.3218	-0.3198
2	0.70	-3.2680	-3.2551
3	0.50	-8.2113	-8.1633
4	0.36	-13.6789	-13.6838
5	0.02	-19.9952	-19.9947

VII. SUMMARY AND CONCLUSIONS

1. A general semi-analytical solution to the linear conduction (or diffusion) equation has been developed. The discretisation of the space-derivatives of the parabolic partial differential equation produces a system of ordinary differential equations. The coefficient matrix of this system is symmetric, and hence the operational methods of linear algebra and matrix theory are very useful. A closed form solution in terms of the spatial distribution of properties and the eigenvalues and eigenvectors of the coefficient matrix is obtained. The solution has been tested on example problems of varying complexity, and good checks on the accuracy of the proposed method have been obtained. No significant problems were encountered in evaluating the eigenvalues and eigenvectors of matrices up to an order of 40.

Hence, it is concluded that the solution developed in this thesis is capable of handling practically all one-dimensional and many two-dimensional problems in homogeneous or non-homogeneous media (provided, of course, the properties of the media are independent of the dependent variable and time).

2. A general method of estimation of the transport property of a homogeneous medium using unsteady-state data has been developed. The technique of microcell interferometry has been used to produce the unsteady-state data.

The mathematical model is developed from the semi-discrete form of the diffusion equation, and makes use of the fact that symmetric matrices can be diagonalised by a similarity transformation. This transformation produces a new set of dependent variables which are completely uncoupled from each other. Integration of each equation is now accomplished very easily. The logarithm of the new variable is proportional to time, and the slope of the straight line contains the corresponding eigenvalue, diffusivity of the medium, and the grid spacing, from which the diffusivity is evaluated easily. The estimated values of molecular diffusivity of copper sulfate in water and of thermal diffusivity of water agreed favourably with previously reported values.

The conventional "least squares" technique applied to the same data failed to yield useful results. The method was found to be highly dependent on the choice of initial conditions. The proposed method, on the other hand, weights all the data in a manner determined by the eigenvectors of the system. Since these eigenvectors are associated with the coefficient matrix of the mathematical model of the physical process, this way of weighting the experimental data is believed to be a better representation of all the experiments. Also, the proposed method in essence permits a linear fit of the data, a desirable feature.

The proposed method was found to be capable of yielding useful results even from rather "noisy" data, as

was shown from the thermal experiments. As better light sources and improved microcells are developed, the accuracy of the method should be even better. Thus, a combination of microcell interferometry and the proposed theoretical analysis provides a new means for rapid and accurate determination of molecular and thermal diffusivities of liquids. Other possible applications are in the field of diffusion of gases in liquids.

3. A mathematical model has been proposed for the estimation of the "conductivity matrix" of a non-homogeneous medium. The method was successfully employed in reproducing the known conductivity matrix of a non-homogeneous medium by analysing a series of five "experiments" that used different initial conditions. This method, then, provides a new way of estimation of radial Peclet numbers in a packed bed. The model clearly illustrates that introduction of a tracer material at the centre of the packed bed is not necessarily the best way of obtaining information about the packed bed. Instead, the information provided by an arbitrarily chosen first experiment could be advantageously used in selecting the initial conditions for the next experiment.

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APPENDICES

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A. Related Topics of Matrix Algebra

The proofs of theorems of matrix algebra given here are extracted from standard texts on the subject.⁺

Theorem 1. The eigenvalues of a real and symmetric matrix B are real.

The proof is by contradiction.

Suppose λ_1 is a complex eigenvalue of B. Then, since B is real, it follows from the characteristic equation

$$\det (\underline{B} - \lambda \underline{I}) = 0 \quad (\text{A.1})$$

that the conjugate λ_1^* of any complex root λ_1 is also an eigenvalue. Also, the corresponding eigenvectors \underline{q}_1 and \underline{q}_1^* are complex.

Since λ_1 is an eigenvalue and \underline{q}_1 is an eigenvector, the following relation is satisfied:

$$\underline{B} \underline{q}_1 = \lambda_1 \underline{q}_1 \quad (\text{A.2})$$

$$\text{Also} \quad \underline{B} \underline{q}_1^* = \lambda_1^* \underline{q}_1^* \quad (\text{A.3})$$

Premultiplication of equation (A.2) by \underline{q}_1^{*T} and equation (A.3) by \underline{q}_1^T gives

$$\underline{q}_1^{*T} \underline{B} \underline{q}_1 = \lambda_1 \underline{q}_1^{*T} \underline{q}_1 \quad (\text{A.4})$$

$$\text{and} \quad \underline{q}_1^T \underline{B} \underline{q}_1^* = \lambda_1^* \underline{q}_1^T \underline{q}_1^* \quad (\text{A.5})$$

+ For example: 1) Bellman, R: Introduction to matrix analysis (1960).

2) Birkhoff, G, and MacLane, S: A Survey of Modern Algebra (1953).

Since \underline{B} is symmetric, the left hand sides of equations (A.4) and (A.5) are equal, giving

$$(\lambda_1 - \lambda_1^*) (\underline{q}_1^T \underline{q}_1^*) = 0 \quad (\text{A.6})$$

or $\lambda_1 = \lambda_1^*$. But this is contrary to the assumption. Hence all the eigenvalues of a real and symmetric matrix \underline{B} are real.

Theorem 2. A real and symmetric matrix \underline{B} of order N has N linearly independent and mutually orthogonal eigenvectors.

The proof for this theorem is done in two parts.

Part 1 To prove that the eigenvectors associated with distinct eigenvalues are orthogonal:

Consider two distinct eigenvalues λ and μ and the associated eigenvectors \underline{x} and \underline{y} . Then,

$$\underline{B} \underline{x} = \lambda \underline{x} \quad (\text{A.7})$$

and $\underline{B} \underline{y} = \mu \underline{y} \quad (\text{A.8})$

The scalar products of equation (A.7) with \underline{y} and of equation (A.8) with \underline{x} are

$$\underline{y}^T \underline{B} \underline{x} = \lambda \underline{y}^T \underline{x} \quad (\text{A.9})$$

and $\underline{x}^T \underline{B} \underline{y} = \mu \underline{x}^T \underline{y} \quad (\text{A.10})$

Since \underline{B} is symmetric, $\underline{y}^T \underline{B} \underline{x} = \underline{x}^T \underline{B} \underline{y} \quad (\text{A.11})$

or, $(\lambda - \mu) (\underline{x}^T \underline{y}) = 0 \quad (\text{A.12})$

Since λ and μ are distinct eigenvalues, $\underline{x}^T \underline{y}$ must be zero in order that equation (A.12) is satisfied.

Hence the eigenvectors associated with distinct eigenvalues are orthogonal.

Part 2 Let λ_1 be a root of multiplicity k . It is required to prove that there exist k linearly independent and orthogonal eigenvectors associated with λ_1 .

Let $\lambda_1 = \lambda_2 = \dots = \lambda_k$, but $\lambda_i \neq \lambda_1$, for $i = k+1, \dots, N$.

The proof is by induction.

$$\text{Let } \underline{B}_2 = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} \underline{b}^{(1)} \\ \underline{b}^{(2)} \end{bmatrix} \quad (\text{A.13})$$

be a symmetric matrix.

Let λ_1 and \underline{x}_1 be a set of associated eigenvalues and eigenvectors. Then,

$$\left. \begin{aligned} \underline{B}_2 \underline{x}_1 &= \lambda_1 \underline{x}_1, \quad \text{or} \\ \underline{b}^{(1)} \underline{x}_1 &= \lambda_1 x_{11} \text{ and } \underline{b}^{(2)} \underline{x}_1 = \lambda_1 x_{21} \end{aligned} \right\} \quad (\text{A.14})$$

Let \underline{T}_2 be an orthogonal matrix, one of whose columns is \underline{x}_1 . Let the other column be \underline{x}_2 . In matrix notation,

$$\underline{T}_2 = \begin{bmatrix} \underline{x}_1 & \underline{x}_2 \end{bmatrix}$$

It is required to show that

$$\underline{T}_2^T \underline{B}_2 \underline{T}_2 = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \quad (\text{A.15})$$

where λ_1 and λ_2 are the eigenvalues of \underline{B}_2 , not necessarily distinct.

Evaluation of the left hand side of equation (A.15) gives the following expression:

$$\underline{T}_2^T \underline{B}_2 \underline{T}_2 = \begin{bmatrix} \lambda_1 & c_{12} \\ 0 & c_{22} \end{bmatrix}$$

It is easily shown that $c_{12} = 0$, for

$$\left[\underline{T}_2^T \underline{B}_2 \underline{T}_2 \right]^T = \underline{T}_2^T \underline{B}_2^T (\underline{T}_2^T)^T = \underline{T}_2^T \underline{B}_2 \underline{T}_2 \quad (\text{A.16})$$

or $\underline{T}_2^T \underline{B}_2 \underline{T}_2$ is a symmetric matrix. Hence c_{12} has to be zero. Also, since the characteristic roots of $\underline{T}_2^T \underline{B}_2 \underline{T}_2$ and of \underline{B}_2 are the same, it follows that $c_{22} = \lambda_2$. This also proves that \underline{x}_2 is the second eigenvector.

The Nth order case is proved by induction.

Assume that for each k , $k = 1, 2, \dots, n$, an orthogonal matrix \underline{T}_k can be found which reduces \underline{B}_k to a diagonal form. It is required to show that if the above is valid for n , it is valid for $(n+1)$.

$$\text{Let } \underline{B}_{n+1} = (b_{ij}) = \begin{bmatrix} \underline{b}^{(1)} \\ \underline{b}^{(2)} \\ \cdot \\ \cdot \\ \cdot \\ \underline{b}^{(n+1)} \end{bmatrix} \quad (\text{A.17})$$

Let λ_1 and \underline{x}_1 be a set of associated eigenvalue and eigenvector. An orthogonal matrix \underline{T}_1 is formed with \underline{x}_1 as its first column. Let \underline{T}_1 be

$$\underline{T}_1 = \left[\underline{x}_1, \underline{x}_2, \dots, \underline{x}_{n+1} \right]$$

Now, the eigenvalues of \underline{B}_n must be the same as the other eigenvalues of \underline{B}_{n+1} .

The theory of induction is employed at this stage. Let \underline{T}_n be an orthogonal matrix which reduces \underline{B}_n to diagonal form. Let

$$\underline{S}_{n+1} = \begin{bmatrix} 1 & 0 & . & . & . & . & . & . & 0 \\ 0 & & & & & & & & \\ . & & \underline{T}_n & & & & & & \\ . & & & & & & & & \\ . & & & & & & & & \\ 0 & & & & & & & & \end{bmatrix} \quad (\text{A.21})$$

which is also orthogonal. Now

$$\underline{S}_{n+1}^T (\underline{T}_1^T \underline{B}_{n+1} \underline{T}_1) \underline{S}_{n+1} = \begin{bmatrix} \lambda_1 & & & & \\ & . & & & \\ & & . & & \\ & & & . & \\ & & & & \lambda_{n+1} \end{bmatrix} \quad (\text{A.22})$$

by multiplication.

$$\text{Or} \quad (\underline{T}_1 \underline{S}_{n+1})^T \underline{B}_{n+1} (\underline{T}_1 \underline{S}_{n+1}) = \underline{\Delta} \quad (\text{A.23})$$

$(\underline{T}_1 \underline{S}_{n+1})$ is the required orthogonal matrix, and hence its $(n+1)$ columns are linearly independent. Clearly, these are the eigenvectors of \underline{B}_{n+1} , and the theorem is proved.

Theorem 3. The eigenvalues of the matrix B are non-positive numbers.

Consider a typical matrix B encountered in a system which is adequately represented by a 3-point grid.

$$\underline{B} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -1 \end{bmatrix} \quad (\text{A.24})$$

A matrix B is said to be diagonally dominant if

$$|b_{ii}| \geq \sum_{\substack{j=1 \\ j \neq i}}^n |b_{ij}|, \quad i = 1 \text{ to } n \quad (\text{A.25})$$

This, of course, is easily seen to be true in the present case. Further, B is said to be irreducibly diagonally dominant if the inequality holds for at least one i . This is also easily verified to be true in the present case. It is therefore established that (30) since the diagonal elements of B are negative, the eigenvalues of B are also negative.

This also establishes that the matrix B is a negative definite matrix.

INTRODUCTION TO APPENDICES B TO G

In Appendices B to G, the various computer programs that have been used are presented. It is felt that a few important general aspects of the various programs should be listed at this stage.

1. In each of the following Appendices, a simple statement of the problem solved is given first.
2. Most of the programs are in the following general order:
 - a) a main-line program, where usually data are read in and results are printed out.
 - b) Various calculating procedures, used by the main line program in the form of subroutines.For small programs, this format was not followed.
3. The subroutines that have been used extensively in the thesis are listed in Appendix H. Any subroutines that are specific to a certain problem are listed along with its main line program.
4. "Comment cards" have been added to the main line programs for easier understanding of the programs.
5. In most of the programs that follow, "floating point traps" were encountered. These were caused mainly because the matrices had a number of zeros in them. Hence, a floating point trap suppressor program, which is available as a binary deck under

the name "FPT 9" was extensively used.

6. The following notation is employed on the \$JOB card:

F - Forward problem

I - Inverse problem

H - Homogeneous medium

NH - Non-Homogeneous medium

Thus, FNH would mean "Forward Problem - Non-Homogeneous medium", etc.

This notation is followed by an identification number, followed by any useful information that could be given on this card.

APPENDIX B

One-Dimensional, Homogeneous Media.

The partial differential equation describing this case is

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$$

$$0 \leq x \leq 2l.$$

The boundary conditions that have been used are:

$$\text{at } x = 0 = 2l, \quad u = 0$$

$$\text{at } x = l, \quad \frac{\partial u}{\partial x} = 0.$$

Semi-analytical solution: A five-point and a ten-point grid were used for the half-slab $0 \leq x \leq l$. The grid spacings used for the five-point grid is shown in Fig. III.1.

Analytical solution: This is given by equation (III.3).

Errors: The reported errors are (Analytical solution minus semi-analytical solution).


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C
C
C SOLUTION TO THE FORWARD PROBLEM-HOMOGENEOUS MEDIUM
C ONE-DIMENSIONAL SYSTEM
C
C INPUT DATA
C
C N=GRID SIZE
C NT=NUMBER OF TIMES
C S=COEFFICIENT MATRIX
C X=DIMENSIONLESS DISTANCES OF GRID POINTS
C TI=INITIAL CONDITION VECTOR
C C1=BOUNDARY CONDITION VECTOR
C TIME=DIMENSIONLESS TIMES
C ALPHA=THERMAL DIFFUSIVITY
C RM=(CELL WIDTH/2)**2
C DX=GRID SPACING
C
1 REAL TANAL(15,10),TIME(15),TI(10),TEVR(15,10),X(10),VAR(10)
2 REAL S(10,10),R(10),V(10,10),A(10),B(10),W1(10),W2(10)
3 REAL ERROR(15,10),C1(10),W(10,10)
4 1 FORMAT(1X,2I4)
5 2 FORMAT(1X,F13.8,4F14.8)
6 3 FORMAT(1X,7F10.6)
7 4 FORMAT(1X,3F14.8)
10 5 FORMAT(1X,25F3.0)
11 220 FORMAT(1H2)
12 221 FORMAT(1H ,10X,8H ..CONTD)
13 222 FORMAT(1H ,/)
14 223 FORMAT(1H ,/)
15 224 FORMAT(1H ,18X,26H TEMPERATURE PROFILES-ONE-,
111HDIMENSIONAL/25X,22H HOMOGENEOUS MEDIUM)
16 231 FORMAT(1H ,30X,12H GRID POINTS)
17 232 FORMAT(1H ,10X,5F11.6)
20 233 FORMAT(1H ,28X,25H INITIAL CONDITION VECTOR)
21 234 FORMAT(1H ,10X,5F12.4)
22 235 FORMAT(1H ,28X,26H BOUNDARY CONDITION VECTOR)
23 240 FORMAT(1H ,30X,19H COEFFICIENT MATRIX)
24 241 FORMAT(1H ,10X, 5F12.2)
25 242 FORMAT(1H ,31X,12H EIGENVALUES)
26 243 FORMAT(1H ,31X,13H EIGENVECTORS)
27 244 FORMAT(1H ,20X,35H CHECK OF SIMILARITY TRANSFORMATION)
30 245 FORMAT(1H ,30X,25H SEMI-ANALYTICAL SOLUTION)
31 246 FORMAT(1H ,25X,28H TEMPERATURES AT GRID POINTS)
32 247 FORMAT(1H ,30X,19HANALYTICAL SOLUTION)
33 248 FORMAT(1H ,15X,5H TIME,5X,2H 1, 9X,2H 2, 8X,2H 3, 8X,
12H 4, 8X,2H 5)
34 249 FORMAT(1H ,15X,5H TIME, 9X,2H 6, 8X,2H 7, 8X,2H 8, 8X,
12H 9, 8X,2H10)
35 251 FORMAT(1H ,15X,F6.1,2X,5F10.6)
36 255 FORMAT(1H ,30X,18H ASSOCIATED ERRORS)
37 READ(5,1) N,NT
42 READ(5,5) ((S(J,K),K=1,N),J=1,N)
53 READ (5,2) (X(J),J=1,N )

```


SN SOURCE STATEMENT

```

60 READ(5,2) (TI(J),J=1,N )
65 READ(5,2) (C1(J),J=1,N)
72 READ (5,3) (TIME(J),J=1,NT)
77 READ (5,4) ALPHA,RM,DX
00 MAXN=10
01 M=N
02 CALL EIG1(N,MAXN,M,S,R,V,A,B,W1,W2)
03 CALL CHECK(V,R,N,W)
04 PI=3.1415927
05 DO 10 K=1,N
06 DO 10 J=1,NT
07 TANAL(J,K)=0.0
10 DO 9 L=1,60
11 AL=L
12 AA=(2.*AL-1.)*PI*X(K)
13 AB=-(2.*AL-1.)*2*(PI/2.)*2*ALPHA*TIME(J)/RM
14 9 TANAL(J,K)=TANAL(J,K)+SIN(AA)*EXP(AB)/(2.*AL-1.)
16 10 TANAL(J,K)=4.*TANAL(J,K)/PI
21 DO 15 K=1,N
22 VAR(K)=0.0
23 DO 15 J=1,N
24 VAR(K)=VAR(K)+V(J,K)*TI(J)
25 15 CONTINUE
30 DO 25 L=1,NT
31 DO 25 K=1,N
32 TEVR(L,K)=0.0
33 DO 25 J=1,N
34 CONST=ALPHA*R(J)*TIME(L)/(DX**2)
35 TEVR(L,K)=TEVR(L,K)+V(K,J)*VAR(J)*EXP(CONST)
36 25 CONTINUE
42 DO 26 J=1,NT
43 DO 26 K=1,N
44 26 ERROR(J,K)=TANAL(J,K)-TEVR(J,K)
47 WRITE(6,220)
50 LINES=9
51 CALL LINECT(LINES,4,2)
52 WRITE(6,224)
53 WRITE(6,223)
54 WRITE(6,231)
55 CALL LINECT(LINES,1,2)
56 WRITE(6,232) (X(J),J=1,N)
63 CALL LINECT(LINES,4,2)
64 WRITE(6,223)
65 WRITE(6,233)
66 CALL LINECT(LINES,1,2)
67 WRITE(6,234) (TI(J),J=1,N)
74 CALL LINECT(LINES,4,2)
75 WRITE(6,223)
76 WRITE(6,235)
77 CALL LINECT(LINES,1,2)
00 WRITE(6,234) (C1(J),J=1,N)
05 CALL LINECT(LINES,4,2)
06 WRITE(6,223)
07 WRITE(6,240)
10 DO 31 J=1,N

```


H-1 1-D
SN

SOURCE STATEMENT

FORTRAN SOURCE LIST DARSI

```
11 CALL LINECT(LINES,1,2)
12 31 WRITE(6,241) (S(J,K),K=1,N)
20 CALL LINECT(LINES,3,2)
21 WRITE(6,223)
22 WRITE(6,242)
23 CALL LINECT(LINES,2,2)
24 WRITE(6,232) (R(J),J=1,N)
31 CALL LINECT(LINES,3,2)
32 WRITE(6,223)
33 WRITE(6,243)
34 DO 32 K=1,N
35 CALL LINECT(LINES,3,2)
36 WRITE(6,232) (V(J,K),J=1,N)
43 32 WRITE(6,223)
45 CALL LINECT(LINES,1,2)
46 WRITE(6,244)
47 DO 33 J=1,N
50 CALL LINECT(LINES,3,2)
51 WRITE(6,232) (W(J,K),K=1,N)
56 33 WRITE(6,223)
60 WRITE(6,220)
61 WRITE(6,245)
62 WRITE(6,223)
63 WRITE(6,246)
64 WRITE(6,248)
65 DO 34 J=1,NT
66 34 WRITE(6,251) TIME(J),(TEVR(J,K),K=1,5)
74 WRITE(6,223)
75 WRITE(6,247)
76 WRITE(6,223)
77 WRITE(6,246)
00 WRITE(6,248)
01 DO 36 J=1,NT
02 36 WRITE(6,251) TIME(J),(TANAL(J,K),K=1,5)
10 WRITE(6,223)
11 WRITE(6,255)
12 WRITE(6,223)
13 WRITE(6,248)
14 DO 38 J=1,NT
15 38 WRITE(6,251) TIME (J),(ERROR(J,K),K=1,5)
23 END
```


PROGRAM IS BEING ENTERED INTO STORAGE.

TEMPERATURE PROFILES-ONE-DIMENSIONAL
HOMOGENEOUS MEDIUM

GRID POINTS

0.090909 0.181818 0.272727 0.363636 0.454545

INITIAL CONDITION VECTOR

1.0000 1.0000 1.0000 1.0000 1.0000

BOUNDARY CONDITION VECTOR

-0.0000 -0.0000 -0.0000 -0.0000 -0.0000

COEFFICIENT MATRIX

-2.00	1.00	0.00	0.00	0.00
1.00	-2.00	1.00	0.00	0.00
0.00	1.00	-2.00	1.00	0.00
0.00	0.00	1.00	-2.00	1.00
0.00	0.00	0.00	1.00	-1.00

EIGENVALUES

-0.081014 -0.690279 -1.715370 -2.830830 -3.682507

EIGENVECTORS

-0.169891 -0.326019 -0.455734 -0.548529 -0.596885

0.455734 0.596885 0.326019 -0.169891 -0.548529

0.596885 0.169891 -0.548529 -0.326019 0.455734

0.548529 -0.455734 -0.169891 0.596885 -0.326019

-0.326019 0.548529 -0.596885 0.455734 -0.169891

CHECK OF SIMILARITY TRANSFORMATION

-2.000000 1.000000 0.000000 -0.000000 0.000000

-5.00000 1.00000 0.00000 -0.00000
CHECK OF SIMILARITY TRANSFORMATION

-0.525019 0.248259 -0.248259 -0.248259

0.248259 -0.248259 -0.148881 0.248259

0.248259 0.148881 -0.248259 -0.248259

0.422734 0.248259 0.32014 -0.148881

-0.148881 -0.32014 -0.422734 -0.248259

-0.081014 -0.490259 -1.112310 -5.830930

EIGENVALUES

0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00

COEFFICIENT MATRIX

-0.00000 -0.00000 -0.00000 -0.00000

BOUNDARY CONDITIONS

1.0000 1.0000 1.0000 1.0000

INITIAL CONDITIONS

0.00000 0.101014 0.248259 0.248259

CRIT. POINTS

TEMPERATURE MONITORING POINTS

GRAM 12 BEING ENTERED INTO MEMORY

• • CONTD

```
1.000000 -2.000000 1.000000 0.000000 -0.000000
```

0.000000 1.000000 -2.000001 1.000000 0.000000

-0.000000 0.000000 1.000000 -2.000000 1.000000

0.000000 -0.000000 0.000000 1.000000 -1.000000

0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

SEMI-ANALYTICAL SOLUTION

TEMPERATURES AT GRID POINTS

TIME	1	2	3	4	5
0.0	1.000000	1.000000	1.000000	1.000000	1.000000
1.0	0.523777	0.832285	0.954311	0.989934	0.997882
2.0	0.385742	0.682074	0.860607	0.946734	0.978333
4.0	0.276900	0.519418	0.704579	0.824697	0.882513
6.0	0.223908	0.426750	0.591200	0.705680	0.764083
9.0	0.172444	0.330549	0.461395	0.554598	0.603008
12.0	0.134840	0.258710	0.361559	0.435085	0.473380
15.0	0.105697	0.202824	0.283513	0.341229	0.371303
15.0	0.105697	0.202824	0.283513	0.341229	0.371303
20.0	0.070486	0.135262	0.189079	0.227578	0.247640

ANALYTICAL SOLUTION

TEMPERATURES AT GRID POINTS

TIME	1	2	3	4	5
0.0	1.017888	0.991817	1.004558	0.997597	1.000756
1.0	0.520500	0.842701	0.966105	0.995321	0.999571
2.0	0.382924	0.682683	0.866322	0.954034	0.984881
4.0	0.275942	0.519041	0.706479	0.829373	0.889005
6.0	0.223810	0.427097	0.592656	0.708494	0.767826
9.0	0.172593	0.330939	0.462128	0.555690	0.604332
12.0	0.134838	0.258723	0.361610	0.435180	0.473505
15.0	0.105538	0.202522	0.283095	0.340731	0.370765
15.0	0.105538	0.202522	0.283095	0.340731	0.370765
20.0	0.070189	0.134691	0.188281	0.226618	0.246595

ASSOCIATED ERRORS

TIME	1	2	3	4	5
0.0	0.017888	-0.008183	0.004558	-0.002403	0.000756
1.0	-0.003278	0.010415	0.011794	0.005387	0.001689
2.0	-0.002818	0.000609	0.005715	0.007300	0.006548
4.0	-0.000958	-0.000377	0.001899	0.004676	0.006493
6.0	-0.000098	0.000347	0.001456	0.002814	0.003743
9.0	0.000149	0.000390	0.000734	0.001093	0.001324
12.0	-0.000002	0.000014	0.000050	0.000095	0.000126
15.0	-0.000159	-0.000302	-0.000418	-0.000498	-0.000538
15.0	-0.000159	-0.000302	-0.000418	-0.000498	-0.000538
20.0	-0.000298	-0.000571	-0.000798	-0.000960	-0.001045

1111000 112171400-103

		TIME IN THE ZEPHYRUS			TIME
		S	I		
1.000000	0.000000	0.000000	0.000000	0.000000	0.0
0.000000	0.000000	0.000000	0.000000	0.000000	0.1
0.000000	0.000000	0.000000	0.000000	0.000000	0.2
0.000000	0.000000	0.000000	0.000000	0.000000	0.3
0.000000	0.000000	0.000000	0.000000	0.000000	0.4
0.000000	0.000000	0.000000	0.000000	0.000000	0.5
0.000000	0.000000	0.000000	0.000000	0.000000	0.6
0.000000	0.000000	0.000000	0.000000	0.000000	0.7
0.000000	0.000000	0.000000	0.000000	0.000000	0.8
0.000000	0.000000	0.000000	0.000000	0.000000	0.9
0.000000	0.000000	0.000000	0.000000	0.000000	1.0

FOI(b)(7) - D

TIME	1	2	3	4	5
0.0	1.017488	0.991217	1.004577	0.997577	1.000000
1.0	0.952000	0.965701	0.978702	0.990703	0.999704
2.0	0.888274	0.905275	0.920276	0.933277	0.944278
3.0	0.824548	0.843549	0.860550	0.875551	0.888552
4.0	0.760822	0.780823	0.798824	0.814825	0.828826
5.0	0.697096	0.718097	0.735098	0.749099	0.760099
6.0	0.633370	0.655371	0.673372	0.688373	0.699374
7.0	0.569644	0.592645	0.610646	0.625647	0.636648
8.0	0.505918	0.529919	0.547920	0.562921	0.573922
9.0	0.442192	0.467193	0.485194	0.499195	0.509196
10.0	0.378466	0.404467	0.422468	0.437469	0.447470
11.0	0.314740	0.341741	0.360742	0.375743	0.385744
12.0	0.251014	0.279015	0.298016	0.313017	0.323018
13.0	0.187288	0.216289	0.235290	0.250291	0.260292
14.0	0.123562	0.153563	0.172564	0.187565	0.197566
15.0	0.059836	0.090837	0.109838	0.124839	0.134840

0226C14110 248943 0110130230

TIME	1	2	3	4	5
0.0	0.01788	-0.00018	0.00000	-0.00000	0.00000
1.0	-0.00000	0.00000	0.00000	0.00000	0.00000
2.0	-0.00000	0.00000	0.00000	0.00000	0.00000
3.0	-0.00000	0.00000	0.00000	0.00000	0.00000
4.0	-0.00000	0.00000	0.00000	0.00000	0.00000
5.0	-0.00000	0.00000	0.00000	0.00000	0.00000
6.0	-0.00000	0.00000	0.00000	0.00000	0.00000
7.0	-0.00000	0.00000	0.00000	0.00000	0.00000
8.0	-0.00000	0.00000	0.00000	0.00000	0.00000
9.0	-0.00000	0.00000	0.00000	0.00000	0.00000
10.0	-0.00000	0.00000	0.00000	0.00000	0.00000
11.0	-0.00000	0.00000	0.00000	0.00000	0.00000
12.0	-0.00000	0.00000	0.00000	0.00000	0.00000
13.0	-0.00000	0.00000	0.00000	0.00000	0.00000
14.0	-0.00000	0.00000	0.00000	0.00000	0.00000
15.0	-0.00000	0.00000	0.00000	0.00000	0.00000

SOURCE STATEMENT

```
0 $IBFTC DARS I   NODECK
1     REAL TANAL(15,10),TIME(15),TI(10),TEVR(15,10),X(10),VAR(10)
2     REAL S(10,10),R(10),V(10,10),A(10),B(10),W1(10),W2(10)
3     REAL ERROR(15,10),C1(10) ,W(10,10)
4     1 FORMAT(1X,2I4)
5     2 FORMAT(1X,F13.8,4F14.8)
6     3 FORMAT(1X,7F10.6)
7     4 FORMAT(1X,3F14.8)
10    5 FORMAT(1X,25F3.0)
11    220 FORMAT(1H2)
12    221 FORMAT(1H ,10X,8H ..CONTD)
13    222 FORMAT(1H ,//)
14    223 FORMAT(1H ,/)
15    224 FORMAT(1H ,18X,26H TEMPERATURE PROFILES-ONE-,
111HDIMENSIONAL/25X,22H   HOMOGENEOUS MEDIUM)
16    231 FORMAT(1H ,30X,12H GRID POINTS)
17    232 FORMAT(1H ,10X,5F11.6)
20    233 FORMAT(1H ,28X,25H INITIAL CONDITION VECTOR)
21    234 FORMAT(1H ,10X,5F12.4)
22    235 FORMAT(1H ,28X,26H BOUNDARY CONDITION VECTOR)
23    240 FORMAT(1H ,30X,19H COEFFICIENT MATRIX)
24    241 FORMAT(1H ,10X,10F6.1)
25    242 FORMAT(1H ,31X,12H EIGENVALUES)
26    243 FORMAT(1H ,31X,13H EIGENVECTORS)
27    244 FORMAT(1H ,20X,35H CHECK OF SIMILARITY TRANSFORMATION)
30    245 FORMAT(1H ,30X,25H SEMI-ANALYTICAL SOLUTION)
31    246 FORMAT(1H ,25X,28H TEMPERATURES AT GRID POINTS)
32    247 FORMAT(1H ,30X,19HANALYTICAL SOLUTION)
33    248 FORMAT(1H ,15X,5H TIME,5X,2H 1, 9X,2H 2, 8X,2H 3, 8X,
12H 4, 8X,2H 5)
34    249 FORMAT(1H ,15X,5H TIME, 9X,2H 6, 8X,2H 7, 8X,2H 8, 8X,
12H 9, 8X,2H10)
35    251 FORMAT(1H ,15X,F6.1,2X,5F10.6)
36    255 FORMAT(1H ,30X,18H ASSOCIATED ERRORS)
37    READ(5,1) N,NT
42    READ(5,5) ((S(J,K),K=1,N),J=1,N)
53    READ (5,2) (X(J),J=1,N )
60    READ(5,2) (TI(J),J=1,N )
65    READ(5,2) (C1(J),J=1,N)
72    READ (5,3) (TIME(J),J=1,NT)
77    READ (5,4) ALPHA,RM,DX
00    MAXN=10
01    M=N
02    CALL EIG1(N,MAXN,M,S,R,V,A,B,W1,W2)
03    CALL CHECK(V,R,N,W)
04    PI=3.1415927
05    DO 10 K=1,N
06    DO 10 J=1,NT
07    TANAL(J,K)=0.0
10    DO 9 L=1,60
11    AL=L
12    AA=(2.*AL-1.)*PI*X(K)
13    AB=-(2.*AL-1.)*2*(PI/2.)*2*ALPHA*TIME(J)/RM
14    9 TANAL(J,K)=TANAL(J,K)+SIN(AA)*EXP(AB)/(2.*AL-1.)
16    10 TANAL(J,K)=4.*TANAL(J,K)/PI
```

Table 1: Summary of data

Year	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030	2031	2032	2033	2034	2035	2036	2037	2038	2039	2040	2041	2042	2043	2044	2045	2046	2047	2048	2049	2050	2051	2052	2053	2054	2055	2056	2057	2058	2059	2060	2061	2062	2063	2064	2065	2066	2067	2068	2069	2070	2071	2072	2073	2074	2075	2076	2077	2078	2079	2080	2081	2082	2083	2084	2085	2086	2087	2088	2089	2090	2091	2092	2093	2094	2095	2096	2097	2098	2099	2100																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																	
Value	1.2	1.5	1.8	2.1	2.4	2.7	3.0	3.3	3.6	3.9	4.2	4.5	4.8	5.1	5.4	5.7	6.0	6.3	6.6	6.9	7.2	7.5	7.8	8.1	8.4	8.7	9.0	9.3	9.6	9.9	10.2	10.5	10.8	11.1	11.4	11.7	12.0	12.3	12.6	12.9	13.2	13.5	13.8	14.1	14.4	14.7	15.0	15.3	15.6	15.9	16.2	16.5	16.8	17.1	17.4	17.7	18.0	18.3	18.6	18.9	19.2	19.5	19.8	20.1	20.4	20.7	21.0	21.3	21.6	21.9	22.2	22.5	22.8	23.1	23.4	23.7	24.0	24.3	24.6	24.9	25.2	25.5	25.8	26.1	26.4	26.7	27.0	27.3	27.6	27.9	28.2	28.5	28.8	29.1	29.4	29.7	30.0	30.3	30.6	30.9	31.2	31.5	31.8	32.1	32.4	32.7	33.0	33.3	33.6	33.9	34.2	34.5	34.8	35.1	35.4	35.7	36.0	36.3	36.6	36.9	37.2	37.5	37.8	38.1	38.4	38.7	39.0	39.3	39.6	39.9	40.2	40.5	40.8	41.1	41.4	41.7	42.0	42.3	42.6	42.9	43.2	43.5	43.8	44.1	44.4	44.7	45.0	45.3	45.6	45.9	46.2	46.5	46.8	47.1	47.4	47.7	48.0	48.3	48.6	48.9	49.2	49.5	49.8	50.1	50.4	50.7	51.0	51.3	51.6	51.9	52.2	52.5	52.8	53.1	53.4	53.7	54.0	54.3	54.6	54.9	55.2	55.5	55.8	56.1	56.4	56.7	57.0	57.3	57.6	57.9	58.2	58.5	58.8	59.1	59.4	59.7	60.0	60.3	60.6	60.9	61.2	61.5	61.8	62.1	62.4	62.7	63.0	63.3	63.6	63.9	64.2	64.5	64.8	65.1	65.4	65.7	66.0	66.3	66.6	66.9	67.2	67.5	67.8	68.1	68.4	68.7	69.0	69.3	69.6	69.9	70.2	70.5	70.8	71.1	71.4	71.7	72.0	72.3	72.6	72.9	73.2	73.5	73.8	74.1	74.4	74.7	75.0	75.3	75.6	75.9	76.2	76.5	76.8	77.1	77.4	77.7	78.0	78.3	78.6	78.9	79.2	79.5	79.8	80.1	80.4	80.7	81.0	81.3	81.6	81.9	82.2	82.5	82.8	83.1	83.4	83.7	84.0	84.3	84.6	84.9	85.2	85.5	85.8	86.1	86.4	86.7	87.0	87.3	87.6	87.9	88.2	88.5	88.8	89.1	89.4	89.7	90.0	90.3	90.6	90.9	91.2	91.5	91.8	92.1	92.4	92.7	93.0	93.3	93.6	93.9	94.2	94.5	94.8	95.1	95.4	95.7	96.0	96.3	96.6	96.9	97.2	97.5	97.8	98.1	98.4	98.7	99.0	99.3	99.6	99.9	100.2	100.5	100.8	101.1	101.4	101.7	102.0	102.3	102.6	102.9	103.2	103.5	103.8	104.1	104.4	104.7	105.0	105.3	105.6	105.9	106.2	106.5	106.8	107.1	107.4	107.7	108.0	108.3	108.6	108.9	109.2	109.5	109.8	110.1	110.4	110.7	111.0	111.3	111.6	111.9	112.2	112.5	112.8	113.1	113.4	113.7	114.0	114.3	114.6	114.9	115.2	115.5	115.8	116.1	116.4	116.7	117.0	117.3	117.6	117.9	118.2	118.5	118.8	119.1	119.4	119.7	120.0	120.3	120.6	120.9	121.2	121.5	121.8	122.1	122.4	122.7	123.0	123.3	123.6	123.9	124.2	124.5	124.8	125.1	125.4	125.7	126.0	126.3	126.6	126.9	127.2	127.5	127.8	128.1	128.4	128.7	129.0	129.3	129.6	129.9	130.2	130.5	130.8	131.1	131.4	131.7	132.0	132.3	132.6	132.9	133.2	133.5	133.8	134.1	134.4	134.7	135.0	135.3	135.6	135.9	136.2	136.5	136.8	137.1	137.4	137.7	138.0	138.3	138.6	138.9	139.2	139.5	139.8	140.1	140.4	140.7	141.0	141.3	141.6	141.9	142.2	142.5	142.8	143.1	143.4	143.7	144.0	144.3	144.6	144.9	145.2	145.5	145.8	146.1	146.4	146.7	147.0	147.3	147.6	147.9	148.2	148.5	148.8	149.1	149.4	149.7	150.0	150.3	150.6	150.9	151.2	151.5	151.8	152.1	152.4	152.7	153.0	153.3	153.6	153.9	154.2	154.5	154.8	155.1	155.4	155.7	156.0	156.3	156.6	156.9	157.2	157.5	157.8	158.1	158.4	158.7	159.0	159.3	159.6	159.9	160.2	160.5	160.8	161.1	161.4	161.7	162.0	162.3	162.6	162.9	163.2	163.5	163.8	164.1	164.4	164.7	165.0	165.3	165.6	165.9	166.2	166.5	166.8	167.1	167.4	167.7	168.0	168.3	168.6	168.9	169.2	169.5	169.8	170.1	170.4	170.7	171.0	171.3	171.6	171.9	172.2	172.5	172.8	173.1	173.4	173.7	174.0	174.3	174.6	174.9	175.2	175.5	175.8	176.1	176.4	176.7	177.0	177.3	177.6	177.9	178.2	178.5	178.8	179.1	179.4	179.7	180.0	180.3	180.6	180.9	181.2	181.5	181.8	182.1	182.4	182.7	183.0	183.3	183.6	183.9	184.2	184.5	184.8	185.1	185.4	185.7	186.0	186.3	186.6	186.9	187.2	187.5	187.8	188.1	188.4	188.7	189.0	189.3	189.6	189.9	190.2	190.5	190.8	191.1	191.4	191.7	192.0	192.3	192.6	192.9	193.2	193.5	193.8	194.1	194.4	194.7	195.0	195.3	195.6	195.9	196.2	196.5	196.8	197.1	197.4	197.7	198.0	198.3	198.6	198.9	199.2	199.5	199.8	200.1	200.4	200.7	201.0	201.3	201.6	201.9	202.2	202.5	202.8	203.1	203.4	203.7	204.0	204.3	204.6	204.9	205.2	205.5	205.8	206.1	206.4	206.7	207.0	207.3	207.6	207.9	208.2	208.5	208.8	209.1	209.4	209.7	210.0	210.3	210.6	210.9	211.2	211.5	211.8	212.1	212.4	212.7	213.0	213.3	213.6	213.9	214.2	214.5	214.8	215.1	215.4	215.7	216.0	216.3	216.6	216.9	217.2	217.5	217.8	218.1	218.4	218.7	219.0	219.3	219.6	219.9	220.2	220.5	220.8	221.1	221.4	221.7	222.0	222.3	222.6	222.9	223.2	223.5	223.8	224.1	224.4	224.7	225.0	225.3	225.6	225.9	226.2	226.5	226.8	227.1	227.4	227.7	228.0	228.3	228.6	228.9	229.2	229.5	229.8	230.1	230.4	230.7	231.0	231.3	231.6	231.9	232.2	232.5	232.8	233.1	233.4	233.7	234.0	234.3	234.6	234.9	235.2	235.5	235.8	236.1	236.4	236.7	237.0	237.3	237.6	237.9	238.2	238.5	238.8	239.1	239.4	239.7	240.0	240.3	240.6	240.9	241.2	241.5	241.8	242.1	242.4	242.7	243.0	243.3	243.6	243.9	244.2	244.5	244.8	245.1	245.4	245.7	246.0	246.3	246.6	246.9	247.2	247.5	247.8	248.1	248.4	248.7	249.0	249.3	249.6	249.9	250.2	250.5	250.8	251.1	251.4	251.7	252.0	252.3	252.6	252.9	253.2	253.5	253.8	254.1	254.4	254.7	255.0	255.3	255.6	255.9	256.2	256.5	256.8	257.1	257.4	257.7	258.0	258.3	258.6	258.9	259.2	259.5	259.8	260.1	260.4	260.7	261.0	261.3	261.6	261.9	262.2	262.5	262.8	263.1	263.4	263.7	264.0	264.3	264.6	264.9	265.2	265.5	265.8	266.1	266.4	266.7	267.0	267.3	267.6	267.9	268.2	268.5	268.8	269.1	269.4	269.7	270.0	270.3	270.6	270.9	271.2	271.5	271.8	272.1	272.4	272.7	273.0	273.3	273.6	273.9	274.2	274.5	274.8	275.1	275.4	275.7	276.0	276.3	276.6	276.9	277.2	277.5	277.8	278.1	278.4	278.7	279.0	279.3	279.6	279.9	280.2	280.5	280.8	281.1	281.4	281.7	282.0	282.3	282.6	282.9	283.2	283.5	283.8	284.1	284.4	284.7	285.0	285.3	285.6	285.9	286.2	286.5	286.8	287.1	287.4	287.7	288.0	288.3	288.6	288.9	289.2	289.5	289.8	290.1	290.4	290.7	291.0	291.3	291.6	291.9	292.2	292.5	292.8	293.1	293.4	293.7	294.0	294.3	294.6	294.9	295.2	295.5	295.8	296.1	296.4	296.7	297.0	297.3	297.6	297.9	298.2	298.5	298.8	299.1	299.4	299.7	300.0	300.3	300.6	300.9	301.2	301.5	301.8	302.1	302.4	302.7	303.0	303.3	303.6	303.9	304.2	304.5	304.8	305.1	305.4	305.7	306.0	306.3	306.6	306.9	307.2	307.5	307.8	308.1	308.4	308.7	309.0	309.3	309.6	309.9	310.2	310.5	310.8	311.1	311.4	311.7	312.0	312.3	312.6	312.9	313.2	313.5	313.8	314.1	314.4	314.7	315.0	315.3	315.6	315.9	316.2	316.5	316.8	317.1	317.4	317.7	318.0	318.3	318.6	318.9	319.2	319.5	319.8	320.1	320.4	320.7	321.0	321.3	321.6	321.9	322.2	322.5	322.8	323.1	323.4	323.7	324.0	324.3	324.6	324.9	325.2	325.5	325.8	326.1	326.4	326.7	327.0	327.3	327.6	327.9	328.2	328.5	328.8	329.1	329.4	329.7	330.0	330.3	330.6	330.9	331.2	331.5	331.8	332.1	332.4	332.7	333.0	333.3	333.6	333.9	334.2	334.5	334.8	335.1	335.4	335.7	336.0	336.3	336.6	336.9	337.2	337.5	337.8	338.1	338.4	338.7	339.0	339.3	339.6	339.9	340.2	340.5	340.8	341.1	341.4	341.7	342.0	342.3	342.6	342.9	343.2	343.5	343.8	344.1	344.4	344.7	345.0	345.3	345.6	345.9	346.2	346.5	346.8	347.1	347.4	347.7	348.0	348.3	348.6	348.9	349.2	349.5	349.8	350.1	350.4	350.7	351.0	351.3	351.6	351.9	352.2	352.5	352.8	353.1	353.4	353.7	354.0	354.3	354.6	354.9


```
SN      SOURCE STATEMENT

21      DO 15 K=1,N
22      VAR(K)=0.0
23      DO 15 J=1,N
24      VAR(K)=VAR(K)+V(J,K)*TI(J)
25      15 CONTINUE
30      DO 25 L=1,NT
31      DO 25 K=1,N
32      TEVR(L,K)=0.0
33      DO 25 J=1,N
34      CONST=ALPHA*R(J)*TIME(L)/(DX**2)
35      TEVR(L,K)=TEVR(L,K)+V(K,J)*VAR(J)*EXP(CONST)
36      25 CONTINUE
42      DO 26 J=1,NT
43      DO 26 K=1,N
44      26 ERROR(J,K)=TANAL(J,K)-TEVR(J,K)
47      WRITE(6,220)
50      LINES=9
51      CALL LINECT(LINES,4,2)
52      WRITE(6,224)
53      WRITE(6,223)
54      WRITE(6,231)
55      CALL LINECT(LINES,2,2)
56      WRITE(6,232) (X(J),J=1,N)
63      CALL LINECT(LINES,4,2)
64      WRITE(6,223)
65      WRITE(6,233)
66      CALL LINECT(LINES,2,2)
67      WRITE(6,234) (TI(J),J=1,N)
74      CALL LINECT(LINES,4,2)
75      WRITE(6,223)
76      WRITE(6,235)
77      CALL LINECT(LINES,2,2)
80      WRITE(6,234) (C1(J),J=1,N)
85      CALL LINECT(LINES,4,2)
86      WRITE(6,223)
87      WRITE(6,240)
90      DO 31 J=1,N
91      CALL LINECT(LINES,1,2)
92      31 WRITE(6,241) (S(J,K),K=1,N)
93      CALL LINECT(LINES,3,2)
94      WRITE(6,223)
95      WRITE(6,242)
96      CALL LINECT(LINES,2,2)
97      WRITE(6,232) (R(J),J=1,N)
98      CALL LINECT(LINES,3,2)
99      WRITE(6,223)
100     WRITE(6,243)
101     DO 32 K=1,N
102     CALL LINECT(LINES,4,2)
103     WRITE(6,232) (V(J,K),J=1,N)
104     32 WRITE(6,223)
105     CALL LINECT(LINES,1,2)
106     WRITE(6,244)
107     DO 33 J=1,N
108     CALL LINECT(LINES,4,2)
```


SOURCE STATEMENT

```
251 WRITE(6,232) (W(J,K),K=1,N)
256 33 WRITE(6,223)
260 WRITE(6,220)
261 WRITE(6,245)
262 WRITE(6,223)
263 WRITE(6,246)
264 WRITE(6,248)
265 DO 34 J=1,NT
266 34 WRITE(6,251) TIME(J),(TEVR(J,K),K=1,5)
274 WRITE(6,223)
275 WRITE(6,246)
276 WRITE(6,249)
277 DO 35 J=1,NT
300 35 WRITE(6,251) TIME(J),(TEVR(J,K),K=6,10)
306 WRITE(6,220)
307 WRITE(6,247)
310 WRITE(6,223)
311 WRITE(6,246)
312 WRITE(6,248)
313 DO 36 J=1,NT
314 36 WRITE(6,251) TIME(J),(TANAL(J,K),K=1,5)
322 WRITE(6,223)
323 WRITE(6,246)
324 WRITE(6,249)
325 DO 37 J=1,NT
326 37 WRITE(6,251) TIME(J),(TANAL(J,K),K=6,10)
334 WRITE(6,220)
335 WRITE(6,255)
336 WRITE(6,223)
337 WRITE(6,248)
340 DO 38 J=1,NT
341 38 WRITE(6,251) TIME(J),(ERROR(J,K),K=1,5)
347 WRITE(6,223)
350 WRITE(6,249)
351 DO 39 J=1,NT
352 39 WRITE(6,251) TIME(J),(ERROR(J,K),K=6,10)
360 END
```

WRITE(0,521) (1,1) 1
WRITE(0,521) (1,1) 2
WRITE(0,521) (1,1) 3
WRITE(0,521) (1,1) 4
WRITE(0,521) (1,1) 5
WRITE(0,521) (1,1) 6
WRITE(0,521) (1,1) 7
WRITE(0,521) (1,1) 8
WRITE(0,521) (1,1) 9
WRITE(0,521) (1,1) 10
WRITE(0,521) (1,1) 11
WRITE(0,521) (1,1) 12
WRITE(0,521) (1,1) 13
WRITE(0,521) (1,1) 14
WRITE(0,521) (1,1) 15
WRITE(0,521) (1,1) 16
WRITE(0,521) (1,1) 17
WRITE(0,521) (1,1) 18
WRITE(0,521) (1,1) 19
WRITE(0,521) (1,1) 20
WRITE(0,521) (1,1) 21
WRITE(0,521) (1,1) 22
WRITE(0,521) (1,1) 23
WRITE(0,521) (1,1) 24
WRITE(0,521) (1,1) 25
WRITE(0,521) (1,1) 26
WRITE(0,521) (1,1) 27
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WRITE(0,521) (1,1) 30
WRITE(0,521) (1,1) 31
WRITE(0,521) (1,1) 32
WRITE(0,521) (1,1) 33
WRITE(0,521) (1,1) 34
WRITE(0,521) (1,1) 35
WRITE(0,521) (1,1) 36
WRITE(0,521) (1,1) 37
WRITE(0,521) (1,1) 38
WRITE(0,521) (1,1) 39
WRITE(0,521) (1,1) 40
WRITE(0,521) (1,1) 41
WRITE(0,521) (1,1) 42
WRITE(0,521) (1,1) 43
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WRITE(0,521) (1,1) 53
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WRITE(0,521) (1,1) 57
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WRITE(0,521) (1,1) 59
WRITE(0,521) (1,1) 60
WRITE(0,521) (1,1) 61
WRITE(0,521) (1,1) 62
WRITE(0,521) (1,1) 63
WRITE(0,521) (1,1) 64
WRITE(0,521) (1,1) 65
WRITE(0,521) (1,1) 66
WRITE(0,521) (1,1) 67
WRITE(0,521) (1,1) 68
WRITE(0,521) (1,1) 69
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WRITE(0,521) (1,1) 71
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WRITE(0,521) (1,1) 81
WRITE(0,521) (1,1) 82
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WRITE(0,521) (1,1) 84
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WRITE(0,521) (1,1) 86
WRITE(0,521) (1,1) 87
WRITE(0,521) (1,1) 88
WRITE(0,521) (1,1) 89
WRITE(0,521) (1,1) 90
WRITE(0,521) (1,1) 91
WRITE(0,521) (1,1) 92
WRITE(0,521) (1,1) 93
WRITE(0,521) (1,1) 94
WRITE(0,521) (1,1) 95
WRITE(0,521) (1,1) 96
WRITE(0,521) (1,1) 97
WRITE(0,521) (1,1) 98
WRITE(0,521) (1,1) 99
WRITE(0,521) (1,1) 100

PROGRAM IS BEING ENTERED INTO STORAGE.

TEMPERATURE PROFILES-ONE-DIMENSIONAL HOMOGENEOUS MEDIUM

GRID POINTS

0.047619	0.095238	0.142857	0.190476	0.238095
0.285714	0.333333	0.380952	0.428571	0.476190

INITIAL CONDITION VECTOR

1.0000	1.0000	1.0000	1.0000	1.0000
1.0000	1.0000	1.0000	1.0000	1.0000

BOUNDARY CONDITION VECTOR

-0.0000	-0.0000	-0.0000	-0.0000	-0.0000
-0.0000	-0.0000	-0.0000	-0.0000	-0.0000

COEFFICIENT MATRIX

-2.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	-2.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	1.0	-2.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	1.0	-2.0	1.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	1.0	-2.0	1.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	1.0	-2.0	1.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	1.0	-2.0	1.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	1.0	-2.0	1.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	-2.0	1.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	-1.0

EIGENVALUES

-0.022338	-0.198062	-0.533896	-1.000000	-1.554958
-2.149460	-2.730682	-3.246980	-3.652478	-3.911146

EIGENVECTORS

-0.065047	-0.128642	-0.189362	-0.245853	-0.296852
-0.341219	-0.377964	-0.406267	-0.425494	-0.435216

0.189362	0.341219	0.425493	0.425493	0.341219
0.189362	-0.000000	-0.189362	-0.341219	-0.425494

0.296852	0.435215	0.341219	0.065047	-0.245853
-0.425493	-0.377964	-0.128642	0.189362	0.406267

TEMPERATURE PROFILE-1111-11111111
TEMPERATURE PROFILE

0.58214	0.37344	0.38025	0.14284	0.14284	0.14284
0.047019	0.04538	0.04538	0.04538	0.04538	0.04538

1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000
-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000

0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

-5.14460	-5.13085	-3.94080	-3.92548	-1.00000	-1.00000
-0.055338	-0.13005	-0.23308	-0.23308	-1.00000	-1.00000

-0.341519	-0.341519	-0.40844	-0.40844	-0.40844	-0.40844
-0.062047	-0.15064	-0.18035	-0.18035	-0.18035	-0.18035

0.183385	-0.00000	-0.18035	-0.34151	0.40844	0.40844
0.183385	0.34151	0.40844	0.40844	0.40844	0.40844

-0.452493	-0.341519	-0.18035	-0.18035	-0.18035	-0.18035
0.500925	0.40844	0.40844	0.40844	0.40844	0.40844

..CONTD

0.377964	0.377964	-0.000000	-0.377965	-0.377965
-0.000000	0.377964	0.377964	0.000000	-0.377964

0.425493	0.189362	-0.341219	-0.341219	0.189362
0.425493	0.000000	-0.425493	-0.189362	0.341219

-0.435215	0.065048	0.425493	-0.128642	-0.406267
0.189363	0.377964	-0.245853	-0.341219	0.296852

0.406267	-0.296852	-0.189362	0.435215	-0.128642
-0.341219	0.377965	0.065047	-0.425493	0.245853

-0.341219	0.425493	-0.189362	-0.189362	0.425493
-0.341219	0.000000	0.341219	-0.425493	0.189362

-0.245853	0.406267	-0.425493	0.296852	-0.065047
-0.189363	0.377965	-0.435216	0.341219	-0.128642

0.128642	-0.245853	0.341219	-0.406267	0.435215
-0.425493	0.377964	-0.296852	0.189362	-0.065047

CHECK OF SIMILARITY TRANSFORMATION

-1.999999	1.000000	-0.000000	-0.000000	0.000000
0.000000	-0.000000	0.000000	-0.000000	0.000000

1.000000	-2.000000	1.000000	0.000000	-0.000000
0.000000	-0.000000	0.000000	0.000000	-0.000000

-0.000000	1.000000	-2.000000	1.000000	0.000000
-0.000001	0.000001	-0.000000	-0.000000	0.000000

-0.000000	0.000000	1.000000	-2.000000	1.000000
0.000000	0.000000	-0.000000	0.000000	0.000000

0.000000	-0.000000	0.000000	1.000000	-2.000000
1.000000	-0.000000	0.000000	-0.000000	0.000000

0.000000	0.000000	-0.000001	0.000000	1.000000
-2.000000	1.000000	-0.000000	0.000000	-0.000000

3TV03..

[illegible]

CHICK OF STIMULATING THE SPONTANEOUS

.1CONTD

-0.000000	-0.000000	0.000001	0.000000	-0.000000
1.000000	-2.000000	1.000001	-0.000000	-0.000000

Continuation of data set

0.000000	0.000000	-0.000000	-0.000000	0.000000
-0.000000	1.000001	-2.000001	1.000000	0.000000

1.00	0.000000	0.000000	0.000000	0.000000	0.000000
2.00	0.000000	0.000000	0.000000	0.000000	0.000000

-0.000000	0.000000	-0.000000	-0.000000	-0.000000
0.000000	-0.000000	1.000000	-2.000000	1.000000

3.00	0.000000	0.000000	0.000000	0.000000	0.000000
4.00	0.000000	0.000000	0.000000	0.000000	0.000000

0.000000	-0.000000	0.000000	0.000000	0.000000
-0.000000	-0.000000	0.000000	1.000000	-1.000000

5.00	0.000000	0.000000	0.000000	0.000000	0.000000
6.00	0.000000	0.000000	0.000000	0.000000	0.000000
7.00	0.000000	0.000000	0.000000	0.000000	0.000000
8.00	0.000000	0.000000	0.000000	0.000000	0.000000
9.00	0.000000	0.000000	0.000000	0.000000	0.000000
10.00	0.000000	0.000000	0.000000	0.000000	0.000000

Continuation of data set

Time	1	2	3	4	5
1.00	0.000000	0.000000	0.000000	0.000000	0.000000
2.00	0.000000	0.000000	0.000000	0.000000	0.000000
3.00	0.000000	0.000000	0.000000	0.000000	0.000000
4.00	0.000000	0.000000	0.000000	0.000000	0.000000
5.00	0.000000	0.000000	0.000000	0.000000	0.000000
6.00	0.000000	0.000000	0.000000	0.000000	0.000000
7.00	0.000000	0.000000	0.000000	0.000000	0.000000
8.00	0.000000	0.000000	0.000000	0.000000	0.000000
9.00	0.000000	0.000000	0.000000	0.000000	0.000000
10.00	0.000000	0.000000	0.000000	0.000000	0.000000
11.00	0.000000	0.000000	0.000000	0.000000	0.000000
12.00	0.000000	0.000000	0.000000	0.000000	0.000000
13.00	0.000000	0.000000	0.000000	0.000000	0.000000
14.00	0.000000	0.000000	0.000000	0.000000	0.000000
15.00	0.000000	0.000000	0.000000	0.000000	0.000000
16.00	0.000000	0.000000	0.000000	0.000000	0.000000
17.00	0.000000	0.000000	0.000000	0.000000	0.000000
18.00	0.000000	0.000000	0.000000	0.000000	0.000000
19.00	0.000000	0.000000	0.000000	0.000000	0.000000
20.00	0.000000	0.000000	0.000000	0.000000	0.000000

SEMI-ANALYTICAL SOLUTION

TIME	TEMPERATURES AT GRID POINTS				
	1	2	3	4	5
0.0	0.999999	0.999999	0.999999	0.999999	0.999999
1.0	0.523777	0.832285	0.954315	0.989972	0.998167
2.0	0.385752	0.682129	0.860880	0.947945	0.983129
4.0	0.277574	0.521612	0.710703	0.840398	0.919592
6.0	0.227890	0.437203	0.613899	0.751169	0.849484
9.0	0.186738	0.363244	0.520963	0.654225	0.760715
12.0	0.161958	0.317204	0.459836	0.585398	0.691216
15.0	0.144818	0.284742	0.415327	0.532937	0.634972
18.0	0.131835	0.259820	0.380370	0.490394	0.587440
20.0	0.124617	0.245842	0.360481	0.465704	0.559163
25.0	0.109642	0.216609	0.318332	0.412438	0.496826
30.0	0.097385	0.192511	0.283186	0.367347	0.443115
40.0	0.077575	0.153405	0.225787	0.293100	0.353840
50.0	0.062001	0.122616	0.180489	0.234326	0.282926
70.0	0.039656	0.078426	0.115445	0.149884	0.180976

TIME	TEMPERATURES AT GRID POINTS				
	6	7	8	9	10
0.0	1.000000	0.999999	1.000000	1.000000	1.000001
1.0	0.999714	0.999961	0.999995	1.000000	1.000001
2.0	0.995203	0.998789	0.999726	0.999944	0.999988
4.0	0.962918	0.984292	0.993853	0.997719	0.999026
6.0	0.914534	0.954362	0.976892	0.988450	0.993215
9.0	0.841158	0.898451	0.936537	0.959336	0.969908
12.0	0.776388	0.841461	0.887915	0.917547	0.931917
15.0	0.719908	0.787172	0.836888	0.869552	0.885712
18.0	0.669747	0.736200	0.786215	0.819570	0.836236
20.0	0.639041	0.704042	0.753317	0.786372	0.802950
25.0	0.569710	0.629643	0.675510	0.706513	0.722140
30.0	0.508834	0.563102	0.604792	0.633055	0.647328
40.0	0.406655	0.450374	0.484032	0.506890	0.518445
50.0	0.325202	0.360213	0.387177	0.405493	0.414755
70.0	0.208024	0.230426	0.247680	0.259401	0.265328

[illegible]

TIME	U	V	W	X	Y
0.0	1.00000.1	0.00000.0	0.00000.1	0.00000.1	0.00000.1
1.0	0.99971.4	0.00000.0	0.00000.0	0.00000.0	0.00000.1
2.0	0.99928.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
3.0	0.99879.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
4.0	0.99824.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
5.0	0.99763.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
6.0	0.99696.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
7.0	0.99623.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
8.0	0.99544.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
9.0	0.99459.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
10.0	0.99368.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
11.0	0.99271.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
12.0	0.99168.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
13.0	0.99059.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
14.0	0.98944.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
15.0	0.98823.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
16.0	0.98696.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
17.0	0.98563.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
18.0	0.98424.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
19.0	0.98279.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
20.0	0.98128.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
21.0	0.97971.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
22.0	0.97808.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
23.0	0.97639.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
24.0	0.97464.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
25.0	0.97283.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
26.0	0.97096.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
27.0	0.96903.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
28.0	0.96704.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
29.0	0.96500.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
30.0	0.96291.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
31.0	0.96076.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
32.0	0.95856.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
33.0	0.95631.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
34.0	0.95400.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
35.0	0.95164.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
36.0	0.94923.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
37.0	0.94676.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
38.0	0.94424.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
39.0	0.94167.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
40.0	0.93905.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
41.0	0.93638.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
42.0	0.93366.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
43.0	0.93089.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
44.0	0.92807.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
45.0	0.92520.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
46.0	0.92228.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
47.0	0.91931.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
48.0	0.91628.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
49.0	0.91320.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1
50.0	0.91007.0	0.00000.0	0.00000.0	0.00000.0	0.00000.1

ANALYTICAL SOLUTION

TIME	TEMPERATURES AT GRID POINTS				
	1	2	3	4	5
0.0	0.979451	1.004470	1.011100	1.008432	1.001667
1.0	0.520500	0.842701	0.966105	0.995322	0.999593
2.0	0.382925	0.682689	0.866386	0.954500	0.987581
4.0	0.276326	0.520500	0.711156	0.842701	0.922900
6.0	0.227170	0.436297	0.613524	0.751786	0.851081
9.0	0.186334	0.362641	0.520478	0.654160	0.761245
12.0	0.161706	0.316806	0.459471	0.585264	0.691475
15.0	0.144666	0.284504	0.415114	0.532885	0.635205
18.0	0.131755	0.259702	0.380288	0.490439	0.587697
20.0	0.124572	0.245783	0.360465	0.465798	0.559432
25.0	0.109648	0.216636	0.318406	0.412590	0.497084
30.0	0.097407	0.192560	0.283274	0.367487	0.443319
40.0	0.077583	0.153422	0.225816	0.293142	0.353897
50.0	0.061987	0.122587	0.180447	0.234272	0.282862
70.0	0.039614	0.078344	0.115323	0.149727	0.180785

TIME	TEMPERATURES AT GRID POINTS				
	6	7	8	9	10
0.0	0.995734	0.993874	0.996461	1.001220	1.004794
1.0	0.999978	0.999999	1.000000	1.000000	1.000000
2.0	0.997300	0.999535	0.999937	0.999993	0.999999
4.0	0.966105	0.986671	0.995318	0.998515	0.999492
6.0	0.916721	0.956639	0.978904	0.990093	0.994611
9.0	0.842294	0.900072	0.938470	0.961427	0.972056
12.0	0.777129	0.842691	0.889566	0.919501	0.934028
15.0	0.720509	0.788171	0.838251	0.871192	0.887501
18.0	0.670277	0.737027	0.787319	0.820886	0.837667
20.0	0.639532	0.704771	0.754269	0.787494	0.804165
25.0	0.570096	0.630162	0.676152	0.707247	0.722924
30.0	0.509109	0.563449	0.605203	0.633515	0.647813
40.0	0.406728	0.450463	0.484136	0.507004	0.518565
50.0	0.325130	0.360134	0.387093	0.405407	0.414667
70.0	0.207805	0.230183	0.247419	0.259128	0.265049

ANALYSTS

[illegible]

TIME	6	7	8	9	10
0.0	0.00000	0.00000	0.00000	0.00000	0.00000
1.0	0.00000	0.00000	0.00000	0.00000	0.00000
2.0	0.00000	0.00000	0.00000	0.00000	0.00000
3.0	0.00000	0.00000	0.00000	0.00000	0.00000
4.0	0.00000	0.00000	0.00000	0.00000	0.00000
5.0	0.00000	0.00000	0.00000	0.00000	0.00000
6.0	0.00000	0.00000	0.00000	0.00000	0.00000
7.0	0.00000	0.00000	0.00000	0.00000	0.00000
8.0	0.00000	0.00000	0.00000	0.00000	0.00000
9.0	0.00000	0.00000	0.00000	0.00000	0.00000
10.0	0.00000	0.00000	0.00000	0.00000	0.00000
11.0	0.00000	0.00000	0.00000	0.00000	0.00000
12.0	0.00000	0.00000	0.00000	0.00000	0.00000
13.0	0.00000	0.00000	0.00000	0.00000	0.00000
14.0	0.00000	0.00000	0.00000	0.00000	0.00000
15.0	0.00000	0.00000	0.00000	0.00000	0.00000
16.0	0.00000	0.00000	0.00000	0.00000	0.00000
17.0	0.00000	0.00000	0.00000	0.00000	0.00000
18.0	0.00000	0.00000	0.00000	0.00000	0.00000
19.0	0.00000	0.00000	0.00000	0.00000	0.00000
20.0	0.00000	0.00000	0.00000	0.00000	0.00000
21.0	0.00000	0.00000	0.00000	0.00000	0.00000

ASSOCIATED ERRORS

TIME	1	2	3	4	5
0.0	-0.020549	0.004472	0.011101	0.008432	0.001667
1.0	-0.003277	0.010416	0.011790	0.005350	0.001426
2.0	-0.002828	0.000560	0.005505	0.006555	0.004452
4.0	-0.001248	-0.001113	0.000453	0.002303	0.003308
6.0	-0.000720	-0.000906	-0.000375	0.000617	0.001597
9.0	-0.000404	-0.000604	-0.000485	-0.000065	0.000530
12.0	-0.000252	-0.000398	-0.000365	-0.000134	0.000259
15.0	-0.000152	-0.000239	-0.000212	-0.000052	0.000232
18.0	-0.000080	-0.000118	-0.000081	0.000045	0.000257
20.0	-0.000045	-0.000058	-0.000016	0.000094	0.000269
25.0	0.000006	0.000027	0.000074	0.000152	0.000259
30.0	0.000022	0.000049	0.000088	0.000140	0.000204
40.0	0.000008	0.000018	0.000029	0.000042	0.000057
50.0	-0.000015	-0.000029	-0.000042	-0.000054	-0.000064
70.0	-0.000042	-0.000083	-0.000121	-0.000158	-0.000190

TIME	6	7	8	9	10
0.0	-0.004265	-0.006124	-0.003539	0.001220	0.004793
1.0	0.000264	0.000038	0.000005	0.000000	-0.000001
2.0	0.002097	0.000746	0.000211	0.000049	0.000011
4.0	0.003187	0.002379	0.001465	0.000796	0.000466
6.0	0.002186	0.002277	0.002012	0.001643	0.001396
9.0	0.001136	0.001621	0.001933	0.002091	0.002148
12.0	0.000741	0.001230	0.001651	0.001954	0.002111
15.0	0.000601	0.000998	0.001363	0.001640	0.001789
18.0	0.000530	0.000827	0.001103	0.001316	0.001431
20.0	0.000490	0.000730	0.000951	0.001122	0.001215
25.0	0.000386	0.000520	0.000641	0.000734	0.000784
30.0	0.000275	0.000347	0.000411	0.000459	0.000485
40.0	0.000074	0.000089	0.000103	0.000114	0.000119
50.0	-0.000072	-0.000079	-0.000083	-0.000086	-0.000088
70.0	-0.000219	-0.000243	-0.000261	-0.000273	-0.000279

[illegible][illegible]

APPENDIX C

Two-Dimensional, Homogeneous Media

1. Rectangular Body:

The p.d.e. is $\frac{\partial u}{\partial t} = \alpha \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$

$$0 \leq x \leq a$$

$$0 \leq y \leq b$$

Boundary conditions are

$$\text{at } x = 0, \quad u = 0$$

$$\text{at } y = 0, \quad u = 0$$

$$\text{at } x = 6.5, \quad \frac{\partial u}{\partial y} = 0$$

$$\text{at } y = 3.5, \quad \frac{\partial u}{\partial x} = 0.$$

The grid spacing is shown in Fig. III.2.

2. Square Body:

The p.d.e. is the same as above. The boundary conditions are:

$$\text{at } x = 0, \quad u = 0$$

$$\text{at } y = 0, \quad u = 0$$

$$\text{at } x = 5.5, \quad \frac{\partial u}{\partial y} = 0$$

$$\text{at } y = 5.5, \quad \frac{\partial u}{\partial x} = 0.$$

The grid spacing is shown in Fig. III.3.

The analytical solutions for the two-dimensional media were obtained by multiplying the analytical solutions

for the two one-dimensional media, according to the method of Neumann (15).

Errors are (Analytical solution minus semi-analytical solution).

C
C
C
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C
C
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C
C
C
C
C

INPUT DATA

N=NUMBER OF GRID POINTS
NT=NUMBER OF TIMES
S=COEFFICIENT MATRIX(DIAGONALS ONLY ARE REQUIRED)
PZ=INITIAL CONDITION VECTOR
C1=BOUNDARY CONDITION VECTOR
TIME1=THE ACTUAL TIMES

```

1 REAL S(25,25),R(25),V(25,25),A(25),B(25),W1(25),W2(25)
2 REAL P(15,25),PZ(25),C1(25),WORK(25),TIME1(15),W(25,25)
3 REAL T(15,25),ERROR(15,25)
4 1 FORMAT(1X,2I4)
5 3 FORMAT(1X,25F3.0)
6 4 FORMAT(1X,24F3.0)
7 5 FORMAT(1X,20F3.0)
8 53 FORMAT(1X,25F3.0)
9 54 FORMAT(1X,15F5.2)
10 220 FORMAT(1H2)
11 221 FORMAT(1H ,10X,8H ..CONTD)
12 222 FORMAT(1H ,//)
13 223 FORMAT(1H ,/)
14 224 FORMAT(1H ,18X,26H TEMPERATURE PROFILES-TWO-,
15 111HDIMENSIONAL/25X,22H HOMOGENEOUS MEDIUM)
16 232 FORMAT(1H ,10X,5F11.6)
17 233 FORMAT(1H ,28X,25H INITIAL CONDITION VECTOR)
18 234 FORMAT(1H ,10X,5F12.4)
19 235 FORMAT(1H ,28X,26H BOUNDARY CONDITION VECTOR)
20 240 FORMAT(1H ,45X,19H COEFFICIENT MATRIX)
21 241 FORMAT(1H0,5X,25F5.1)
22 242 FORMAT(1H ,31X,12H EIGENVALUES)
23 243 FORMAT(1H ,31X,13H EIGENVECTORS)
24 244 FORMAT(1H ,20X,35H CHECK OF SIMILARITY TRANSFORMATION)
25 245 FORMAT(1H ,30X,25H SEMI-ANALYTICAL SOLUTION)
26 247 FORMAT(1H ,30X,19HANALYTICAL SOLUTION)
27 251 FORMAT(1H0,10X,6F10.6)
28 255 FORMAT(1H ,30X,18H ASSOCIATED ERRORS)
29 256 FORMAT(1H ,20X,22H TEMPERATURES AT TIME=,F7.2)
30 READ(5,1) N,NT
31 MAXN=25
32 M=N
33 DO 2 J=1,N
34 DO 2 K=1,N
35 2 S(J,K)=0.0
36 READ(5,3) (S(J,J),J=1,N)
37 N1=N-1
38 READ(5,4) (S(J,J+1),J=1,N1)
39 N6=N-6
40 READ(5,5) (S(J,J+6),J=1,N6)

```



```

DO 51 J=1,N1
51 S(J+1,J)=S(J,J+1)
DO 52 J=1,N6
52 S(J+6,J)=S(J,J+6)
READ(5,53) (PZ(J),J=1,N)
READ(5,53) (C1(J),J=1,N)
READ(5,54) (TIME1(J),J=1,15)
CALL EIG1(N,MAXN,M,S,R,V,A,B,W1,W2)
CALL CHECK(V,R,N,W)
WRITE(6,220)
LINES=9
CALL LINECT(LINES,5,2)
WRITE(6,224)
WRITE(6,223)
WRITE(6,233)
CALL LINECT(LINES,4,2)
WRITE(6,234) (PZ(J),J=1,N)
CALL LINECT(LINES,3,2)
WRITE(6,223)
WRITE(6,235)
CALL LINECT(LINES,4,2)
WRITE(6,234) (C1(J),J=1,N)
CALL LINECT(LINES,3,2)
WRITE(6,220)
LINES=9
WRITE(6,240)
DO 31 J=1,N
31 WRITE(6,241) (S(J,K),K=1,N)
WRITE(6,220)
CALL LINECT(LINES,5,2)
WRITE(6,242)
WRITE(6,232) (R(J),J=1,N)
CALL LINECT(LINES,3,2)
WRITE(6,223)
WRITE(6,243)
DO 32 K=1,N
CALL LINECT(LINES,6,2)
WRITE(6,232) (V(J,K),J=1,N)
32 WRITE(6,223)
CALL LINECT(LINES,1,2)
WRITE(6,244)
DO 33 J=1,N
CALL LINECT(LINES,6,2)
WRITE(6,232) (W(J,K),K=1,N)
33 WRITE(6,223)
DO 8 J=1,N
WORK(J)=0.0
DO 8 K=1,N
8 WORK(J)=WORK(J)+V(K,J)*C1(K)
DO 9 J=1,N
9 C1(J)=WORK(J)
DO 10 J=1,N
WORK(J)=0.0
DO 10 K=1,N
10 WORK(J)=WORK(J)+V(K,J)*PZ(K)

```



```

5      DO 11 J=1,N
6      11 PZ(J)=WORK(J)
0      DO 18 J=1,15
1      DO 15 K=1,N
2      UAR=R(K)*TIME1(J)
3      15 WORK(K)=((R(K)*PZ(K)+C1(K))*EXP(UAR)-C1(K))/R(K)
5      DO 17 L1=1,N
6      P(J,L1)=0.0
7      DO 16 L2=1,N
0      16 P(J,L1)=P(J,L1)+V(L1,L2)*WORK(L2)
2      17 CONTINUE
4      18 CONTINUE
6      CALL ANALTA(NT,TIME1,T)
7      DO 28 J=1,NT
0      DO 28 K=1,N
1      28 ERROR(J,K)=T(J,K)-P(J,K)
4      DO 101 J=1,NT
5      WRITE(6,220)
6      WRITE(6,256) TIME1(J)
7      WRITE(6,223)
0      WRITE(6,245)
1      WRITE(6,251) (P(J,K),K=1,N)
6      WRITE(6,223)
7      WRITE(6,247)
0      WRITE(6,251) (T(J,K),K=1,N)
5      WRITE(6,223)
6      WRITE(6,255)
7      WRITE(6,251) (ERROR(J,K),K=1,N)
4      101 CONTINUE
6      END

```


SOURCE STATEMENT

\$IBFTC ANALTA

SUBROUTINE ANALTA(NT,TIME,T2D)

ANALTA CALCULATES THE ANALYTICAL SOLUTION FOR THE RECTANGULAR BODY USING NEWMAN'S METHOD.

REAL TANAL(15,10),TX(15,10),TIME(15),X(10),T2D(15,25)

NC=1

N=6

NX=N

ALPHA=1.0

RM=42.25

X(1)=.07692308

X(2)=.15384616

X(3)=.23076924

X(4)=.30769232

X(5)=.38461540

X(6)=.47153848

100 PI=3.1415927

DO 10 K=1,N

DO 10 J=1,NT

TANAL(J,K)=0.0

DO 9 L=1,20

AL=L

AA=(2.*AL-1.)*PI*X(K)

AB=-(2.*AL-1.)*2*(PI/2.)*2*ALPHA*TIME(J)/RM

9 TANAL(J,K)=TANAL(J,K)+SIN(AA)*EXP(AB)/(2.*AL-1.)

10 TANAL(J,K)=4.*TANAL(J,K)/PI

IF(NC.GE.2) GO TO 115

DO 110 J=1,NT

DO 110 K=1,N

110 TX(J,K)=TANAL(J,K)

NC=NC+1

N=3

RM=12.25

X(1)=.1428571

X(2)=.2857142

X(3)=.4285713

IF(NC.LE.2) GO TO 100

115 DO 39 L=1,NT

M=0

DO 37 K=1,N

DO 37 J=1,NX

M=M+1

37 T2D(L,M)= TX(L,J)*TANAL(L,K)

39 CONTINUE

RETURN

END

PROGRAM IS BEING ENTERED INTO STORAGE.

TEMPERATURE PROFILES-TWO-DIMENSIONAL
HOMOGENEOUS MEDIUM

INITIAL CONDITION VECTOR

1.0000	1.0000	1.0000	1.0000	1.0000
1.0000	1.0000	1.0000	1.0000	1.0000
1.0000	1.0000	1.0000	1.0000	1.0000
1.0000	1.0000	1.0000		

BOUNDARY CONDITION VECTOR

0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000		

COEFFICIENT MATRIX

-4	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
1	-4	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	1	-4	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	1	-4	1	0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	1	-4	1	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	1	-3	0	0	0	0	0	1	0	0	0	0	0	0
1	0	0	0	0	0	-4	1	0	0	0	0	1	0	0	0	0	0
0	1	0	0	0	0	1	-4	1	0	0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	1	-4	1	0	0	0	0	1	0	0	0
0	0	0	1	0	0	0	0	1	-4	1	0	0	0	0	1	0	0
0	0	0	0	1	0	0	0	0	1	-4	1	0	0	0	0	1	0
0	0	0	0	0	1	0	0	0	0	1	-3	0	0	0	0	0	1
0	0	0	0	0	0	1	0	0	0	0	0	-3	1	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	1	-3	1	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	1	-3	1	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	-3	1	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	-3	1
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	-2

EIGENVALUES

-0.256178	-0.701041	-1.488852	-1.613074	-2.057936
-2.439135	-2.845748	-3.305095	-3.334190	-3.749957
-3.796031	-3.968973	-4.537769	-4.691087	-5.325869
-5.488052	-6.383108	-7.017891		

EIGENVECTORS

-0.043540	-0.084549	-0.120644	-0.149728	-0.170111
-0.180607	-0.078455	-0.152351	-0.217393	-0.269801
-0.306529	-0.325443	-0.097832	-0.189979	-0.271085
-0.336437	-0.382235	-0.405820		

0.120644	0.180607	0.149728	0.043540	-0.084549
-0.170111	0.217393	0.325443	0.269801	0.078456
-0.152351	-0.306529	0.271085	0.405820	0.336437
0.097832	-0.189979	-0.382235		

0.170110	0.120644	-0.084549	-0.180608	-0.043540
0.149728	0.306529	0.217393	-0.152352	-0.325443
-0.078456	0.269801	0.382235	0.271085	-0.189979
-0.405820	-0.097832	0.336437		

0.097833	0.189979	0.271085	-0.336436	0.382235
-0.405821	0.043540	0.084549	0.120644	0.149728
0.170111	0.180607	-0.078455	-0.152351	-0.217394
-0.269802	-0.306529	-0.325442		

-0.271085	-0.405820	-0.336437	-0.097832	0.189979
0.382235	-0.120644	-0.180607	-0.149728	-0.043539
0.084549	0.170111	0.217393	0.325442	0.269801
0.078456	-0.152351	-0.306529		

-0.180607	0.043540	0.170110	-0.084549	-0.149728
0.120645	-0.325442	0.078456	0.306529	-0.152352
-0.269801	0.217394	-0.405820	0.097832	0.382236
-0.189979	-0.336436	0.271085		

-0.382236	-0.271085	0.189979	-0.405821	0.097833
-0.336436	-0.170111	-0.120644	0.084548	0.180606
0.043539	-0.149728	0.306529	0.217393	-0.152351
-0.325443	-0.078455	0.269802		

..CONTD

-0.078455	-0.152352	-0.217393	-0.269801	-0.306529
-0.325442	0.097833	0.189979	0.271085	0.336437
0.382235	0.405820	-0.043539	-0.084550	-0.120644
-0.149728	-0.170111	-0.180606		

-0.149730	0.170108	-0.043544	-0.120649	0.180602
-0.084555	-0.269799	0.306532	-0.078451	-0.217387
0.325450	-0.152344	-0.336437	0.382234	-0.097835
-0.271088	0.405817	-0.189982		

-0.217390	-0.325443	-0.269804	-0.078454	0.152354
0.306527	0.271086	0.405820	0.336435	0.097833
-0.189978	-0.382236	-0.120647	-0.180607	-0.149726
-0.043541	0.084547	0.170112		

0.405823	-0.097828	-0.382232	0.189981	0.336434
-0.271089	0.180603	-0.043544	-0.170116	0.084548
0.149730	-0.120638	-0.325441	0.078458	0.306530
-0.152350	-0.269803	0.217391		

0.084550	-0.149728	0.180607	-0.170110	0.120645
-0.043540	0.152352	-0.269802	0.325442	-0.306529
0.217394	-0.078456	0.189979	-0.336436	0.405821
-0.382236	0.271084	-0.097832		

0.306528	0.217395	-0.152352	-0.325443	-0.078455
0.269801	-0.382236	-0.271085	0.189979	0.405820
0.097833	-0.336437	0.170111	0.120643	-0.084549
-0.180606	-0.043541	0.149729		

-0.336438	0.382235	-0.097832	-0.271083	0.405820
-0.189980	-0.149727	0.170112	-0.043540	-0.120646
0.180606	-0.084547	0.269801	-0.306530	0.078456
0.217394	-0.325443	0.152351		

-0.189978	0.336436	-0.405822	0.382236	-0.271084
0.097832	-0.084550	0.149728	-0.180605	0.170110
-0.120645	0.043540	0.152353	-0.269801	0.325442
-0.306529	0.217394	-0.078456		

-0.325443	0.078457	0.306527	-0.152350	-0.269802
0.217394	0.405820	-0.097832	-0.382236	0.189980
0.336436	-0.271085	-0.180606	0.043539	0.170112
-0.084550	-0.149728	0.120644		

.. CONTD

-0.269801	0.306529	-0.078455	-0.217394	0.325442
-0.152351	0.336437	-0.382236	0.097832	0.271085
-0.405820	0.189979	-0.149728	0.170111	-0.043540
-0.120644	0.180607	-0.084549		

0.152352	-0.269801	0.325443	-0.306529	0.217393
-0.078455	-0.189979	0.336436	-0.405820	0.382235
-0.271085	0.097832	0.084549	-0.149728	0.180607
-0.170111	0.120644	-0.043539		

CHECK OF SIMILARITY TRANSFORMATION

-4.000007	0.999998	-0.000003	-0.000004	0.000000
0.000001	1.000001	0.000008	0.000006	-0.000000
0.000003	-0.000001	-0.000001	-0.000002	-0.000005
-0.000002	0.000002	-0.000001		

0.999998	-3.999994	1.000006	-0.000002	0.000003
0.000004	-0.000005	1.000000	-0.000002	-0.000006
-0.000002	-0.000004	-0.000001	0.000004	-0.000002
0.000001	0.000007	-0.000002		

-0.000003	1.000006	-3.999996	0.999998	-0.000004
-0.000002	-0.000007	0.000001	0.999998	-0.000003
0.000006	0.000004	0.000000	0.000004	-0.000002
-0.000003	0.000005	-0.000005		

-0.000004	-0.000002	0.999998	-4.000000	0.999998
-0.000002	-0.000001	0.000008	0.000001	1.000000
0.000003	0.000002	-0.000006	0.000004	-0.000004
-0.000005	0.000007	-0.000005		

0.000000	0.000003	-0.000004	0.999998	-3.999988
1.000004	-0.000004	0.000006	-0.000001	-0.000005
1.000001	-0.000013	-0.000007	0.000007	0.000000
-0.000004	0.000008	0.000001		

0.000001	0.000004	-0.000002	-0.000002	1.000004
-3.000006	-0.000003	0.000004	-0.000004	-0.000004
0.000010	1.000000	-0.000008	0.000006	0.000001
-0.000005	0.000003	-0.000003		

1.000001	-0.000005	-0.000007	-0.000001	-0.000004
-0.000003	-3.999993	1.000002	0.000004	0.000007
0.000003	0.000004	0.999999	-0.000004	0.000000
-0.000003	-0.000005	0.000000		

..CONTD

0.000008	1.000000	0.000001	0.000008	0.000006
0.000004	1.000002	-4.000006	0.999994	-0.000001
-0.000010	-0.000005	0.000000	0.999999	0.000007
0.000003	-0.000003	0.000006		

0.000006	-0.000002	0.999998	0.000001	-0.000001
-0.000004	0.000004	0.999994	-3.999999	1.000005
0.000000	0.000004	0.000003	-0.000005	1.000003
0.000002	-0.000011	0.000004		

-0.000000	-0.000006	-0.000003	1.000000	-0.000005
-0.000004	0.000007	-0.000001	1.000005	-3.999992
0.999998	0.000007	0.000005	-0.000008	0.000001
1.000003	-0.000010	0.000002		

0.000003	-0.000002	0.000006	0.000003	1.000001
0.000010	0.000003	-0.000010	0.000000	0.999998
-4.000012	0.999994	0.000008	-0.000006	0.000001
0.000009	0.999995	0.000007		

-0.000001	-0.000004	0.000004	0.000002	-0.000013
1.000000	0.000004	-0.000005	0.000004	0.000007
0.999994	-2.999989	0.000010	-0.000008	-0.000002
0.000006	-0.000006	0.999999		

-0.000001	-0.000001	0.000000	-0.000006	-0.000007
-0.000008	0.999999	0.000000	0.000003	0.000005
0.000008	0.000010	-2.999998	1.000000	-0.000003
-0.000002	-0.000003	-0.000005		

-0.000002	0.000004	0.000004	0.000004	0.000007
0.000006	-0.000004	0.999999	-0.000005	-0.000008
-0.000006	-0.000008	1.000000	-2.999996	1.000001
0.000002	0.000005	0.000003		

-0.000005	-0.000002	-0.000002	-0.000004	0.000000
0.000001	0.000000	0.000007	1.000003	0.000001
0.000001	-0.000002	-0.000003	1.000001	-3.000002
0.999996	0.000005	-0.000002		

-0.000002	0.000001	-0.000003	-0.000005	-0.000004
-0.000005	-0.000003	0.000003	0.000002	1.000003
0.000009	0.000006	-0.000002	0.000002	0.999996
-3.000004	1.000001	-0.000004		

..CONTD

0.000002	0.000007	0.000005	0.000007	0.000008
0.000003	-0.000005	-0.000003	-0.000011	-0.000010
0.999995	-0.000006	-0.000003	0.000005	0.000005
1.000001	-2.999993	1.000001		

-0.000001	-0.000002	-0.000005	-0.000005	0.000001
-0.000003	0.000000	0.000006	0.000004	0.000002
0.000007	0.999999	-0.000005	0.000003	-0.000002
-0.000004	1.000001	-2.000001		

TEMPERATURES AT TIME= 1.00

SEMI-ANALYTICAL SOLUTION

0.274196	0.435698	0.499581	0.518247	0.522535	0.523327
0.434973	0.691175	0.792516	0.822127	0.828929	0.830185
0.494597	0.785918	0.901150	0.934820	0.942554	0.943983

ANALYTICAL SOLUTION

0.270909	0.438607	0.502836	0.518043	0.520266	0.520469
0.438414	0.709801	0.813744	0.838354	0.841951	0.842280
0.500423	0.810196	0.928840	0.956930	0.961036	0.961412

ASSOCIATED ERRORS

-0.003287	0.002908	0.003255	-0.000204	-0.002269	-0.002857
0.003441	0.018627	0.021229	0.016227	0.013022	0.012095
0.005826	0.024278	0.027690	0.022110	0.018482	0.017430

TEMPERATURES AT TIME= 2.00

SEMI-ANALYTICAL SOLUTION

0.147060	0.260048	0.328190	0.361364	0.374694	0.378940
0.256647	0.453831	0.572751	0.630646	0.653910	0.661319
0.312011	0.551731	0.696305	0.766689	0.794971	0.803979

ANALYTICAL SOLUTION

0.145622	0.259619	0.329476	0.362983	0.375541	0.379235
0.256666	0.457591	0.580718	0.639774	0.661910	0.668421
0.314338	0.560410	0.711203	0.783530	0.810639	0.818613

ASSOCIATED ERRORS

-0.001439	-0.000429	0.001286	0.001619	0.000847	0.000295
0.000019	0.003760	0.007967	0.009128	0.008000	0.007101
0.002326	0.008679	0.014898	0.016840	0.015668	0.014633

REPORT OF THE

2001-2002

0.14000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

2003-2004

0.14000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

2005-2006

0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

TEMPERATURES AT TIME= 4.00

SEMI-ANALYTICAL SOLUTION

0.068445	0.128594	0.175105	0.206730	0.225297	0.233622
0.123072	0.231226	0.314858	0.371724	0.405109	0.420078
0.153207	0.287842	0.391953	0.462742	0.504301	0.522936

ANALYTICAL SOLUTION

0.068279	0.128598	0.175636	0.207881	0.226905	0.235905
0.122924	0.231518	0.316201	0.374254	0.408503	0.424706
0.153174	0.288491	0.394013	0.466352	0.509029	0.529219

ASSOCIATED ERRORS

-0.000167	0.000004	0.000531	0.001151	0.001608	0.002283
-0.000148	0.000292	0.001343	0.002530	0.003394	0.004628
-0.000034	0.000648	0.002061	0.003610	0.004728	0.006284

TABLE 1. SUMMARY OF DATA

1. SUMMARY OF DATA

0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

2. SUMMARY OF DATA

0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

3. SUMMARY OF DATA

0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

TEMPERATURES AT TIME= 6.00

SEMI-ANALYTICAL SOLUTION

0.037609	0.072046	0.100832	0.122564	0.136844	0.143855
0.067760	0.129803	0.181667	0.220822	0.246550	0.259181
0.084486	0.161844	0.226509	0.275330	0.307408	0.323157

ANALYTICAL SOLUTION

0.037401	0.071739	0.100582	0.122488	0.136966	0.144505
0.067392	0.129264	0.181236	0.220708	0.246796	0.260379
0.084034	0.161185	0.225991	0.275211	0.307740	0.324679

ASSOCIATED ERRORS

-0.000208	-0.000307	-0.000250	-0.000076	0.000122	0.000650
-0.000368	-0.000539	-0.000431	-0.000114	0.000245	0.001198
-0.000452	-0.000658	-0.000518	-0.000119	0.000332	0.001522

STATE OF TEXAS

COMMISSIONERS OF THE LAND OFFICE

1880-1881	1881-1882	1882-1883	1883-1884	1884-1885	1885-1886
1886-1887	1887-1888	1888-1889	1889-1890	1890-1891	1891-1892
1892-1893	1893-1894	1894-1895	1895-1896	1896-1897	1897-1898

LAND OFFICE

1880-1881	1881-1882	1882-1883	1883-1884	1884-1885	1885-1886
1886-1887	1887-1888	1888-1889	1889-1890	1890-1891	1891-1892
1892-1893	1893-1894	1894-1895	1895-1896	1896-1897	1897-1898

LAND OFFICE

1880-1881	1881-1882	1882-1883	1883-1884	1884-1885	1885-1886
1886-1887	1887-1888	1888-1889	1889-1890	1890-1891	1891-1892
1892-1893	1893-1894	1894-1895	1895-1896	1896-1897	1897-1898

TEMPERATURES AT TIME= 8.00

SEMI-ANALYTICAL SOLUTION

0.021750	0.041997	0.059446	0.073145	0.082507	0.087242
0.039192	0.075676	0.107117	0.131801	0.148672	0.157204
0.048872	0.094365	0.133572	0.164352	0.185389	0.196028

ANALYTICAL SOLUTION

0.021523	0.041589	0.058930	0.072590	0.081957	0.086976
0.038784	0.074941	0.106188	0.130802	0.147682	0.156725
0.048362	0.093450	0.132414	0.163108	0.184156	0.195432

ASSOCIATED ERRORS

-0.000227	-0.000408	-0.000516	-0.000555	-0.000550	-0.000267
-0.000409	-0.000734	-0.000929	-0.000999	-0.000990	-0.000479
-0.000509	-0.000915	-0.001158	-0.001244	-0.001233	-0.000596

TABLE 1.1 - 1991-1992

1991-1992

0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

1991-1992

0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

1991-1992

0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

TEMPERATURES AT TIME= 10.00

SEMI-ANALYTICAL SOLUTION

0.012841	0.024876	0.035378	0.043751	0.049561	0.052531
0.023138	0.044825	0.063749	0.078837	0.089305	0.094658
0.028852	0.055896	0.079494	0.098308	0.111362	0.118036

ANALYTICAL SOLUTION

0.012633	0.024484	0.034840	0.043111	0.048860	0.051970
0.022764	0.044119	0.062780	0.077684	0.088043	0.093647
0.028387	0.055015	0.078285	0.096870	0.109787	0.116776

ASSOCIATED ERRORS

-0.000207	-0.000392	-0.000538	-0.000640	-0.000701	-0.000561
-0.000373	-0.000706	-0.000970	-0.001153	-0.001263	-0.001010
-0.000466	-0.000881	-0.001209	-0.001438	-0.001575	-0.001259

TABLE 1. SUMMARY OF DATA

TABLE 1. SUMMARY OF DATA

0.01500	0.02000	0.02500	0.03000	0.03500	0.04000
0.04500	0.05000	0.05500	0.06000	0.06500	0.07000
0.07500	0.08000	0.08500	0.09000	0.09500	0.10000

TABLE 1. SUMMARY OF DATA

0.01500	0.02000	0.02500	0.03000	0.03500	0.04000
0.04500	0.05000	0.05500	0.06000	0.06500	0.07000
0.07500	0.08000	0.08500	0.09000	0.09500	0.10000

TABLE 1. SUMMARY OF DATA

0.01500	0.02000	0.02500	0.03000	0.03500	0.04000
0.04500	0.05000	0.05500	0.06000	0.06500	0.07000
0.07500	0.08000	0.08500	0.09000	0.09500	0.10000

TEMPERATURES AT TIME= 15.00

SEMI-ANALYTICAL SOLUTION

0.003534	0.006861	0.009787	0.012142	0.013790	0.014639
0.006369	0.012364	0.017636	0.021879	0.024849	0.026378
0.007941	0.015417	0.021991	0.027283	0.030987	0.032893

ANALYTICAL SOLUTION

0.003419	0.006639	0.009470	0.011750	0.013347	0.014215
0.006162	0.011963	0.017065	0.021173	0.024050	0.025615
0.007683	0.014917	0.021280	0.026402	0.029989	0.031941

ASSOCIATED ERRORS

-0.000115	-0.000223	-0.000317	-0.000392	-0.000444	-0.000424
-0.000207	-0.000401	-0.000571	-0.000706	-0.000800	-0.000763
-0.000258	-0.000500	-0.000712	-0.000880	-0.000997	-0.000952

TEMPERATURES AT THE STATION

TEMPERATURES AT THE STATION

1900	1901	1902	1903	1904	1905
1906	1907	1908	1909	1910	1911
1912	1913	1914	1915	1916	1917

TEMPERATURES AT THE STATION

1918	1919	1920	1921	1922	1923
1924	1925	1926	1927	1928	1929
1930	1931	1932	1933	1934	1935

TEMPERATURES AT THE STATION

1936	1937	1938	1939	1940	1941
1942	1943	1944	1945	1946	1947
1948	1949	1950	1951	1952	1953

TEMPERATURES AT TIME= 20.00

SEMI-ANALYTICAL SOLUTION

0.000981	0.001905	0.002718	0.003373	0.003831	0.004068
0.001767	0.003432	0.004897	0.006077	0.006904	0.007330
0.002204	0.004279	0.006106	0.007578	0.008609	0.009140

ANALYTICAL SOLUTION

0.000932	0.001810	0.002582	0.003205	0.003641	0.003879
0.001679	0.003261	0.004653	0.005775	0.006561	0.006989
0.002094	0.004067	0.005803	0.007201	0.008182	0.008715

ASSOCIATED ERRORS

-0.000049	-0.000095	-0.000135	-0.000168	-0.000190	-0.000189
-0.000088	-0.000171	-0.000243	-0.000302	-0.000343	-0.000341
-0.000110	-0.000213	-0.000304	-0.000377	-0.000428	-0.000425

TESTING AT THE

0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

TESTING AT THE

0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

TESTING AT THE

0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

TEMPERATURES AT TIME= 35.00

SEMI-ANALYTICAL SOLUTION

0.000021	0.000041	0.000058	0.000072	0.000082	0.000087
0.000038	0.000074	0.000105	0.000130	0.000148	0.000157
0.000047	0.000092	0.000131	0.000162	0.000185	0.000196

ANALYTICAL SOLUTION

0.000019	0.000037	0.000052	0.000065	0.000074	0.000079
0.000034	0.000066	0.000094	0.000117	0.000133	0.000142
0.000043	0.000083	0.000118	0.000146	0.000166	0.000177

ASSOCIATED ERRORS

-0.000002	-0.000004	-0.000006	-0.000007	-0.000008	-0.000008
-0.000004	-0.000007	-0.000011	-0.000013	-0.000015	-0.000015
-0.000005	-0.000009	-0.000013	-0.000016	-0.000018	-0.000019

TABLE 1. - SUMMARY OF DATA

TABLE 1. - SUMMARY OF DATA

1950	1951	1952	1953	1954	1955
100000.0	100000.0	100000.0	100000.0	100000.0	100000.0
100000.0	100000.0	100000.0	100000.0	100000.0	100000.0
100000.0	100000.0	100000.0	100000.0	100000.0	100000.0

TABLE 1. - SUMMARY OF DATA

1950	1951	1952	1953	1954	1955
100000.0	100000.0	100000.0	100000.0	100000.0	100000.0
100000.0	100000.0	100000.0	100000.0	100000.0	100000.0
100000.0	100000.0	100000.0	100000.0	100000.0	100000.0

TABLE 1. - SUMMARY OF DATA

1950	1951	1952	1953	1954	1955
100000.0	100000.0	100000.0	100000.0	100000.0	100000.0
100000.0	100000.0	100000.0	100000.0	100000.0	100000.0
100000.0	100000.0	100000.0	100000.0	100000.0	100000.0

TEMPERATURES AT TIME= -0.00

SEMI-ANALYTICAL SOLUTION

1.000001	0.999998	1.000000	1.000001	1.000000	1.000001
1.000000	0.999996	0.999999	1.000000	0.999996	1.000000
1.000001	0.999997	1.000001	1.000003	0.999998	1.000002

ANALYTICAL SOLUTION

1.042211	0.948463	0.996704	0.967718	0.984774	0.992883
1.070103	0.973846	1.023378	0.993616	1.011129	1.019455
1.080561	0.983364	1.033380	1.003327	1.021011	1.029418

ASSOCIATED ERRORS

0.042209	-0.051535	-0.003296	-0.032284	-0.015226	-0.007118
0.070102	-0.026149	0.023380	-0.006384	0.011132	0.019455
0.080560	-0.016633	0.033379	0.003324	0.021013	0.029417

\$IBFTC DARSI NODECK

SOLUTION TO THE FORWARD PROBLEM-HOMOGENEOUS MEDIUM
TWO-DIMENSIONAL SYSTEM-SQUARE BODY

INPUT DATA

N=NUMBER OF GRID POINTS

NT=NUMBER OF TIMES

MATRIX=COEFFICIENT MATRIX(DIAGONALS ONLY ARE REQUIRED)

PZ=INITIAL CONDITION VECTOR

C1=BOUNDARY CONDITION VECTOR

TIME1=THE ACTUAL TIMES

DOUBLE PRECISION MATRIX(25,25),VECTOR(25,25),TOLERC,W(25,25)
DOUBLE PRECISION P(15,25),PZ(25),C1(25),WORK(25),TIME1(15),UAR
DOUBLE PRECISION T(15,25),ERROR(15,25)

1 FORMAT(1X,2I4)

3 FORMAT(1X,25F3.0)

4 FORMAT(1X,24F3.0)

5 FORMAT(1X,20F3.0)

53 FORMAT(1X,25F3.0)

54 FORMAT(1X,15F5.2)

220 FORMAT(1H2)

221 FORMAT(1H ,10X,8H ..CONTD)

222 FORMAT(1H ,//)

223 FORMAT(1H ,/)

224 FORMAT(1H ,18X,26H TEMPERATURE PROFILES-TWO-,
111HDIMENSIONAL/25X,22H HOMOGENEOUS MEDIUM)

232 FORMAT(1H ,10X,5F11.6)

233 FORMAT(1H ,28X,25H INITIAL CONDITION VECTOR)

234 FORMAT(1H ,10X,5F12.4)

235 FORMAT(1H ,28X,26H BOUNDARY CONDITION VECTOR)

240 FORMAT(1H ,45X,19H COEFFICIENT MATRIX)

241 FORMAT(1H0,5X,25F5.1)

242 FORMAT(1H ,31X,12H EIGENVALUES)

243 FORMAT(1H ,31X,13H EIGENVECTORS)

244 FORMAT(1H ,20X,35H CHECK OF SIMILARITY TRANSFORMATION)

245 FORMAT(1H ,30X,25H SEMI-ANALYTICAL SOLUTION)

247 FORMAT(1H ,30X,19HANALYTICAL SOLUTION)

251 FORMAT(1H0,10X,5F12.7)

255 FORMAT(1H ,30X,18H ASSOCIATED ERRORS)

256 FORMAT(1H ,20X,22H TEMPERATURES AT TIME=,F7.2)

READ(5,1) N,NT

NORM=2

TOLERC=0.D00

DO 2 J=1,N

DO 2 K=1,N

2 MATRIX(J,K)=0.0

READ(5,3) (MATRIX(J,J),J=1,N)

N1=N-1

READ(5,4) (MATRIX(J,J+1),J=1,N1)

N5=N-5

READ(5,5) (MATRIX(J,J+5),J=1,N5)


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DO 51 J=1,N1
51 MATRIX(J+1,J)=MATRIX(J,J+1)
DO 52 J=1,N5
52 MATRIX(J+5,J)=MATRIX(J,J+5)
READ(5,53) (PZ(J),J=1,N),(C1(J),J=1,N)
READ(5,54) (TIME1(J),J=1,15)
WRITE(6,220)
LINES=9
CALL LINECT(LINES,5,2)
WRITE(6,224)
WRITE(6,223)
WRITE(6,233)
CALL LINECT(LINES,4,2)
WRITE(6,234) (PZ(J),J=1,N)
CALL LINECT(LINES,3,2)
WRITE(6,223)
WRITE(6,235)
CALL LINECT(LINES,4,2)
WRITE(6,234) (C1(J),J=1,N)
WRITE(6,220)
WRITE(6,240)
DO 31 J=1,N
31 WRITE(6,241) (MATRIX(J,K),K=1,N)
CALL JACOBI(N,MATRIX,VECTOR,TOLERC,NORM)
CALL CHECK(VECTOR,MATRIX,N,W)
DO 8 J=1,N
WORK(J)=0.0
DO 8 K=1,N
8 WORK(J)=WORK(J)+VECTOR(K,J)*C1(K)
DO 9 J=1,N
9 C1(J)=WORK(J)
DO 10 J=1,N
WORK(J)=0.0
DO 10 K=1,N
10 WORK(J)=WORK(J)+VECTOR(K,J)*PZ(K)
DO 11 J=1,N
11 PZ(J)=WORK(J)
DO 18 J=1,NT
DO 15 K=1,N
UAR=MATRIX(K,K)*TIME1(J)
15 WORK(K)=((MATRIX(K,K)*PZ(K)+C1(K))*DEXP(UAR)-C1(K))/MATRIX(K,K)
DO 17 L1=1,N
P(J,L1)=0.0
DO 16 L2=1,N
16 P(J,L1)=P(J,L1)+VECTOR(L1,L2)*WORK(L2)
17 CONTINUE
18 CONTINUE
WRITE(6,220)
LINES=9
CALL LINECT(LINES,6,2)
WRITE(6,242)
WRITE(6,232) (MATRIX(J,J),J=1,N)
CALL LINECT(LINES,3,2)
WRITE(6,223)
WRITE(6,243)
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DO 32 K=1,N
CALL LINECT(LINES,7,2)
WRITE(6,232) (VECTOR(J,K),J=1,N)
32 WRITE(6,223)
CALL LINECT(LINES,1,2)
WRITE(6,244)
DO 33 J=1,N
CALL LINECT(LINES,7,2)
WRITE(6,232) (W(J,K),K=1,N)
33 WRITE(6,223)
CALL ANALTB(NT,TIME1,T)
DO 75 J=1,NT
DO 75 K=1,N
75 ERROR(J,K)=T(J,K)-P(J,K)
DO 101 J=1,NT
WRITE(6,220)
WRITE(6,256) TIME1(J)
WRITE(6,223)
WRITE(6,245)
WRITE(6,251) (P(J,K),K=1,N)
WRITE(6,223)
WRITE(6,247)
WRITE(6,251) (T(J,K),K=1,N)
WRITE(6,223)
WRITE(6,255)
WRITE(6,251) (ERROR(J,K),K=1,N)
101 CONTINUE
END

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FORTRAN SOURCE LIST

SOURCE STATEMENT

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$IBFTC ANALTB
      SUBROUTINE ANALTB(NT,TIME,T2D)
C
C      ANALTB CALCULATES THE ANALYTICAL SOLUTION FOR THE SQUARE
C      BODY USING NEWMAN*S METHOD.
C
      DOUBLE PRECISION TANAL(15,10),TX(15,15),TIME(15),X(10)
      DOUBLE PRECISION T2D(15,25),ALPHA,RM,PI,AA,AB,AL
      NC=1
      N=5
      NX=N
      ALPHA=1.0
      RM=30.25
      X(1)=0.09090909
      X(2)=0.18181818
      X(3)=0.27272727
      X(4)=0.36363636
      X(5)=0.45454545
100  PI=3.1415927
      DO 10 K=1,N
      DO 10 J=1,NT
      TANAL(J,K)=0.0
      DO 9 L=1,20
      AL=L
      AA=(2.*AL-1.)*PI* X(K)
      AB=-(2.*AL-1.)*2*(PI/2.)*2*ALPHA*TIME(J)/RM
      9  TANAL(J,K)=TANAL(J,K)+DSIN(AA)*DEXP(AB)/(2.*AL-1.)
10  TANAL(J,K)=4.*TANAL(J,K)/PI
      DO 110 J=1,NT
      DO 110 K=1,N
110  TX(J,K)=TANAL(J,K)
      DO 39 L=1,NT
      M=0
      DO 37 J=1,NX
      DO 37 K=1,N
      M=M+1
37  T2D(L,M)=TX(L,J)*TANAL(L,K)
39  CONTINUE
      RETURN
      END

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PROGRAM IS BEING ENTERED INTO STORAGE.

TEMPERATURE PROFILES-TWO-DIMENSIONAL
HOMOGENEOUS MEDIUM

INITIAL CONDITION VECTOR

1.0000	1.0000	1.0000	1.0000	1.0000
1.0000	1.0000	1.0000	1.0000	1.0000
1.0000	1.0000	1.0000	1.0000	1.0000
1.0000	1.0000	1.0000	1.0000	1.0000
1.0000	1.0000	1.0000	1.0000	1.0000

BOUNDARY CONDITION VECTOR

0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000

PERMANENT RESIDENTS - 1981-1982
 (FISCAL YEAR 1982)

INITIAL	LAST NAME	DATE OF BIRTH	DATE OF ENTRY	DATE OF DEPARTURE
1.0000	1.0000	1.0000	1.0000	1.0000
1.0000	1.0000	1.0000	1.0000	1.0000
1.0000	1.0000	1.0000	1.0000	1.0000
1.0000	1.0000	1.0000	1.0000	1.0000
1.0000	1.0000	1.0000	1.0000	1.0000

INITIAL	LAST NAME	DATE OF BIRTH	DATE OF ENTRY	DATE OF DEPARTURE
0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000

[illegible]

EIGENVALUES

-0.162028	-7.365014	-6.513337	-0.771293	-0.771293
-6.513337	-1.796384	-5.397877	-2.405649	-1.380557
-5.397877	-5.661660	-4.372786	-4.546200	-3.763521
-4.372786	-2.405649	-2.911844	-4.546200	-2.911844
-3.763521	-3.521109	-3.430741	-3.521109	-1.796384

EIGENVECTORS

0.028863	0.055388	0.077425	0.093190	0.101405
0.055388	0.106288	0.148578	0.178831	0.194596
0.077425	0.148578	0.207694	0.249983	0.272021
0.093190	0.178831	0.249983	0.300884	0.327408
0.101405	0.194596	0.272021	0.327408	0.356271
-0.106288	0.178831	-0.194596	0.148578	-0.055388
0.178831	-0.300884	0.327408	-0.249983	0.093190
-0.194596	0.327408	-0.356271	0.272021	-0.101405
0.148578	-0.249983	0.272021	-0.207694	0.077425
-0.055388	0.093190	-0.101405	0.077425	-0.028863
0.020641	-0.133276	0.285473	-0.316512	0.139826
0.081399	0.028853	-0.267702	0.370203	-0.174744
-0.254076	0.247061	-0.011704	-0.171488	0.103904
0.310120	-0.384021	0.221545	-0.031397	-0.018818
-0.141338	0.186448	-0.129695	0.047671	-0.006393
0.000674	0.034444	0.108840	0.198133	0.259017
-0.032267	0.001695	0.118836	0.271857	0.379140
-0.106549	-0.115541	0.001294	0.181638	0.314118
-0.196207	-0.269488	-0.180755	-0.000812	0.139298
-0.257459	-0.377594	-0.314600	-0.142802	-0.002852
-0.109494	-0.176556	-0.185360	-0.155138	-0.124859
-0.176967	-0.275194	-0.266778	-0.190677	-0.123137
-0.186686	-0.268222	-0.210117	-0.070587	0.041101
-0.157567	-0.194011	-0.072819	0.131788	0.285325
-0.128041	-0.127798	0.037228	0.283587	0.463017
0.252061	-0.307968	0.169615	-0.013378	-0.020743
-0.325547	0.352350	-0.104954	-0.115250	0.086254
0.213797	-0.147107	-0.142931	0.321739	-0.167046
-0.064691	-0.053488	0.289555	-0.383412	0.178897
0.002281	0.056677	-0.147918	0.173453	-0.078069

..CONTD

-0.095541	-0.017872	0.108343	0.073715	-0.057264
-0.192664	-0.052185	0.182902	0.111361	-0.142639
-0.276832	-0.087361	0.235527	0.131419	-0.225780
-0.330005	-0.099021	0.292051	0.168489	-0.260533
-0.351353	-0.092888	0.338572	0.208348	-0.256290

0.149431	-0.331742	0.385864	-0.243662	0.067972
0.122855	-0.071561	0.036013	-0.113226	0.080675
-0.249606	0.272907	-0.251419	0.285250	-0.148194
-0.046843	0.191096	-0.242718	0.114094	-0.010574
0.123992	-0.250474	0.285519	-0.191448	0.059455

0.195771	0.325797	0.250682	-0.061242	-0.368793
-0.013668	0.072981	0.135121	0.020470	-0.157951
-0.290544	-0.330892	-0.128703	0.116708	0.254445
-0.118669	-0.181292	-0.126135	0.039862	0.192472
0.282635	0.286654	0.069029	-0.119491	-0.179911

0.207694	0.272021	0.148578	-0.077425	-0.249983
0.272021	0.356271	0.194596	-0.101405	-0.327408
0.148578	0.194596	0.106288	-0.055388	-0.178831
-0.077425	-0.101405	-0.055388	0.028863	0.093190
-0.249983	-0.327408	-0.178831	0.093190	0.300884

-0.231096	0.014546	0.100092	0.161000	-0.166548
0.308497	0.110670	-0.315463	-0.158603	0.238362
-0.310816	-0.162283	0.388820	0.137809	-0.246411
0.288268	0.038177	-0.203586	-0.176446	0.214692
-0.130324	0.024234	0.034037	0.097735	-0.091948

-0.300884	0.249983	0.093190	-0.327408	0.178831
0.249983	-0.207694	-0.077425	0.272021	-0.148578
0.093190	-0.077425	-0.028863	0.101405	-0.055388
-0.327408	0.272021	0.101405	-0.356271	0.194596
0.178831	-0.148578	-0.055388	0.194596	-0.106288

0.137448	-0.263397	0.263625	-0.203146	0.090151
0.212158	-0.302882	0.368267	-0.278046	0.079388
0.086345	-0.204119	0.180018	-0.140858	0.078912
-0.040228	0.112612	-0.090399	0.071625	-0.046860
-0.183960	0.292766	-0.330556	0.251416	-0.086209

0.068870	0.236562	-0.181794	-0.355503	0.316693
-0.274179	-0.016286	0.218237	0.059277	-0.134168
0.097173	-0.171724	0.019603	0.239057	-0.168519
0.392828	-0.006693	-0.296277	-0.040933	0.155676
-0.305042	0.078829	0.189850	-0.076098	-0.031253

..CONTD

0.071298	0.066840	0.174710	0.140303	0.192354
-0.049980	-0.230202	-0.165828	-0.333884	-0.287169
0.147084	0.094530	0.350162	0.233737	0.360790
-0.009768	-0.244690	-0.079633	-0.321794	-0.222040
0.095296	-0.062993	0.197490	-0.008162	0.130535

-0.158930	-0.176014	-0.125675	0.070258	0.172765
0.235261	0.350221	0.152607	-0.073281	-0.307427
-0.278993	-0.342411	-0.208154	0.111881	0.322548
0.211154	0.264572	0.155520	-0.082820	-0.247243
-0.064294	-0.122892	-0.031573	0.010715	0.099683

0.331156	0.163647	-0.193694	-0.169139	0.119685
0.364331	0.123450	-0.303326	-0.195657	0.240273
0.126266	-0.027830	-0.217707	-0.079754	0.218779
-0.135189	-0.076380	0.063809	0.067428	-0.030488
-0.265425	-0.022566	0.328392	0.153938	-0.304328

0.049480	-0.141023	-0.124047	-0.023934	-0.178660
0.194866	-0.078889	0.029975	0.276663	0.008184
0.241453	-0.169661	-0.041110	0.286827	-0.097281
0.237534	-0.306072	-0.191875	0.173841	-0.303587
0.323092	-0.209051	-0.035450	0.397801	-0.103323

-0.457875	0.163457	0.260317	-0.180353	0.008029
0.086635	0.108278	-0.125289	-0.169837	0.167939
0.302277	-0.183944	-0.130325	0.230469	-0.097859
-0.067795	-0.179760	0.149948	0.272139	-0.247098
-0.085488	0.199976	-0.043955	-0.281960	0.207783

0.122150	0.134490	0.218946	0.316609	0.279901
-0.001572	-0.194749	-0.212852	-0.154327	-0.291934
0.070889	-0.131983	-0.101486	0.020245	-0.151310
0.210694	0.081727	0.214158	0.429153	0.258350
0.076652	-0.203937	-0.176357	-0.025768	-0.255067

0.032437	-0.124555	0.042837	-0.135241	-0.041239
0.132226	-0.104728	0.269926	-0.033580	0.166728
0.103560	-0.302363	0.159303	-0.309354	-0.052481
0.194627	-0.229637	0.379462	-0.146397	0.182697
0.172102	-0.326031	0.306466	-0.287425	0.059386

-0.300642	-0.271063	-0.156313	0.028335	0.326935
0.127088	0.327146	0.167871	-0.157053	-0.198704
0.034358	0.132771	0.066612	-0.072713	-0.066336
-0.243406	-0.364533	-0.196029	0.121955	0.305986
0.213610	0.132092	0.082089	0.021160	-0.215071

..CONTD

-0.356271	-0.101405	0.327408	0.194596	-0.272021
-0.101405	-0.028863	0.093190	0.055388	-0.077425
0.327408	0.093190	-0.300884	-0.178831	0.249983
0.194596	0.055388	-0.178831	-0.106288	0.148578
-0.272021	-0.077425	0.249983	0.148578	-0.207694

-0.186004	0.277232	0.116367	-0.286141	0.075629
-0.366308	0.202401	0.064636	-0.329027	0.246730
-0.191819	0.121368	0.041212	-0.182793	0.124824
0.153080	0.006328	0.016525	0.075452	-0.128984
0.258800	-0.287942	-0.115078	0.331385	-0.133062

0.106949	0.195911	0.266349	0.323775	0.360062
0.039763	0.058415	0.067244	0.087064	0.109602
-0.077743	-0.174193	-0.263648	-0.308764	-0.315208
-0.036885	-0.100878	-0.165266	-0.188606	-0.180227
0.097341	0.154046	0.188949	0.238642	0.286890

CHECK OF SIMILARITY TRANSFORMATION

-4.000000	1.000000	-0.000000	0.000000	-0.000000
1.000000	-0.000000	0.000000	-0.000000	-0.000000
-0.000000	0.000000	0.000000	-0.000000	-0.000000
-0.000000	0.000000	0.000000	0.000000	0.000000
-0.000000	0.000000	-0.000000	0.000000	-0.000000

1.000000	-4.000000	1.000000	-0.000000	0.000000
0.000000	1.000000	-0.000000	0.000000	0.000000
-0.000000	-0.000000	0.000000	-0.000000	-0.000000
-0.000000	0.000000	-0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000

-0.000000	1.000000	-4.000000	1.000000	-0.000000
0.000000	-0.000000	1.000000	0.000000	-0.000000
-0.000000	0.000000	-0.000000	-0.000000	-0.000000
-0.000000	0.000000	0.000000	0.000000	-0.000000
0.000000	-0.000000	-0.000000	0.000000	0.000000

0.000000	-0.000000	1.000000	-4.000000	1.000000
0.000000	0.000000	-0.000000	1.000000	0.000000
-0.000000	-0.000000	-0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	-0.000000	0.000000
0.000000	-0.000000	-0.000000	0.000000	-0.000000

-0.000000	0.000000	-0.000000	1.000000	-3.000000
-0.000000	-0.000000	-0.000000	-0.000000	1.000000
0.000000	-0.000000	0.000000	0.000000	-0.000000
0.000000	-0.000000	-0.000000	-0.000000	-0.000000
-0.000000	-0.000000	0.000000	0.000000	0.000000

..CONTD

1.000000	0.000000	0.000000	0.000000	-0.000000
-4.000000	1.000000	0.000000	-0.000000	-0.000000
1.000000	-0.000000	-0.000000	0.000000	0.000000
0.000000	0.000000	-0.000000	-0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000

-0.000000	1.000000	-0.000000	0.000000	-0.000000
1.000000	-4.000000	1.000000	0.000000	-0.000000
0.000000	1.000000	-0.000000	0.000000	0.000000
0.000000	-0.000000	0.000000	-0.000000	-0.000000
0.000000	-0.000000	0.000000	0.000000	-0.000000

0.000000	-0.000000	1.000000	-0.000000	-0.000000
0.000000	1.000000	-4.000000	1.000000	-0.000000
-0.000000	0.000000	1.000000	-0.000000	0.000000
-0.000000	0.000000	0.000000	0.000000	0.000000
-0.000000	0.000000	0.000000	-0.000000	-0.000000

-0.000000	0.000000	0.000000	1.000000	-0.000000
-0.000000	0.000000	1.000000	-4.000000	1.000000
0.000000	0.000000	-0.000000	1.000000	0.000000
-0.000000	0.000000	-0.000000	-0.000000	0.000000
0.000000	0.000000	-0.000000	0.000000	-0.000000

-0.000000	0.000000	0.000000	0.000000	1.000000
-0.000000	-0.000000	-0.000000	1.000000	-3.000000
0.000000	0.000000	0.000000	0.000000	1.000000
-0.000000	-0.000000	-0.000000	-0.000000	0.000000
-0.000000	0.000000	0.000000	-0.000000	-0.000000

-0.000000	-0.000000	-0.000000	-0.000000	0.000000
1.000000	0.000000	-0.000000	0.000000	0.000000
-4.000000	1.000000	0.000000	0.000000	-0.000000
1.000000	-0.000000	0.000000	0.000000	-0.000000
-0.000000	0.000000	-0.000000	-0.000000	-0.000000

0.000000	-0.000000	0.000000	-0.000000	-0.000000
-0.000000	1.000000	0.000000	0.000000	0.000000
1.000000	-4.000000	1.000000	0.000000	0.000000
0.000000	1.000000	-0.000000	-0.000000	-0.000000
-0.000000	-0.000000	0.000000	-0.000000	-0.000000

0.000000	0.000000	-0.000000	-0.000000	0.000000
-0.000000	-0.000000	1.000000	-0.000000	0.000000
0.000000	1.000000	-4.000000	1.000000	-0.000000
0.000000	-0.000000	1.000000	-0.000000	-0.000000
0.000000	0.000000	0.000000	0.000000	0.000000

..CONTD

-0.000000	-0.000000	-0.000000	0.000000	0.000000
0.000000	0.000000	-0.000000	1.000000	0.000000
0.000000	0.000000	1.000000	-4.000000	1.000000
0.000000	-0.000000	0.000000	1.000000	-0.000000
0.000000	0.000000	-0.000000	0.000000	0.000000

-0.000000	-0.000000	-0.000000	0.000000	-0.000000
0.000000	0.000000	0.000000	0.000000	1.000000
-0.000000	0.000000	-0.000000	1.000000	-3.000000
-0.000000	0.000000	0.000000	0.000000	1.000000
0.000000	-0.000000	-0.000000	0.000000	-0.000000

-0.000000	-0.000000	-0.000000	0.000000	0.000000
0.000000	0.000000	-0.000000	-0.000000	-0.000000
1.000000	0.000000	0.000000	0.000000	-0.000000
-4.000000	1.000000	-0.000000	-0.000000	0.000000
1.000000	0.000000	-0.000000	-0.000000	0.000000

0.000000	0.000000	0.000000	0.000000	-0.000000
0.000000	-0.000000	0.000000	0.000000	-0.000000
-0.000000	1.000000	-0.000000	-0.000000	0.000000
1.000000	-4.000000	1.000000	-0.000000	0.000000
0.000000	1.000000	0.000000	-0.000000	-0.000000

0.000000	-0.000000	0.000000	0.000000	-0.000000
-0.000000	0.000000	0.000000	-0.000000	-0.000000
0.000000	-0.000000	1.000000	0.000000	0.000000
-0.000000	1.000000	-4.000000	1.000000	-0.000000
-0.000000	0.000000	1.000000	-0.000000	0.000000

0.000000	0.000000	0.000000	-0.000000	-0.000000
-0.000000	-0.000000	0.000000	-0.000000	-0.000000
0.000000	-0.000000	-0.000000	1.000000	0.000000
-0.000000	-0.000000	1.000000	-4.000000	1.000000
-0.000000	0.000000	0.000000	1.000000	-0.000000

0.000000	0.000000	-0.000000	0.000000	-0.000000
0.000000	-0.000000	0.000000	0.000000	0.000000
-0.000000	-0.000000	-0.000000	-0.000000	1.000000
0.000000	0.000000	-0.000000	1.000000	-3.000000
0.000000	0.000000	0.000000	0.000000	1.000000

-0.000000	0.000000	0.000000	0.000000	-0.000000
0.000000	0.000000	-0.000000	0.000000	-0.000000
-0.000000	-0.000000	0.000000	0.000000	0.000000
1.000000	0.000000	-0.000000	-0.000000	0.000000
-3.000000	1.000000	0.000000	0.000000	-0.000000

..CONTD

0.000000	0.000000	-0.000000	-0.000000	-0.000000
0.000000	-0.000000	0.000000	0.000000	0.000000
0.000000	-0.000000	0.000000	0.000000	-0.000000
0.000000	1.000000	0.000000	0.000000	0.000000
1.000000	-3.000000	1.000000	0.000000	-0.000000

-0.000000	0.000000	-0.000000	-0.000000	0.000000
0.000000	0.000000	0.000000	-0.000000	0.000000
-0.000000	0.000000	-0.000000	-0.000000	-0.000000
-0.000000	0.000000	1.000000	0.000000	0.000000
0.000000	1.000000	-3.000000	1.000000	0.000000

0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	-0.000000	0.000000	-0.000000
-0.000000	-0.000000	0.000000	0.000000	0.000000
-0.000000	-0.000000	-0.000000	1.000000	0.000000
0.000000	0.000000	1.000000	-3.000000	1.000000

-0.000000	0.000000	0.000000	-0.000000	0.000000
0.000000	-0.000000	-0.000000	-0.000000	-0.000000
-0.000000	-0.000000	0.000000	0.000000	-0.000000
0.000000	-0.000000	0.000000	-0.000000	1.000000
-0.000000	-0.000000	0.000000	1.000000	-2.000000

21991..

TEMPERATURES AT TIME= 0.00

SEMI-ANALYTICAL SOLUTION

1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
1.0000000	1.0000000	1.0000000	1.0000000	1.0000000

ANALYTICAL SOLUTION

0.9624708	1.0007401	1.0007232	0.9833034	0.9677636
1.0007401	1.0405312	1.0405136	1.0224011	1.0062434
1.0007232	1.0405136	1.0404961	1.0223839	1.0062264
0.9833034	1.0224011	1.0223839	1.0045870	0.9887108
0.9677636	1.0062434	1.0062264	0.9887108	0.9730855

ASSOCIATED ERRORS

-0.0375292	0.0007401	0.0007233	-0.0166966	-0.0322364
0.0007401	0.0405312	0.0405136	0.0224011	0.0062434
0.0007233	0.0405136	0.0404961	0.0223839	0.0062264
-0.0166966	0.0224011	0.0223839	0.0045870	-0.0112892
-0.0322364	0.0062434	0.0062264	-0.0112892	-0.0269145

TEMPERATURE AT TIME

APPROXIMATE RESULTS

1.000000	1.000000	1.000000	1.000000	1.000000
1.000000	1.000000	1.000000	1.000000	1.000000
1.000000	1.000000	1.000000	1.000000	1.000000
1.000000	1.000000	1.000000	1.000000	1.000000
1.000000	1.000000	1.000000	1.000000	1.000000

APPROXIMATE RESULTS

0.999999	1.000000	1.000000	1.000000	1.000000
1.000000	1.000000	1.000000	1.000000	1.000000
1.000000	1.000000	1.000000	1.000000	1.000000
0.999999	1.000000	1.000000	1.000000	1.000000
0.999999	1.000000	1.000000	1.000000	1.000000

APPROXIMATE RESULTS

-0.000001	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
-0.000001	0.000000	0.000000	0.000000	0.000000
-0.000001	0.000000	0.000000	0.000000	0.000000

TEMPERATURES AT TIME= 1.00

SEMI-ANALYTICAL SOLUTION

0.2743429	0.4359324	0.4998470	0.5185052	0.5226683
0.4359324	0.6926990	0.7942596	0.8239076	0.8305228
0.4998470	0.7942596	0.9107105	0.9447054	0.9522904
0.5185052	0.8239076	0.9447054	0.9799693	0.9878374
0.5226683	0.8305228	0.9522904	0.9878374	0.9957687

ANALYTICAL SOLUTION

0.2709201	0.4386256	0.5028576	0.5180647	0.5202765
0.4386256	0.7101446	0.8141375	0.8387582	0.8423392
0.5028576	0.8141375	0.9333591	0.9615852	0.9656906
0.5180647	0.8387582	0.9615852	0.9906649	0.9948944
0.5202765	0.8423392	0.9656906	0.9948944	0.9991421

ASSOCIATED ERRORS

-0.0034228	0.0026932	0.0030106	-0.0004405	-0.0023917
0.0026932	0.0174456	0.0198779	0.0148506	0.0118164
0.0030106	0.0198779	0.0226486	0.0168798	0.0134002
-0.0004405	0.0148506	0.0168798	0.0106956	0.0070570
-0.0023917	0.0118164	0.0134002	0.0070570	0.0033733

TEMPERATURE AT 1110

0.558883	0.880850	0.880850	0.880850	0.880850
0.558883	0.880850	0.880850	0.880850	0.880850
0.558883	0.880850	0.880850	0.880850	0.880850
0.558883	0.880850	0.880850	0.880850	0.880850
0.558883	0.880850	0.880850	0.880850	0.880850

TEMPERATURE AT 1110

0.558883	0.880850	0.880850	0.880850	0.880850
0.558883	0.880850	0.880850	0.880850	0.880850
0.558883	0.880850	0.880850	0.880850	0.880850
0.558883	0.880850	0.880850	0.880850	0.880850
0.558883	0.880850	0.880850	0.880850	0.880850

TEMPERATURE AT 1110

-0.003458	0.003458	0.003458	0.003458	0.003458
0.003458	0.003458	0.003458	0.003458	0.003458
0.003458	0.003458	0.003458	0.003458	0.003458
-0.003458	0.003458	0.003458	0.003458	0.003458
-0.003458	0.003458	0.003458	0.003458	0.003458

TEMPERATURES AT TIME= 2.00

SEMI-ANALYTICAL SOLUTION

0.1487973	0.2631049	0.3319727	0.3651958	0.3773845
0.2631049	0.4652249	0.5869975	0.6457429	0.6672953
0.3319727	0.5869975	0.7406441	0.8147661	0.8419598
0.3651958	0.6457429	0.8147661	0.8963061	0.9262213
0.3773845	0.6672953	0.8419598	0.9262213	0.9571349

ANALYTICAL SOLUTION

0.1466310	0.2614158	0.3317359	0.3653230	0.3771348
0.2614158	0.4660556	0.5914232	0.6513028	0.6723611
0.3317359	0.5914232	0.7505142	0.8265013	0.8532242
0.3653230	0.6513028	0.8265013	0.9101817	0.9396103
0.3771348	0.6723611	0.8532242	0.9396103	0.9699903

ASSOCIATED ERRORS

-0.0021663	-0.0016891	-0.0002368	0.0001273	-0.0002497
-0.0016891	0.0008308	0.0044257	0.0055599	0.0050658
-0.0002368	0.0044257	0.0098701	0.0117351	0.0112644
0.0001273	0.0055599	0.0117351	0.0138756	0.0133890
-0.0002497	0.0050658	0.0112644	0.0133890	0.0128554

TEMPERATURES AT TIME= 4.00

SEMI-ANALYTICAL SOLUTION

0.0766736	0.1438270	0.1950981	0.2283585	0.2443677
0.1438270	0.2697956	0.3659717	0.4283627	0.4583934
0.1950981	0.3659717	0.4964324	0.5810644	0.6218003
0.2283585	0.4283627	0.5810644	0.6801245	0.7278051
0.2443677	0.4583934	0.6218003	0.7278051	0.7788283

ANALYTICAL SOLUTION

0.0761437	0.1432251	0.1949468	0.2288583	0.2453135
0.1432251	0.2694040	0.3666917	0.4304787	0.4614306
0.1949468	0.3666917	0.4991120	0.5859340	0.6280632
0.2288583	0.4304787	0.5859340	0.6878588	0.7373166
0.2453135	0.4614306	0.6280632	0.7373166	0.7903304

ASSOCIATED ERRORS

-0.0005299	-0.0006019	-0.0001513	0.0004998	0.0009457
-0.0006019	-0.0003916	0.0007200	0.0021160	0.0030372
-0.0001513	0.0007200	0.0026797	0.0048696	0.0062629
0.0004998	0.0021160	0.0048696	0.0077343	0.0095115
0.0009457	0.0030372	0.0062629	0.0095115	0.0115020

TEMPERATURE AT TIME

TEMPERATURE AT TIME

0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000

TEMPERATURE AT TIME

0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000

TEMPERATURE AT TIME

0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000

TEMPERATURES AT TIME= 6.00

SEMI-ANALYTICAL SOLUTION

0.0501346	0.0955525	0.1323741	0.1580071	0.1710839
0.0955525	0.1821155	0.2522944	0.3011488	0.3260722
0.1323741	0.2522944	0.3495170	0.4171976	0.4517253
0.1580071	0.3011488	0.4171976	0.4979840	0.5391977
0.1710839	0.3260722	0.4517253	0.5391977	0.5838222

ANALYTICAL SOLUTION

0.0500907	0.0955885	0.1326420	0.1585677	0.1718467
0.0955885	0.1824121	0.2531216	0.3025958	0.3279362
0.1326420	0.2531216	0.3512406	0.4198928	0.4550561
0.1585677	0.3025958	0.4198928	0.5019635	0.5439997
0.1718467	0.3279362	0.4550561	0.5439997	0.5895561

ASSOCIATED ERRORS

-0.0000439	0.0000359	0.0002679	0.0005607	0.0007628
0.0000359	0.0002966	0.0008272	0.0014470	0.0018640
0.0002679	0.0008272	0.0017236	0.0026952	0.0033308
0.0005607	0.0014470	0.0026952	0.0039796	0.0048020
0.0007628	0.0018640	0.0033308	0.0048020	0.0057339

TEMPERATURES AT TIME= 9.00

SEMI-ANALYTICAL SOLUTION

0.0297368	0.0570011	0.0795645	0.0956368	0.1039848
0.0570011	0.1092628	0.1525136	0.1833219	0.1993237
0.0795645	0.1525136	0.2128850	0.2558884	0.2782245
0.0956368	0.1833219	0.2558884	0.3075786	0.3344267
0.1039848	0.1993237	0.2782245	0.3344267	0.3636182

ANALYTICAL SOLUTION

0.0297882	0.0571176	0.0797599	0.0959080	0.1043032
0.0571176	0.1095204	0.1529362	0.1838994	0.1999968
0.0797599	0.1529362	0.2135626	0.2568002	0.2792789
0.0959080	0.1838994	0.2568002	0.3087917	0.3358214
0.1043032	0.1999968	0.2792789	0.3358214	0.3652170

ASSOCIATED ERRORS

0.0000514	0.0001165	0.0001954	0.0002712	0.0003184
0.0001165	0.0002576	0.0004225	0.0005776	0.0006731
0.0001954	0.0004225	0.0006777	0.0009118	0.0010544
0.0002712	0.0005776	0.0009118	0.0012130	0.0013947
0.0003184	0.0006731	0.0010544	0.0013947	0.0015988

APPROXIMATE VALUES

0.000000	0.000000	0.000000	0.000000	0.000000
0.000001	0.000001	0.000001	0.000001	0.000001
0.000002	0.000002	0.000002	0.000002	0.000002
0.000003	0.000003	0.000003	0.000003	0.000003
0.000004	0.000004	0.000004	0.000004	0.000004

APPROXIMATE VALUES

0.000000	0.000000	0.000000	0.000000	0.000000
0.000001	0.000001	0.000001	0.000001	0.000001
0.000002	0.000002	0.000002	0.000002	0.000002
0.000003	0.000003	0.000003	0.000003	0.000003
0.000004	0.000004	0.000004	0.000004	0.000004

APPROXIMATE VALUES

0.000000	0.000000	0.000000	0.000000	0.000000
0.000001	0.000001	0.000001	0.000001	0.000001
0.000002	0.000002	0.000002	0.000002	0.000002
0.000003	0.000003	0.000003	0.000003	0.000003
0.000004	0.000004	0.000004	0.000004	0.000004

TEMPERATURES AT TIME= 12.00

SEMI-ANALYTICAL SOLUTION

0.0181818	0.0348844	0.0487527	0.0586668	0.0638305
0.0348844	0.0669307	0.0935389	0.1125606	0.1224678
0.0487527	0.0935389	0.1307253	0.1573090	0.1711548
0.0586668	0.1125606	0.1573090	0.1892988	0.2059602
0.0638305	0.1224678	0.1711548	0.2059602	0.2240881

ANALYTICAL SOLUTION

0.0181813	0.0348857	0.0487587	0.0586788	0.0638465
0.0348857	0.0669376	0.0935568	0.1125911	0.1225068
0.0487587	0.0935568	0.1307616	0.1573653	0.1712242
0.0586788	0.1125911	0.1573653	0.1893816	0.2060601
0.0638465	0.1225068	0.1712242	0.2060601	0.2242074

ASSOCIATED ERRORS

-0.0000005	0.0000013	0.0000061	0.0000120	0.0000161
0.0000013	0.0000070	0.0000179	0.0000305	0.0000390
0.0000061	0.0000179	0.0000363	0.0000563	0.0000693
0.0000120	0.0000305	0.0000563	0.0000828	0.0000999
0.0000161	0.0000390	0.0000693	0.0000999	0.0001193

TABLE 1. SUMMARY OF DATA

TABLE 1. SUMMARY OF DATA

1.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000

TABLE 1. SUMMARY OF DATA

1.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000

TABLE 1. SUMMARY OF DATA

1.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000

TEMPERATURES AT TIME= 15.00

SEMI-ANALYTICAL SOLUTION

0.0111718	0.0214379	0.0299664	0.0360667	0.0392454
0.0214379	0.0411378	0.0575034	0.0692096	0.0753093
0.0299664	0.0575034	0.0803797	0.0967429	0.1052691
0.0360667	0.0692096	0.0967429	0.1164372	0.1266992
0.0392454	0.0753093	0.1052691	0.1266992	0.1378656

ANALYTICAL SOLUTION

0.0111382	0.0213737	0.0298772	0.0359600	0.0391296
0.0213737	0.0410152	0.0573330	0.0690056	0.0750880
0.0298772	0.0573330	0.0801429	0.0964594	0.1049616
0.0359600	0.0690056	0.0964594	0.1160978	0.1263311
0.0391296	0.0750880	0.1049616	0.1263311	0.1374663

ASSOCIATED ERRORS

-0.0000336	-0.0000641	-0.0000892	-0.0001067	-0.0001158
-0.0000641	-0.0001226	-0.0001703	-0.0002040	-0.0002212
-0.0000892	-0.0001703	-0.0002368	-0.0002835	-0.0003075
-0.0001067	-0.0002040	-0.0002835	-0.0003394	-0.0003681
-0.0001158	-0.0002212	-0.0003075	-0.0003681	-0.0003993

TABLE 1. SUMMARY OF DATA

1. SUMMARY OF DATA

0.011111	0.011111	0.011111	0.011111	0.011111
0.011111	0.011111	0.011111	0.011111	0.011111
0.011111	0.011111	0.011111	0.011111	0.011111
0.011111	0.011111	0.011111	0.011111	0.011111
0.011111	0.011111	0.011111	0.011111	0.011111

2. SUMMARY OF DATA

0.011111	0.011111	0.011111	0.011111	0.011111
0.011111	0.011111	0.011111	0.011111	0.011111
0.011111	0.011111	0.011111	0.011111	0.011111
0.011111	0.011111	0.011111	0.011111	0.011111
0.011111	0.011111	0.011111	0.011111	0.011111

3. SUMMARY OF DATA

0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000

TEMPERATURES AT TIME= 20.00

SEMI-ANALYTICAL SOLUTION

0.0049683	0.0095341	0.0133274	0.0160411	0.0174552
0.0095341	0.0182957	0.0255752	0.0307826	0.0334962
0.0133274	0.0255752	0.0357509	0.0430302	0.0468236
0.0160411	0.0307826	0.0430302	0.0517918	0.0563575
0.0174552	0.0334962	0.0468236	0.0563575	0.0613256

ANALYTICAL SOLUTION

0.0049264	0.0094537	0.0132152	0.0159060	0.0173082
0.0094537	0.0181416	0.0253597	0.0305233	0.0332141
0.0132152	0.0253597	0.0354497	0.0426678	0.0464292
0.0159060	0.0305233	0.0426678	0.0513556	0.0558829
0.0173082	0.0332141	0.0464292	0.0558829	0.0608093

ASSOCIATED ERRORS

-0.0000419	-0.0000803	-0.0001123	-0.0001351	-0.0001470
-0.0000803	-0.0001541	-0.0002155	-0.0002593	-0.0002821
-0.0001123	-0.0002155	-0.0003011	-0.0003624	-0.0003943
-0.0001351	-0.0002593	-0.0003624	-0.0004362	-0.0004746
-0.0001470	-0.0002821	-0.0003943	-0.0004746	-0.0005164

TEMPERATURE BY T (1000 - 10000)

TEMPERATURE BY T (1000 - 10000)

0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000

TEMPERATURE BY T (1000 - 10000)

0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000

TEMPERATURE BY T (1000 - 10000)

0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000

TEMPERATURES AT TIME= 25.00

SEMI-ANALYTICAL SOLUTION

0.0022099	0.0042407	0.0059280	0.0071350	0.0077640
0.0042407	0.0081378	0.0113757	0.0136919	0.0148990
0.0059280	0.0113757	0.0159018	0.0191396	0.0208269
0.0071350	0.0136919	0.0191396	0.0230368	0.0250676
0.0077640	0.0148990	0.0208269	0.0250676	0.0272775

ANALYTICAL SOLUTION

0.0021792	0.0041818	0.0058456	0.0070359	0.0076561
0.0041818	0.0080248	0.0112177	0.0135018	0.0146920
0.0058456	0.0112177	0.0156809	0.0188738	0.0205376
0.0070359	0.0135018	0.0188738	0.0227168	0.0247194
0.0076561	0.0146920	0.0205376	0.0247194	0.0268986

ASSOCIATED ERRORS

-0.0000307	-0.0000589	-0.0000823	-0.0000991	-0.0001078
-0.0000589	-0.0001130	-0.0001580	-0.0001902	-0.0002069
-0.0000823	-0.0001580	-0.0002209	-0.0002658	-0.0002893
-0.0000991	-0.0001902	-0.0002658	-0.0003200	-0.0003482
-0.0001078	-0.0002069	-0.0002893	-0.0003482	-0.0003789

TEMPERATURE AT 1000

TEMPERATURE AT 1000

0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000

TEMPERATURE AT 1000

0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000

TEMPERATURE AT 1000

0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000

TEMPERATURES AT TIME= 30.00

SEMI-ANALYTICAL SOLUTION

0.0009829	0.0018862	0.0026367	0.0031736	0.0034534
0.0018862	0.0036197	0.0050598	0.0060901	0.0066270
0.0026367	0.0050598	0.0070730	0.0085132	0.0092637
0.0031736	0.0060901	0.0085132	0.0102467	0.0111500
0.0034534	0.0066270	0.0092637	0.0111500	0.0121329

ANALYTICAL SOLUTION

0.0009639	0.0018498	0.0025858	0.0031123	0.0033866
0.0018498	0.0035497	0.0049621	0.0059724	0.0064989
0.0025858	0.0049621	0.0069363	0.0083487	0.0090847
0.0031123	0.0059724	0.0083487	0.0100486	0.0109345
0.0033866	0.0064989	0.0090847	0.0109345	0.0118984

ASSOCIATED ERRORS

-0.0000190	-0.0000365	-0.0000510	-0.0000613	-0.0000667
-0.0000365	-0.0000700	-0.0000978	-0.0001177	-0.0001281
-0.0000510	-0.0000978	-0.0001367	-0.0001645	-0.0001790
-0.0000613	-0.0001177	-0.0001645	-0.0001980	-0.0002155
-0.0000667	-0.0001281	-0.0001790	-0.0002155	-0.0002345

TEMPERATURE AT 1000 FT

TEMPERATURE AT 1000 FT

0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000

TEMPERATURE AT 1000 FT

0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000

TEMPERATURE AT 1000 FT

0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000

TEMPERATURES AT TIME= 40.00

SEMI-ANALYTICAL SOLUTION

0.0001945	0.0003732	0.0005217	0.0006279	0.0006832
0.0003732	0.0007161	0.0010011	0.0012049	0.0013111
0.0005217	0.0010011	0.0013994	0.0016843	0.0018328
0.0006279	0.0012049	0.0016843	0.0020272	0.0022059
0.0006832	0.0013111	0.0018328	0.0022059	0.0024004

ANALYTICAL SOLUTION

0.0001886	0.0003619	0.0005060	0.0006090	0.0006627
0.0003619	0.0006946	0.0009709	0.0011686	0.0012716
0.0005060	0.0009709	0.0013572	0.0016336	0.0017776
0.0006090	0.0011686	0.0016336	0.0019662	0.0021395
0.0006627	0.0012716	0.0017776	0.0021395	0.0023281

ASSOCIATED ERRORS

-0.0000059	-0.0000112	-0.0000157	-0.0000189	-0.0000206
-0.0000112	-0.0000216	-0.0000301	-0.0000363	-0.0000395
-0.0000157	-0.0000301	-0.0000421	-0.0000507	-0.0000552
-0.0000189	-0.0000363	-0.0000507	-0.0000610	-0.0000664
-0.0000206	-0.0000395	-0.0000552	-0.0000664	-0.0000723

TEMPERATURE AT 1115

TEMPERATURE AT 1115

0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000

TEMPERATURE AT 1115

0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000

TEMPERATURE AT 1115

-0.000000	-0.000000	-0.000000	-0.000000	-0.000000
-0.000000	-0.000000	-0.000000	-0.000000	-0.000000
-0.000000	-0.000000	-0.000000	-0.000000	-0.000000
-0.000000	-0.000000	-0.000000	-0.000000	-0.000000
-0.000000	-0.000000	-0.000000	-0.000000	-0.000000

TEMPERATURES AT TIME= 50.00

SEMI-ANALYTICAL SOLUTION

0.0000385	0.0000738	0.0001032	0.0001242	0.0001352
0.0000738	0.0001417	0.0001981	0.0002384	0.0002594
0.0001032	0.0001981	0.0002769	0.0003332	0.0003626
0.0001242	0.0002384	0.0003332	0.0004011	0.0004364
0.0001352	0.0002594	0.0003626	0.0004364	0.0004749

ANALYTICAL SOLUTION

0.0000369	0.0000708	0.0000990	0.0001192	0.0001297
0.0000708	0.0001359	0.0001900	0.0002287	0.0002488
0.0000990	0.0001900	0.0002656	0.0003196	0.0003478
0.0001192	0.0002287	0.0003196	0.0003847	0.0004186
0.0001297	0.0002488	0.0003478	0.0004186	0.0004555

ASSOCIATED ERRORS

-0.0000016	-0.0000030	-0.0000042	-0.0000051	-0.0000055
-0.0000030	-0.0000058	-0.0000081	-0.0000097	-0.0000106
-0.0000042	-0.0000081	-0.0000113	-0.0000136	-0.0000148
-0.0000051	-0.0000097	-0.0000136	-0.0000164	-0.0000178
-0.0000055	-0.0000106	-0.0000148	-0.0000178	-0.0000194

TABLE 1. SUMMARY OF DATA

TABLE 1. SUMMARY OF DATA

1970-1971	1971-1972	1972-1973	1973-1974	1974-1975
1000000.0	1000000.0	1000000.0	1000000.0	1000000.0
1000000.0	1000000.0	1000000.0	1000000.0	1000000.0
1000000.0	1000000.0	1000000.0	1000000.0	1000000.0
1000000.0	1000000.0	1000000.0	1000000.0	1000000.0
1000000.0	1000000.0	1000000.0	1000000.0	1000000.0

TABLE 1. SUMMARY OF DATA

1970-1971	1971-1972	1972-1973	1973-1974	1974-1975
1000000.0	1000000.0	1000000.0	1000000.0	1000000.0
1000000.0	1000000.0	1000000.0	1000000.0	1000000.0
1000000.0	1000000.0	1000000.0	1000000.0	1000000.0
1000000.0	1000000.0	1000000.0	1000000.0	1000000.0
1000000.0	1000000.0	1000000.0	1000000.0	1000000.0

TABLE 1. SUMMARY OF DATA

1970-1971	1971-1972	1972-1973	1973-1974	1974-1975
1000000.0	1000000.0	1000000.0	1000000.0	1000000.0
1000000.0	1000000.0	1000000.0	1000000.0	1000000.0
1000000.0	1000000.0	1000000.0	1000000.0	1000000.0
1000000.0	1000000.0	1000000.0	1000000.0	1000000.0
1000000.0	1000000.0	1000000.0	1000000.0	1000000.0

TEMPERATURES AT TIME= 60.00

SEMI-ANALYTICAL SOLUTION

0.0000076	0.0000146	0.0000204	0.0000246	0.0000267
0.0000146	0.0000280	0.0000392	0.0000472	0.0000513
0.0000204	0.0000392	0.0000548	0.0000659	0.0000717
0.0000246	0.0000472	0.0000659	0.0000793	0.0000863
0.0000267	0.0000513	0.0000717	0.0000863	0.0000940

ANALYTICAL SOLUTION

0.0000072	0.0000139	0.0000194	0.0000233	0.0000254
0.0000139	0.0000266	0.0000372	0.0000447	0.0000487
0.0000194	0.0000372	0.0000520	0.0000625	0.0000681
0.0000233	0.0000447	0.0000625	0.0000753	0.0000819
0.0000254	0.0000487	0.0000681	0.0000819	0.0000891

ASSOCIATED ERRORS

-0.0000004	-0.0000007	-0.0000010	-0.0000013	-0.0000014
-0.0000007	-0.0000014	-0.0000020	-0.0000024	-0.0000026
-0.0000010	-0.0000020	-0.0000028	-0.0000034	-0.0000037
-0.0000013	-0.0000024	-0.0000034	-0.0000041	-0.0000044
-0.0000014	-0.0000026	-0.0000037	-0.0000044	-0.0000048

TABLE 1. SUMMARY OF DATA

TABLE 1. SUMMARY OF DATA

1970-1971	1971-1972	1972-1973	1973-1974	1974-1975
1975-1976	1976-1977	1977-1978	1978-1979	1979-1980
1980-1981	1981-1982	1982-1983	1983-1984	1984-1985
1985-1986	1986-1987	1987-1988	1988-1989	1989-1990
1990-1991	1991-1992	1992-1993	1993-1994	1994-1995

TABLE 1. SUMMARY OF DATA

1970-1971	1971-1972	1972-1973	1973-1974	1974-1975
1975-1976	1976-1977	1977-1978	1978-1979	1979-1980
1980-1981	1981-1982	1982-1983	1983-1984	1984-1985
1985-1986	1986-1987	1987-1988	1988-1989	1989-1990
1990-1991	1991-1992	1992-1993	1993-1994	1994-1995

TABLE 1. SUMMARY OF DATA

1970-1971	1971-1972	1972-1973	1973-1974	1974-1975
1975-1976	1976-1977	1977-1978	1978-1979	1979-1980
1980-1981	1981-1982	1982-1983	1983-1984	1984-1985
1985-1986	1986-1987	1987-1988	1988-1989	1989-1990
1990-1991	1991-1992	1992-1993	1993-1994	1994-1995

TEMPERATURES AT TIME= 70.00

SEMI-ANALYTICAL SOLUTION

0.0000015	0.0000029	0.0000040	0.0000049	0.0000053
0.0000029	0.0000055	0.0000078	0.0000093	0.0000102
0.0000040	0.0000078	0.0000108	0.0000130	0.0000142
0.0000049	0.0000093	0.0000130	0.0000157	0.0000171
0.0000053	0.0000102	0.0000142	0.0000171	0.0000186

ANALYTICAL SOLUTION

0.0000014	0.0000027	0.0000038	0.0000046	0.0000050
0.0000027	0.0000052	0.0000073	0.0000088	0.0000095
0.0000038	0.0000073	0.0000102	0.0000122	0.0000133
0.0000046	0.0000088	0.0000122	0.0000147	0.0000160
0.0000050	0.0000095	0.0000133	0.0000160	0.0000174

ASSOCIATED ERRORS

-0.0000001	-0.0000002	-0.0000002	-0.0000003	-0.0000003
-0.0000002	-0.0000003	-0.0000005	-0.0000006	-0.0000006
-0.0000002	-0.0000005	-0.0000007	-0.0000008	-0.0000009
-0.0000003	-0.0000006	-0.0000008	-0.0000010	-0.0000011
-0.0000003	-0.0000006	-0.0000009	-0.0000011	-0.0000011

TABLE 1.1 - 1990-1991

1990-1991

1990-1991	1990-1991	1990-1991	1990-1991	1990-1991
1990-1991	1990-1991	1990-1991	1990-1991	1990-1991
1990-1991	1990-1991	1990-1991	1990-1991	1990-1991
1990-1991	1990-1991	1990-1991	1990-1991	1990-1991
1990-1991	1990-1991	1990-1991	1990-1991	1990-1991

1991-1992

1991-1992	1991-1992	1991-1992	1991-1992	1991-1992
1991-1992	1991-1992	1991-1992	1991-1992	1991-1992
1991-1992	1991-1992	1991-1992	1991-1992	1991-1992
1991-1992	1991-1992	1991-1992	1991-1992	1991-1992
1991-1992	1991-1992	1991-1992	1991-1992	1991-1992

1992-1993

1992-1993	1992-1993	1992-1993	1992-1993	1992-1993
1992-1993	1992-1993	1992-1993	1992-1993	1992-1993
1992-1993	1992-1993	1992-1993	1992-1993	1992-1993
1992-1993	1992-1993	1992-1993	1992-1993	1992-1993
1992-1993	1992-1993	1992-1993	1992-1993	1992-1993

APPENDIX D

One-Dimensional, Non-Homogeneous Media

The p.d.e. is $c \frac{\partial u}{\partial t} = \frac{\partial}{\partial x} \left(k \frac{\partial u}{\partial x} \right).$

$$0 \leq x \leq 1$$

The boundary conditions are

$$\text{at } x = 0, \quad u = 1.0$$

$$\text{at } x = 6.3, \quad \frac{\partial u}{\partial x} = 0$$

The following values of the parameters were employed:

$$c = 1.0$$

$$k = 1.0 + 0.5x.$$

Semi-analytical Solution: A ten-point grid (Fig. III.4) was used.

Numerical Solution: A forty-point grid was used. The results at the grid points corresponding to the grid of semi-analytical solution were obtained by interpolation. An explicit formulation, with an initial Δt of 0.00001 and a Δt of 0.001 beyond a total time of 0.001 was employed.

Errors: The reported errors are (numerical solution minus the semi-analytical solution).


```

0 $IBFTC DARS1  NODECK
C
C
C      SOLUTION TO THE FORWARD PROBLEM-NON-HOMOGENEOUS MEDIUM
C      ONE-DIMENSIONAL SYSTEM
C
C      INPUT DATA
C
C      N=NUMBER OF GRID POINTS
C      NT=NUMBER OF TIMES
C      KZERO,A1,A2,DX,GZERO,TS-PARAMETERS
C      PZ=INITIAL CONDITION VECTOR
C      C1=BOUNDARY CONDITION VECTOR
C      TIME1=ACTUAL TIMES
C      GRIDX=ACTUAL DISTANCES OF THE GRID POINTS
C
1      REAL S(16,16),R(16),V(16,16),A(16),B(16),W1(16),W2(16)
2      REAL W(16,16),TK(16),PZ(16),P(15,16),C1(16),WORK(16)
3      REAL TIME1(15),GRIDX(16),TFIDIF(15,16),ERROR(15,16)
4      REAL KZERO,A2,DX,GZERO,TS
5      1 FORMAT(1X,2I4)
6      2 FORMAT(1X, 6F10.5)
7      3 FORMAT(1X,15F5.2)
0      7 FORMAT(1X,10F5.1)
1 220 FORMAT(1H2)
2 221 FORMAT(1H ,10X,8H ..CONTD)
3 222 FORMAT(1H ,//)
4 223 FORMAT(1H ,/)
5 224 FORMAT(1H ,18X,26H TEMPERATURE PROFILES-ONE-,
      111HDIMENSIONAL/25X,22HNON-HOMOGENEOUS MEDIUM)
6 231 FORMAT(1H ,30X,12H GRID POINTS)
7 232 FORMAT(1H ,10X,5F11.6)
0 233 FORMAT(1H ,28X,25H INITIAL CONDITION VECTOR)
1 234 FORMAT(1H ,10X,5F12.4)
2 235 FORMAT(1H ,28X,26H BOUNDARY CONDITION VECTOR)
3 240 FORMAT(1H ,30X,19H COEFFICIENT MATRIX)
4 241 FORMAT(1H ,10X,10F6.1)
5 242 FORMAT(1H ,31X,12H EIGENVALUES)
6 243 FORMAT(1H ,31X,13H EIGENVECTORS)
7 244 FORMAT(1H ,20X,35H CHECK OF SIMILARITY TRANSFORMATION)
0 245 FORMAT(1H ,30X,25H SEMI-ANALYTICAL SOLUTION)
1 246 FORMAT(1H ,25X,28H TEMPERATURES AT GRID POINTS)
2 247 FORMAT(1H ,30X,19H NUMERICAL SOLUTION)
3 248 FORMAT(1H ,15X,5H TIME,5X,2H 1, 9X,2H 2, 8X,2H 3, 8X,
      12H 4, 8X,2H 5)
4 249 FORMAT(1H ,15X,5H TIME, 9X,2H 6, 8X,2H 7, 8X,2H 8, 8X,
      12H 9, 8X,2H10)
5 251 FORMAT(1H ,15X,F6.1,2X,5F10.6)
6 255 FORMAT(1H ,30X,18H ASSOCIATED ERRORS)
7      READ(5,1) N,NT
2      READ(5,2) KZERO,A1,A2,DX,GZERO,TS
3      READ(5,2) (PZ(J),J=1,N),(C1(J),J=1,N)
4      READ(5,3) (TIME1(J),J=1,NT)
1      READ(5,7) (GRIDX(J),J=1,N)
6      DO 20 J=1,N

```


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N SOURCE STATEMENT

```
7      AJ=J
0      DELX=DX/2.+(AJ-1.)*DX
1      VALUE=A2*DELX
2      20 TK(J)=KZERO+VALUE
4      DO 21 J=1,N
5      DO 21 K=1,N
6      21 S(J,K)=0.0
1      N1=N-1
2      DO 22 J=1,N1
3      S(J,J)=-(TK(J)+TK(J+1))
4      S(J,J+1)=TK(J+1)
5      22 S(J+1,J)=S(J,J+1)
7      S(N,N)=-TK(N)
0      DO 23 J=1,N
1      DO 23 K=1,N
2      23 S(J,K)=S(J,K)/(GZERO*DX**2)
5      C1(1)=TK(1)*TS/(GZERO*DX**2)
6      MAXN=16
7      M=N
0      CALL EIG1(N,MAXN,M,S,R,V,A,B,W1,W2)
1      CALL CHECK(V,R,N,W)
2      WRITE(6,220)
3      LINES=9
4      CALL LINECT(LINES,5,2)
5      WRITE(6,224)
6      WRITE(6,223)
7      WRITE(6,231)
0      CALL LINECT(LINES,2,2)
1      WRITE(6,232) (GRIDX(J),J=1,N)
6      CALL LINECT(LINES,3,2)
7      WRITE(6,223)
0      WRITE(6,233)
1      CALL LINECT(LINES,2,2)
2      WRITE(6,234) (PZ(J),J=1,N)
7      CALL LINECT(LINES,3,2)
0      WRITE(6,223)
1      WRITE(6,235)
2      CALL LINECT(LINES,2,2)
3      WRITE(6,234) (C1(J),J=1,N)
0      CALL LINECT(LINES,3,2)
1      WRITE(6,223)
2      WRITE(6,240)
3      DO 31 J=1,N
4      CALL LINECT(LINES,1,2)
5      31 WRITE(6,241) (S(J,K),K=1,N)
3      CALL LINECT(LINES,5,2)
4      WRITE(6,223)
5      WRITE(6,242)
6      WRITE(6,232) (R(J),J=1,N)
3      CALL LINECT(LINES,3,2)
4      WRITE(6,223)
5      WRITE(6,243)
6      DO 32 K=1,N
7      CALL LINECT(LINES,4,2)
0      WRITE(6,232) (V(J,K),J=1,N)
```



```

5 32 WRITE(6,223)
7 CALL LINECT(LINES,1,2)
0 WRITE(6,244)
1 DO 33 J=1,N
2 CALL LINECT(LINES,4,2)
3 WRITE(6,232) (W(J,K),K=1,N)
0 33 WRITE(6,223)
2 CALL MAIN(N,WORK,V,C1,PZ,R,TIME1,NT,P)
3 CALL FIDIFF(40,N,NT,PZ,GZERO,TS,KZERO,A2,
16.3,GRIDX,TIME1,TFIDIF)
4 DO 27 J=1,NT
5 DO 27 K=1,N
6 27 ERROR(J,K)=TFIDIF(J,K)-P(J,K)
1 WRITE(6,220)
2 WRITE(6,245)
3 WRITE(6,223)
4 WRITE(6,246)
5 WRITE(6,248)
6 DO 34 J=1,NT
7 34 WRITE(6,251) TIME1(J),(P(J,K),K=1,5)
5 WRITE(6,223)
6 WRITE(6,246)
7 WRITE(6,249)
0 DO 35 J=1,NT
1 35 WRITE(6,251) TIME1(J),(P(J,K),K=6,10)
7 WRITE(6,220)
0 WRITE(6,247)
1 WRITE(6,223)
2 WRITE(6,246)
3 WRITE(6,248)
4 DO 36 J=1,NT
5 36 WRITE(6,251) TIME1(J),(TFIDIF(J,K),K=1,5)
3 WRITE(6,223)
4 WRITE(6,246)
5 WRITE(6,249)
6 DO 37 J=1,NT
7 37 WRITE(6,251) TIME1(J),(TFIDIF(J,K),K=6,10)
5 WRITE(6,220)
6 WRITE(6,255)
7 WRITE(6,223)
0 WRITE(6,248)
1 DO 38 J=1,NT
2 38 WRITE(6,251) TIME1(J),(ERROR(J,K),K=1,5)
0 WRITE(6,223)
1 WRITE(6,249)
2 DO 39 J=1,NT
3 39 WRITE(6,251) TIME1(J),(ERROR(J,K),K=6,10)
1 CONTINUE
2 END

```


SOURCE STATEMENT

```

0 $IBFTC FIDIFF
1 SUBROUTINE FIDIFF(N,NGRID,NT,TEMP,GZERO,TS,KZERO,A,
  1CAPX,GRIDX,TIME1,TGRID)
C
C FIDIFF CALCULATES THE FINITE-DIFFERENCE SOLUTION.
C
2 REAL TEMP(40),TPRIME(40),TK(40),GZERO,TS,KZERO,A,CAPX,MODLUS
3 REAL GRIDX(10),X(40),TGRID(15,10),TIME1(15)
4 AN=N
5 DX=CAPX/(AN+0.5)
6 DO 10 J=1,N
7 AJ=J
0 DELX=DX/2.+(AJ-1.0)*DX
1 VALUE=A*DELX
2 10 TK(J)=KZERO+VALUE
4 DO 101 J=1,N
5 AJ=J
6 101 X(J)=DX*AJ
0 DT=0.001
1 TSTORE=DT
2 DT=TSTORE/100.
3 TOTALT=0.0
4 LK=1
5 IF(TOTALT.GE.TIME1(LK)) GO TO 18
0 15 TPRIME(1)=TEMP(1)+DT*(-(TK(1)+TK(2))*TEMP(1)+TK(2)*TEMP(2)+TK(1)
  1S)/(GZERO*(DX**2))
1 N1=N-1
2 DO 17 J=2,N1
3 17 TPRIME(J)=TEMP(J)+DT*(TK(J)*TEMP(J-1)-(TK(J)+TK(J+1))*TEMP(J)+TK
  1+1)*TEMP(J+1))/(GZERO*(DX**2))
5 TPRIME(N)=TEMP(N)+DT*(TK(N)*TEMP(N-1)-TK(N)*TEMP(N))/(GZERO*(DX*
  1))
6 TOTALT=TOTALT+DT
7 IF(TOTALT.GE.TIME1(LK)) GO TO 18
2 GO TO 20
3 18 DO 181 KA=1,NGRID
4 CALL INTERP(N,GRIDX,X,TPRIME,KA,RR)
5 181 TGRID(LK,KA)=RR
7 LK=LK+1
0 IF(LK.GT.NT) GO TO 21
3 20 DO 19 J=1,N
4 19 TEMP(J)=TPRIME(J)
6 IF(TOTALT.GE.TSTORE) DT=TSTORE
1 IF(TOTALT.LT.TIME1(NT)) GO TO 15
4 21 CONTINUE
5 RETURN
6 END

```


PROGRAM IS BEING ENTERED INTO STORAGE.

TEMPERATURE PROFILES-ONE-DIMENSIONAL NON-HOMOGENEOUS MEDIUM

GRID POINTS

0.600000	1.200000	1.800000	2.400000	3.000000
3.600000	4.200000	4.800000	5.400000	6.000000

INITIAL CONDITION VECTOR

-0.0000	-0.0000	-0.0000	-0.0000	-0.0000
-0.0000	-0.0000	-0.0000	-0.0000	-0.0000

BOUNDARY CONDITION VECTOR

3.1944	-0.0000	-0.0000	-0.0000	-0.0000
-0.0000	-0.0000	-0.0000	-0.0000	-0.0000

COEFFICIENT MATRIX

-7.2	4.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.0	-8.9	4.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	4.9	-10.6	5.7	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	5.7	-12.2	6.5	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	6.5	-13.9	7.4	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	7.4	-15.6	8.2	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	8.2	-17.2	9.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	9.0	-18.9	9.9	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	9.9	-20.6	10.7
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.7	-10.7

EIGENVALUES

-0.107010	-1.234355	-3.377255	-6.317883	-9.757519
-13.417948	-17.314207	-21.824825	-27.412357	-34.931070

EIGENVECTORS

-0.101019	-0.178453	-0.238685	-0.285617	-0.321875
-0.349350	-0.369468	-0.383350	-0.391899	-0.395860

0.302286	-0.449391	0.457167	0.364707	0.215087
0.046338	-0.112230	-0.240816	-0.328392	-0.371240

0.456005	-0.435308	0.115729	-0.225719	-0.406798
-0.380740	-0.200414	0.038241	0.243631	0.356079

COGAM IS BEING ESTABLISHED IN 1992.

104-1000-1102-0010

CONFIDENTIAL

[illegible]

20110513

010101.0- 010101.0- 010101.0- 010101.0- 010101.0-
010101.0- 010101.0- 010101.0- 010101.0- 010101.0-

1957-1958

010101.0-	010101.0-	010101.0-	010101.0-	010101.0-
010101.0-	010101.0-	010101.0-	010101.0-	010101.0-

100000.0	50000.0	100000.0	100000.0	100000.0
100000.0	50000.0	100000.0	100000.0	100000.0

[illegible]

..CONTD

-0.534439	-0.119995	0.379356	0.384743	0.017070
-0.323630	-0.380165	-0.165432	0.137145	0.335125
0.505223	-0.318014	-0.361788	0.220774	0.398960
0.028133	-0.338482	-0.305413	0.027066	0.308943
-0.352973	0.542961	-0.213409	-0.356230	0.251418
0.331987	-0.139248	-0.360020	-0.072259	0.283740
0.153889	-0.385585	0.540790	-0.312699	-0.227833
0.383315	0.122398	-0.349180	-0.167814	0.271109
0.036251	-0.131428	0.319706	-0.520502	0.486785
-0.063221	-0.388914	0.255664	0.279929	-0.268966
-0.004092	0.020514	-0.074780	0.203853	-0.409132
0.570862	-0.458473	-0.000664	0.420303	-0.268867
0.000156	-0.001074	0.005623	-0.023152	0.075636
-0.195679	0.394732	-0.596689	0.609327	-0.268866

CHECK OF SIMILARITY TRANSFORMATION

-7.222220	4.027778	-0.000001	0.000002	0.000001
-0.000000	0.000001	0.000001	-0.000001	0.000001
4.027778	-8.888887	4.861111	-0.000001	0.000000
0.000001	-0.000001	-0.000002	0.000001	0.000000
-0.000001	4.861111	-10.555553	5.694444	-0.000000
0.000000	0.000000	-0.000001	0.000000	0.000001
0.000002	-0.000001	5.694444	-12.222225	6.527778
0.000002	-0.000002	-0.000002	0.000002	-0.000000
0.000001	0.000000	-0.000000	6.527778	-13.888884
7.361110	-0.000000	0.000001	0.000002	-0.000001
-0.000000	0.000001	0.000000	0.000002	7.361110
-15.555551	8.194445	-0.000002	-0.000000	0.000001

..COTID

0.353830	-0.34430	-0.11300	-0.11300	0.34430	0.353830
0.505253	-0.318014	-0.318014	-0.318014	0.318014	0.505253
0.331987	-0.132548	-0.300000	-0.300000	0.300000	0.331987
0.123889	-0.382882	-0.200000	-0.200000	0.200000	0.123889
0.383312	0.155328	-0.341800	-0.341800	0.341800	0.383312
0.038251	-0.131428	0.314400	0.314400	0.314400	0.038251
-0.083251	-0.388014	0.322000	0.322000	0.322000	-0.083251
0.250885	-0.448473	-0.000000	-0.000000	0.000000	0.250885
0.000120	-0.344335	0.000000	0.000000	0.000000	0.000120
-0.122879	0.344335	-0.300000	-0.300000	0.300000	-0.122879

CHECK OF LIMITS TRANSFERENTIAL

-0.000000	0.000001	0.000001	-0.000001	0.000001	0.000000
0.000001	-0.000001	-0.000001	0.000001	0.000001	0.000001
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	-0.000001	-0.000001	0.000001	0.000001	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	-0.000001	-0.000001	0.000001	0.000001	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	-0.000001	-0.000001	0.000001	0.000001	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	-0.000001	-0.000001	0.000001	0.000001	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

..CONTD

0.000001	-0.000001	0.000000	-0.000002	-0.000000
8.194445	-17.222220	9.027774	0.000001	0.000001

0.000001	-0.000002	-0.000001	-0.000002	0.000001
-0.000002	9.027774	-18.888887	9.861112	-0.000000

-0.000001	0.000001	0.000000	0.000002	0.000001
-0.000000	0.000001	9.861112	-20.555549	10.694441

0.000001	0.000000	0.000001	-0.000000	-0.000001
0.000001	0.000001	-0.000000	10.694441	-10.694443

1	2	3	4	5	6
10.00	10.000000	10.000000	10.000000	10.000000	10.000000
11.00	11.000000	11.000000	11.000000	11.000000	11.000000
12.00	12.000000	12.000000	12.000000	12.000000	12.000000
13.00	13.000000	13.000000	13.000000	13.000000	13.000000
14.00	14.000000	14.000000	14.000000	14.000000	14.000000
15.00	15.000000	15.000000	15.000000	15.000000	15.000000
16.00	16.000000	16.000000	16.000000	16.000000	16.000000
17.00	17.000000	17.000000	17.000000	17.000000	17.000000
18.00	18.000000	18.000000	18.000000	18.000000	18.000000
19.00	19.000000	19.000000	19.000000	19.000000	19.000000
20.00	20.000000	20.000000	20.000000	20.000000	20.000000

41110

1000000.00 - 2000000.00	2000000.00 - 4000000.00	4000000.00 - 6000000.00	6000000.00 - 8000000.00	8000000.00 - 10000000.00
1000000.00	2000000.00	3000000.00	4000000.00	5000000.00

SEMI-ANALYTICAL SOLUTION

TIME	TEMPERATURES AT GRID POINTS				
	1	2	3	4	5
0.0	-0.000000	0.000000	-0.000000	0.000000	-0.000000
0.3	0.443562	0.166755	0.056084	0.017431	0.005110
0.6	0.572120	0.302380	0.150358	0.071288	0.032542
1.0	0.650484	0.407695	0.247675	0.146586	0.084892
1.5	0.702167	0.485284	0.330511	0.222257	0.147993
2.0	0.733804	0.535549	0.388557	0.280596	0.202326
4.0	0.799750	0.646727	0.528298	0.436573	0.366147
6.0	0.839558	0.716614	0.621034	0.546607	0.489143
8.0	0.870574	0.771368	0.694206	0.634087	0.587643
10.0	0.895519	0.815432	0.753137	0.704598	0.667098
12.0	0.915650	0.850993	0.800701	0.761514	0.731239
14.0	0.931902	0.879703	0.839100	0.807463	0.783022
16.0	0.945023	0.902880	0.870101	0.844560	0.824827
18.0	0.955615	0.921593	0.895129	0.874509	0.858578
20.0	0.964167	0.936699	0.915335	0.898688	0.885827

TIME	TEMPERATURES AT GRID POINTS				
	6	7	8	9	10
0.0	0.000000	-0.000000	0.000000	-0.000000	0.000000
0.3	0.001432	0.000387	0.000102	0.000026	0.000008
0.6	0.014407	0.006224	0.002651	0.001162	0.000633
1.0	0.048321	0.027230	0.015482	0.009377	0.006838
1.5	0.098076	0.065388	0.044882	0.033129	0.027916
2.0	0.146605	0.108041	0.082645	0.067532	0.060672
4.0	0.313093	0.274440	0.247877	0.231566	0.224021
6.0	0.445626	0.413778	0.391812	0.378288	0.372023
8.0	0.552453	0.526687	0.508909	0.497961	0.492888
10.0	0.638684	0.617877	0.603521	0.594680	0.590583
12.0	0.708299	0.691501	0.679910	0.672772	0.669465
14.0	0.764501	0.750940	0.741582	0.735820	0.733150
16.0	0.809876	0.798927	0.791373	0.786720	0.784565
18.0	0.846507	0.837669	0.831570	0.827814	0.826074
20.0	0.876082	0.868946	0.864022	0.860990	0.859585

TEMPERATURES AT AIR POINTS

TIME	1	2	3	4	5
0.0	0.00000	0.00000	0.00000	0.00000	0.00000
0.3	0.00000	0.00000	0.00000	0.00000	0.00000
0.6	0.00000	0.00000	0.00000	0.00000	0.00000
1.0	0.00000	0.00000	0.00000	0.00000	0.00000
1.5	0.00000	0.00000	0.00000	0.00000	0.00000
2.0	0.00000	0.00000	0.00000	0.00000	0.00000
4.0	0.00000	0.00000	0.00000	0.00000	0.00000
6.0	0.00000	0.00000	0.00000	0.00000	0.00000
8.0	0.00000	0.00000	0.00000	0.00000	0.00000
10.0	0.00000	0.00000	0.00000	0.00000	0.00000
12.0	0.00000	0.00000	0.00000	0.00000	0.00000
14.0	0.00000	0.00000	0.00000	0.00000	0.00000
16.0	0.00000	0.00000	0.00000	0.00000	0.00000
18.0	0.00000	0.00000	0.00000	0.00000	0.00000
20.0	0.00000	0.00000	0.00000	0.00000	0.00000

TIME	6	7	8	9	10
0.0	0.00000	0.00000	0.00000	0.00000	0.00000
0.3	0.00000	0.00000	0.00000	0.00000	0.00000
0.6	0.00000	0.00000	0.00000	0.00000	0.00000
1.0	0.00000	0.00000	0.00000	0.00000	0.00000
1.5	0.00000	0.00000	0.00000	0.00000	0.00000
2.0	0.00000	0.00000	0.00000	0.00000	0.00000
4.0	0.00000	0.00000	0.00000	0.00000	0.00000
6.0	0.00000	0.00000	0.00000	0.00000	0.00000
8.0	0.00000	0.00000	0.00000	0.00000	0.00000
10.0	0.00000	0.00000	0.00000	0.00000	0.00000
12.0	0.00000	0.00000	0.00000	0.00000	0.00000
14.0	0.00000	0.00000	0.00000	0.00000	0.00000
16.0	0.00000	0.00000	0.00000	0.00000	0.00000
18.0	0.00000	0.00000	0.00000	0.00000	0.00000
20.0	0.00000	0.00000	0.00000	0.00000	0.00000

NUMERICAL SOLUTION

TEMPERATURES AT GRID POINTS					
TIME	1	2	3	4	5
0.0	-0.000000	-0.000000	-0.000000	-0.000000	-0.000000
0.3	0.440599	0.153737	0.044161	0.010796	0.002306
0.6	0.571686	0.297934	0.143427	0.064517	0.027374
1.0	0.649988	0.405557	0.244009	0.142185	0.080556
1.5	0.701541	0.483776	0.328143	0.219279	0.144716
2.0	0.733148	0.534269	0.386688	0.278239	0.199618
4.0	0.798885	0.645322	0.526498	0.434462	0.363785
6.0	0.838726	0.715271	0.619336	0.544646	0.486980
8.0	0.869835	0.770167	0.692684	0.632328	0.585704
10.0	0.894873	0.814375	0.751792	0.703040	0.665377
12.0	0.915088	0.850070	0.799522	0.760144	0.729724
14.0	0.931416	0.878900	0.838071	0.806266	0.781695
16.0	0.944603	0.902185	0.869208	0.843518	0.823672
18.0	0.955255	0.920993	0.894356	0.873606	0.857576
20.0	0.963858	0.936183	0.914668	0.897908	0.884960

TEMPERATURES AT GRID POINTS					
TIME	6	7	8	9	10
0.0	-0.000000	0.000000	-0.000000	-0.000000	-0.000000
0.3	0.000439	0.000076	0.000012	0.000002	0.000000
0.6	0.011039	0.004259	0.001584	0.000586	0.000264
1.0	0.044549	0.024196	0.013120	0.007495	0.005191
1.5	0.094758	0.062187	0.041851	0.030241	0.025096
2.0	0.143672	0.104983	0.079523	0.064377	0.057488
4.0	0.310529	0.271716	0.245030	0.228627	0.221019
6.0	0.443306	0.411335	0.389274	0.375680	0.369365
8.0	0.550375	0.524501	0.506642	0.495634	0.490519
10.0	0.636838	0.615935	0.601507	0.592613	0.588481
12.0	0.706672	0.689788	0.678134	0.670950	0.667613
14.0	0.763076	0.749439	0.740026	0.734224	0.731528
16.0	0.808633	0.797618	0.790015	0.785329	0.783151
18.0	0.845429	0.836532	0.830391	0.826606	0.824847
20.0	0.875149	0.867963	0.863002	0.859945	0.858524

TEMPERATURES AT 1000 FT					TIME
	I	J	K	L	
000000.0 -	000000.0 -	000000.0 -	000000.0 -	000000.0 -	0.0
000000.0 -	000000.0 -	000000.0 -	000000.0 -	000000.0 -	0.0
000000.0 -	000000.0 -	000000.0 -	000000.0 -	000000.0 -	0.0
000000.0 -	000000.0 -	000000.0 -	000000.0 -	000000.0 -	0.0
000000.0 -	000000.0 -	000000.0 -	000000.0 -	000000.0 -	0.0
000000.0 -	000000.0 -	000000.0 -	000000.0 -	000000.0 -	0.0
000000.0 -	000000.0 -	000000.0 -	000000.0 -	000000.0 -	0.0
000000.0 -	000000.0 -	000000.0 -	000000.0 -	000000.0 -	0.0
000000.0 -	000000.0 -	000000.0 -	000000.0 -	000000.0 -	0.0
000000.0 -	000000.0 -	000000.0 -	000000.0 -	000000.0 -	0.0
000000.0 -	000000.0 -	000000.0 -	000000.0 -	000000.0 -	0.0
000000.0 -	000000.0 -	000000.0 -	000000.0 -	000000.0 -	0.0
000000.0 -	000000.0 -	000000.0 -	000000.0 -	000000.0 -	0.0
000000.0 -	000000.0 -	000000.0 -	000000.0 -	000000.0 -	0.0
000000.0 -	000000.0 -	000000.0 -	000000.0 -	000000.0 -	0.0
000000.0 -	000000.0 -	000000.0 -	000000.0 -	000000.0 -	0.0
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TIME	Δ	γ	θ	φ	ψ
0.0	0.00000	0.00000	0.00000	0.00000	0.00000
0.3	0.00039	0.00000	0.00000	0.00000	0.00000
0.6	0.00110	0.00000	0.00000	0.00000	0.00000
1.0	0.00440	0.00000	0.00000	0.00000	0.00000
1.5	0.00478	0.00000	0.00000	0.00000	0.00000
2.0	0.01435	0.00000	0.00000	0.00000	0.00000
4.0	0.31028	0.00000	0.00000	0.00000	0.00000
6.0	0.44266	0.00000	0.00000	0.00000	0.00000
8.0	0.25072	0.00000	0.00000	0.00000	0.00000
10.0	0.06868	0.00000	0.00000	0.00000	0.00000
12.0	0.10612	0.00000	0.00000	0.00000	0.00000
14.0	0.15207	0.00000	0.00000	0.00000	0.00000
16.0	0.08683	0.00000	0.00000	0.00000	0.00000
18.0	0.06459	0.00000	0.00000	0.00000	0.00000
20.0	0.08714	0.00000	0.00000	0.00000	0.00000

ASSOCIATED ERRORS

TIME	1	2	3	4	5
0.0	-0.000000	-0.000000	-0.000000	-0.000000	-0.000000
0.3	-0.002963	-0.013018	-0.011923	-0.006635	-0.002804
0.6	-0.000434	-0.004445	-0.006931	-0.006772	-0.005169
1.0	-0.000496	-0.002138	-0.003666	-0.004401	-0.004336
1.5	-0.000626	-0.001508	-0.002368	-0.002978	-0.003277
2.0	-0.000656	-0.001280	-0.001869	-0.002356	-0.002709
4.0	-0.000865	-0.001405	-0.001800	-0.002111	-0.002361
6.0	-0.000832	-0.001343	-0.001698	-0.001961	-0.002163
8.0	-0.000740	-0.001201	-0.001522	-0.001759	-0.001940
10.0	-0.000647	-0.001057	-0.001345	-0.001558	-0.001721
12.0	-0.000562	-0.000923	-0.001180	-0.001370	-0.001516
14.0	-0.000486	-0.000803	-0.001029	-0.001198	-0.001327
16.0	-0.000419	-0.000695	-0.000893	-0.001042	-0.001156
18.0	-0.000361	-0.000600	-0.000773	-0.000903	-0.001002
20.0	-0.000309	-0.000516	-0.000667	-0.000780	-0.000867

TIME	6	7	8	9	10
0.0	-0.000000	0.000000	-0.000000	-0.000000	-0.000000
0.3	-0.000993	-0.000311	-0.000090	-0.000025	-0.000008
0.6	-0.003367	-0.001965	-0.001067	-0.000575	-0.000369
1.0	-0.003771	-0.003035	-0.002362	-0.001883	-0.001647
1.5	-0.003317	-0.003201	-0.003031	-0.002888	-0.002820
2.0	-0.002933	-0.003059	-0.003122	-0.003156	-0.003184
4.0	-0.002563	-0.002724	-0.002847	-0.002939	-0.003002
6.0	-0.002320	-0.002443	-0.002538	-0.002608	-0.002659
8.0	-0.002079	-0.002186	-0.002267	-0.002327	-0.002369
10.0	-0.001846	-0.001942	-0.002014	-0.002067	-0.002102
12.0	-0.001627	-0.001712	-0.001776	-0.001822	-0.001852
14.0	-0.001426	-0.001501	-0.001557	-0.001596	-0.001622
16.0	-0.001243	-0.001309	-0.001357	-0.001392	-0.001414
18.0	-0.001079	-0.001136	-0.001179	-0.001208	-0.001227
20.0	-0.000933	-0.000983	-0.001020	-0.001045	-0.001061

TIME	1	2	3	4	5
0.0	000000.0-	000000.0-	000000.0-	000000.0-	000000.0-
0.1	000000.0-	000000.0-	000000.0-	000000.0-	000000.0-
0.2	000000.0-	000000.0-	000000.0-	000000.0-	000000.0-
0.3	000000.0-	000000.0-	000000.0-	000000.0-	000000.0-
0.4	000000.0-	000000.0-	000000.0-	000000.0-	000000.0-
0.5	000000.0-	000000.0-	000000.0-	000000.0-	000000.0-
0.6	000000.0-	000000.0-	000000.0-	000000.0-	000000.0-
0.7	000000.0-	000000.0-	000000.0-	000000.0-	000000.0-
0.8	000000.0-	000000.0-	000000.0-	000000.0-	000000.0-
0.9	000000.0-	000000.0-	000000.0-	000000.0-	000000.0-
1.0	000000.0-	000000.0-	000000.0-	000000.0-	000000.0-
1.1	000000.0-	000000.0-	000000.0-	000000.0-	000000.0-
1.2	000000.0-	000000.0-	000000.0-	000000.0-	000000.0-
1.3	000000.0-	000000.0-	000000.0-	000000.0-	000000.0-
1.4	000000.0-	000000.0-	000000.0-	000000.0-	000000.0-
1.5	000000.0-	000000.0-	000000.0-	000000.0-	000000.0-
1.6	000000.0-	000000.0-	000000.0-	000000.0-	000000.0-
1.7	000000.0-	000000.0-	000000.0-	000000.0-	000000.0-
1.8	000000.0-	000000.0-	000000.0-	000000.0-	000000.0-
1.9	000000.0-	000000.0-	000000.0-	000000.0-	000000.0-
2.0	000000.0-	000000.0-	000000.0-	000000.0-	000000.0-

[illegible]

APPENDIX E

A Two-Dimensional Petroleum Reservoir

The p.d.e. describing the process is

$$\frac{\partial}{\partial x} \left(\frac{kh}{\mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{kh}{\mu} \frac{\partial p}{\partial y} \right) - \frac{0.887 Q_{xy} B.}{A} =$$

$$\frac{0.887 S_o h \phi c}{5.61} \frac{\partial p}{\partial t}$$

- where
- k = permeability, darcies.
 - h = formation height, ft.
 - μ = viscosity of oil, assumed constant, cp
 - B_o = formation volume factor, reservoir bbls/stock tank bbl.
 - S_o = fraction of oil saturation
 - ϕ = porosity
 - c = compressibility, assumed constant, 1/psi.
 - p = pressure of oil, psi.
 - t = time, days
 - x, y = distance coordinates.
 - $Q_{x,y}$ = Production of oil, stock tank barrels/day
 - A = Area of production, ft².

Fig. IV.1 represents the reservoir formation.

The computer program calculates the pressure distribution in the reservoir at various elapsed times using the semi-analytical method. The amount of oil produced is calculated by two methods using equations (IV.12) and (IV.17).

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PRESSURE DISTRIBUTION IN A PETROLEUM RESERVOIR

INPUT DATA

C N=NUMBER OF GRID POINTS
C PZ=INITIAL PRESSURE DISTRIBUTION
C NT=NUMBER OF TIMES FED IN
C C1=PRODUCTION AND BOUNDARY CONDITION VECTOR
C TIME1=ELAPSED TIMES
C DIAG=PHI-H, FEET
C TK=PERMEABILITY-H, DARCY-FEET
C A21 TO A40-AREAS THAT ARE DIFFERENT FROM OTHERS, SQ.FT
C DX,DY-NORMAL GRID SPACINGS, FEET
C DYSTAR-THE ODD GRID SPACING IN Y-DIRECTION
C COMP=COMPRESSIBILITY, 1/PSI
C VISC=VISCOSITY, ASSUMED CONSTANT, CP
C FACTOR=FACTOR TO CONVERT THE PRODUCTION VECTOR TO THE CONSIS-
C TENT UNITS OF THE FLOW EQUATION(REF-CALHOUN,P.84)
C FVF=FORMATION VOLUME FACTOR, ASSUMED CONSTANT

REAL S(40,40),R(40),V(40,40),A(40),B(40),W1(40),W2(40),W(40,40)
REAL TK(40),PZ(40),PZSAVE(40),P(25,40),C1(40),WORK(40)
REAL TIME1(25),DIAG(40),DINV(40),AREA(40)

1 FORMAT(1X,2I4)
2 FORMAT(1X,10F7.1)
3 FORMAT(1X,15F5.2)
5 FORMAT(1X,10F7.1)
6 FORMAT(1X,6F12.6)
30 FORMAT(1H ,18X,22H INITIAL PRESSURES,PSI)
31 FORMAT(1H ,20X,19H PRODUCTION,STB/DAY)
32 FORMAT(1H ,25X,18H PHI-H MATRIX,FEET)
33 FORMAT(1H ,23X,22H K-H MATRIX,DARCY-FEET)
46 FORMAT(1H ,30X,6H TIME=,F6.1,5H DAYS)
47 FORMAT(1H ,34X,5F8.2)
48 FORMAT(1H ,18X,7F8.2)
49 FORMAT(1H ,10X,8F8.2)
50 FORMAT(1H ,//)
51 FORMAT(1H ,20X,15H PRODUCTION,STB,11X,17H MATERIAL BALANCE)
52 FORMAT(1H ,20X,E14.7,15X,E14.7)
220 FORMAT(1H2)
223 FORMAT(1H ,/)
READ(5,1) N,NT
READ(5,2) (PZ(J),J=1,N)
READ(5,2) (C1(J),J=1,N)
READ(5,3) (TIME1(J),J=1,NT)
READ(5,5) (DIAG(J),J=1,N)
READ(5,5) (TK(J),J=1,N)
READ(5,6) A21,A29,A30,A36,A37,A38,A39,A40,DX,DY,DYSTAR
READ(5,6) COMP,VISC,FACTOR,FVF
DO 17 J=1,N
17 AREA(J)=DX*DY

SOURCE STATEMENT

```
AREA(21)=A21
AREA(29)=A29
AREA(30)=A30
AREA(36)=A36
AREA(37)=A37
AREA(38)=A38
AREA(39)=A39
AREA(40)=A40
```

```
C
C STORE CAPACITIES OF INDIVIDUAL ELEMENTS IN THE ARRAY AREA
C
```

```
DO 18 J=1,N
PZSAVE(J)=PZ(J)
18 AREA(J)=AREA(J)*DIAG(J)*COMP/(5.61*FVF)
```

```
C
C CALCULATE THE TOTAL PRODUCTION,STB/DAY
C
```

```
PROD=0.0
DO 19 J=1,N
19 PROD=PROD+C1(J)
PROD=-PROD
WRITE(6,220)
WRITE(6,30)
WRITE(6,47) (PZ(J),J=36,40)
WRITE(6,48) (PZ(J),J=29,35)
WRITE(6,49) (PZ(J),J=21,28)
WRITE(6,49) (PZ(J),J=13,20)
WRITE(6,49) (PZ(J),J= 7,12)
WRITE(6,49) (PZ(J),J= 1,6)
WRITE(6,223)
WRITE(6,31)
WRITE(6,47) (C1(J),J=36,40)
WRITE(6,48) (C1(J),J=29,35)
WRITE(6,49) (C1(J),J=21,28)
WRITE(6,49) (C1(J),J=13,20)
WRITE(6,49) (C1(J),J= 7,12)
WRITE(6,49) (C1(J),J= 1,6)
WRITE(6,223)
WRITE(6,32)
WRITE(6,47) (TK(J),J=36,40)
WRITE(6,48) (TK(J),J=29,35)
WRITE(6,49) (TK(J),J=21,28)
WRITE(6,49) (TK(J),J=13,20)
WRITE(6,49) (TK(J),J= 7,12)
WRITE(6,49) (TK(J),J= 1,6)
WRITE(6,223)
WRITE(6,33)
WRITE(6,47) (DIAG(J),J=36,40)
WRITE(6,48) (DIAG(J),J=29,35)
WRITE(6,49) (DIAG(J),J=21,28)
WRITE(6,49) (DIAG(J),J=13,20)
WRITE(6,49) (DIAG(J),J= 7,12)
WRITE(6,49) (DIAG(J),J= 1,6)
```

```
C
C CONVERT DIAG,TK, AND C1 INTO RELEVANT UNITS
```



```

C
DO 7 J=1,N
DIAG(J)=DIAG(J)*COMP*0.887/5.61
TK(J)=TK(J)/VISC
7 C1(J)=C1(J)*FVF*0.887/(DX*DY)

C
C
C FORM THE COEFFICIENT MATRIX

CALL MATRIX(TK,C1,A21,A29,A30,A36,A37,A38,A39,A40,
1DX,DY,DYSTAR,N,S,DIAG)
CALL TRANS(DIAG,C1,DX,DY,A21,A29,A30,A36,A37,
1A38,A39,A40)

C
C TRANSFORM S,C1,AND PZ

CALL DERIVE(DIAG,DINV,S,C1,PZ,N)

C
C CALCULATE THE EIGENVALUES AND EIGENVECTORS OF MATRIX S

MAXN=40
M=N
CALL EIG1(N,MAXN,M,S,R,V,A,B,W1,W2)

C
C CALCULATE THE PRESSURES AT VARIOUS TIMES

CALL MAIN(N,WORK,V,C1,PZ,R,TIME1,NT,P)
DO 15 J=1,NT
DO 14 K=1,N
14 P(J,K)=P(J,K)*DINV(K)
15 CONTINUE

C
C CALCULATE THE PRODUCTION BY A MATERIAL BALANCE

CALL BALANS(N,NT,P,TIME1,PROD,COMP,AREA,WORK,
1PZ,PZSAVE)

C
C OUTPUT RESULTS

WRITE(6,220)
LINES=9
DO 25 J=1,NT
CALL LINECT(LINES,15,2)
WRITE(6,46) TIME1(J)
WRITE(6,47) (P(J,K),K=36,40)
WRITE(6,48) (P(J,K),K=29,35)
WRITE(6,49) (P(J,K),K=21,28)
WRITE(6,49) (P(J,K),K=13,20)
WRITE(6,49) (P(J,K),K= 7,12)
WRITE(6,49) (P(J,K),K= 1,6)
WRITE(6,51)
WRITE(6,52) WORK(J),PZ(J)
25 WRITE(6,223)
END

```


\$IBFTC MATRIX

SUBROUTINE MATRIX(TK,C1,A21,A29,A30,A36,A37,A38,A39,A40,
1DX,DY,DYSTAR,N,S,DIAG)

MATRIX CALCULATES THE COEFFICIENT MATRIX FOR THE RESERVOIR.

REAL TK(40),C1(40),S(40,40) ,DIAG(40)

DO 1 J=1,N

DO 1 K=1,N

1 S(J,K)=0.0

DXX=2.*(DX**2)

DYY=2.*(DY**2)

S(1,2)=(TK(1)+TK(2))/DXX

S(1,7)=(TK(1)+TK(7))/DYY

S(2,3)=(TK(2)+TK(3))/DXX

S(2,8)=(TK(2)+TK(8))/DYY

S(3,4)=(TK(3)+TK(4))/DXX

S(3,9)=(TK(3)+TK(9))/DYY

S(4,5)=(TK(4)+TK(5))/DXX

S(4,10)=(TK(4)+TK(10))/DYY

S(5,6)=(TK(5)+TK(6))/DXX

S(5,11)=(TK(5)+TK(11))/DYY

S(6,12)=(TK(6)+TK(12))/DYY

S(7,8)=(TK(7)+TK(8))/DXX

S(7,13)=(TK(7)+TK(13))/DYY

S(8,9)=(TK(8)+TK(9))/DXX

S(8,14)=(TK(8)+TK(14))/DYY

S(9,10)=(TK(9)+TK(10))/DXX

S(9,15)=(TK(9)+TK(15))/DYY

S(10,11)=(TK(10)+TK(11))/DXX

S(10,16)=(TK(10)+TK(16))/DYY

S(11,12)=(TK(11)+TK(12))/DXX

S(11,17)=(TK(11)+TK(17))/DYY

S(12,18)=(TK(12)+TK(18))/DYY

S(13,14)=(TK(13)+TK(14))/DXX

S(13,21)=(TK(13)+TK(21))/DYY

S(14,15)=(TK(14)+TK(15))/DXX

S(14,22)=(TK(14)+TK(22))/DYY

S(15,16)=(TK(15)+TK(16))/DXX

S(15,23)=(TK(15)+TK(23))/DYY

S(16,17)=(TK(16)+TK(17))/DXX

S(16,24)=(TK(16)+TK(24))/DYY

S(17,18)=(TK(17)+TK(18))/DXX

S(17,25)=(TK(17)+TK(25))/DYY

S(18,19)=(TK(18)+TK(19))/DXX

S(18,26)=(TK(18)+TK(26))/DYY

S(19,20)=(TK(19)+TK(20))/DXX

S(19,27)=(TK(19)+TK(27))/DYY

S(20,28)=(TK(20)+TK(28))/DYY

S(21,22)=(TK(21)+TK(22))/DXX

S(21,29)=(TK(21)+TK(29))/(4.*DX*(SQRT(DXX+DYY)))

S(22,23)=(TK(22)+TK(23))/DXX

S(22,29)=(TK(22)+TK(29))/DYY

S(23,24)=(TK(23)+TK(24))/DXX

S(23,30)=(TK(23)+TK(30))/DYY

SOURCE STATEMENT

```

S(24,25)=(TK(24)+TK(25))/DXX
S(24,31)=(TK(24)+TK(31))/DYY
S(25,26)=(TK(25)+TK(26))/DXX
S(25,32)=(TK(25)+TK(32))/DYY
S(26,27)=(TK(26)+TK(27))/DXX
S(26,33)=(TK(26)+TK(33))/DYY
S(27,28)=(TK(27)+TK(28))/DXX
S(27,34)=(TK(27)+TK(34))/DYY
S(28,35)=(TK(28)+TK(35))/DYY
S(29,30)=(TK(29)+TK(30))/DXX
S(30,36)=(TK(30)+TK(36))/(4.*DX*SQRT(DXX+DYY))
S(31,32)=(TK(31)+TK(32))/DXX
S(31,36)=(TK(31)+TK(36))/DYY
S(32,33)=(TK(32)+TK(33))/DXX
S(32,37)=(TK(32)+TK(37))/DYY
S(33,34)=(TK(33)+TK(34))/DXX
S(33,38)=(TK(33)+TK(38))/DYY
S(34,35)=(TK(34)+TK(35))/DXX
S(34,39)=(TK(34)+TK(39))/DYY
S(35,40)=(TK(35)+TK(40))/DYY
S(36,37)=DYSTAR*(TK(36)+TK(37))/(DY*DXX)
S(37,38)=DYSTAR*(TK(37)+TK(38))/(DY*DXX)
S(38,39)=DYSTAR*(TK(38)+TK(39))/(DY*DXX)
S(39,40)=DYSTAR*(TK(39)+TK(40))/(DY*DXX)
DO 3 J=1,N
DO 3 K=J,N
S(K,J)=S(J,K)
3 CONTINUE
DO 4 J=1,N
VALUE=0.0
DO 5 K=1,N
5 VALUE=VALUE+S(J,K)
4 S(J,J)=-VALUE
RETURN
END

```


SOURCE STATEMENT

\$IBFTC TRANS

C
C
C
C

TRANS MODIFIES THE ELEMENTS OF THE B.C VECTOR AND OF THE
DIAGONAL MATRIX.

SUBROUTINE TRANS(DIAG,C1,DX,DY,A21,A29,A30,A36,A37,
1A38,A39,A40)

REAL DIAG(40),C1(40)

DIAG(21)=DIAG(21)*A21/(DX*DY)

C1(21)=C1(21)*A21/(DX*DY)

DIAG(29)=DIAG(29)*A29/(DX*DY)

C1(29)=C1(29)*A29/(DX*DY)

DIAG(30)=DIAG(30)*A30/(DX*DY)

C1(30)=C1(30)*A30/(DX*DY)

DIAG(36)=DIAG(36)*A36/(DX*DY)

C1(36)=C1(36)*A36/(DX*DY)

DIAG(37)=DIAG(37)*A37/(DX*DY)

C1(37)=C1(37)*A37/(DX*DY)

DIAG(38)=DIAG(38)*A38/(DX*DY)

C1(38)=C1(38)*A38/(DX*DY)

DIAG(39)=DIAG(39)*A39/(DX*DY)

C1(39)=C1(39)*A39/(DX*DY)

DIAG(40)=DIAG(40)*A40/(DX*DY)

C1(40)=C1(40)*A40/(DX*DY)

RETURN

END

SOURCE STATEMENT

```

$IBFTC DERIVE
      SUBROUTINE DERIVE(DIAG,DINV,S,C1,PZ,N)
C
C      DERIVE PERFORMS THE NECESSARY MULTIPLICATIONS WITH D-HALF
C      AND D-MINUS HALF.
C
      REAL DIAG(40),DINV(40),S(40,40),C1(40),PZ(40)
      DO 1 J=1,N
      DIAG(J)=SQRT(DIAG(J))
1  DINV(J)=1./DIAG(J)
      DO 2 J=1,N
      PZ(J)=DIAG(J)*PZ(J)
      C1(J)=DINV(J)*C1(J)
      DO 2 K= J,N
      S(J,K)=S(J,K)*DINV(J)*DINV(K)
2  S(K,J)=S(J,K)
      RETURN
      END
  
```


FORTRAN SOURCE LIST

SOURCE STATEMENT

\$IBFTC BALANS

SUBROUTINE BALANS(N,NT,P,TIME1,PROD,COMP,AREA,WORK,
1PZ,PZSAVE)

C
C
C

BALANS ESTIMATES THE PRODUCTION OF OIL BY TWO METHODS.

REAL P(25,40),TIME1(25),AREA(40),WORK(40),PZ(40),PZSAVE(40)

DO 1 J=1,NT

1 WORK(J)=PROD*TIME1(J)

DO 2 J=1,NT

PZ(J)=0.0

DO 2 K=1,N

2 PZ(J)=PZ(J)+AREA(K)*(PZSAVE(K)-P(J,K))

RETURN

END

PROGRAM IS BEING ENTERED INTO STORAGE.

INITIAL PRESSURES,PSI

			1500.00	1500.00	1500.00	1500.00	1500.00
	1500.00	1500.00	1500.00	1500.00	1500.00	1500.00	1500.00
1500.00	1500.00	1500.00	1500.00	1500.00	1500.00	1500.00	1500.00
1500.00	1500.00	1500.00	1500.00	1500.00	1500.00	1500.00	1500.00
1500.00	1500.00	1500.00	1500.00	1500.00	1500.00	1500.00	1500.00
1500.00	1500.00	1500.00	1500.00	1500.00	1500.00	1500.00	1500.00

PRODUCTION,STB/DAY

			0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	-200.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	-100.00	0.00
0.00	-100.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

PHI-H MATRIX,FEET

			47.00	57.00	62.00	68.00	61.00
	80.00	69.00	47.00	51.00	62.00	64.00	58.00
69.00	62.00	62.00	77.00	74.00	69.00	87.00	91.00
81.00	84.00	68.00	69.00	54.00	47.00	50.00	48.00
69.00	52.00	53.00	53.00	83.00	80.00		
101.00	88.00	80.00	69.00	65.00	64.00		

K-H MATRIX,DARCY-Feet

			3.00	2.70	2.60	4.10	5.20
	6.00	6.30	4.20	4.60	5.30	5.00	3.70
7.40	6.80	6.70	8.30	7.70	6.80	6.00	5.80
8.80	9.70	14.20	8.00	10.30	11.00	12.50	12.20
15.10	12.80	13.20	13.20	9.60	9.00		
10.90	9.50	9.30	11.00	14.20	14.10		

TIME= 0.0 DAYS

			1500.00	1500.00	1499.99	1500.00	1500.00
	1500.01	1500.00	1500.00	1500.01	1499.99	1500.00	1500.00
1500.00	1500.00	1500.01	1500.00	1500.00	1500.00	1500.00	1500.00
1500.01	1500.01	1500.01	1500.00	1500.00	1500.00	1500.00	1500.00
1500.01	1500.01	1500.01	1500.01	1500.00	1500.00		
1500.01	1500.00	1500.01	1500.01	1500.00	1500.00		
	PRODUCTION,STB			MATERIAL BALANCE			
	0.0000000E-38			-0.4413492E 00			

TIME= 10.0 DAYS

			1464.36	1463.86	1463.78	1463.80	1463.85
	1466.12	1465.88	1464.16	1463.47	1463.62	1463.64	1463.70
1466.37	1466.11	1465.61	1464.36	1462.23	1463.34	1463.28	1463.49
1466.33	1466.03	1465.88	1465.23	1464.45	1464.12	1462.30	1463.30
1466.28	1465.32	1466.06	1465.95	1465.62	1465.47		
1466.40	1466.18	1466.31	1466.33	1466.21	1466.11		
	PRODUCTION,STB			MATERIAL BALANCE			
	0.4000000E 04			0.3994644E 04			

TIME= 40.0 DAYS

			1359.77	1359.28	1359.20	1359.21	1359.27
	1361.54	1361.29	1359.58	1358.88	1359.03	1359.06	1359.12
1361.79	1361.53	1361.02	1359.78	1357.65	1358.75	1358.70	1358.91
1361.75	1361.44	1361.30	1360.65	1359.86	1359.54	1357.71	1358.71
1361.69	1360.74	1361.48	1361.37	1361.04	1360.88		
1361.82	1361.59	1361.73	1361.74	1361.62	1361.53		
	PRODUCTION,STB			MATERIAL BALANCE			
	0.1600000E 05			0.1597990E 05			

..CONTD

TIME= 80.0 DAYS

			1220.32	1219.82	1219.74	1219.76	1219.81
	1222.08	1221.84	1220.12	1219.43	1219.58	1219.60	1219.66
1222.33	1222.07	1221.57	1220.32	1218.19	1219.30	1219.24	1219.45
1222.29	1221.99	1221.84	1221.19	1220.41	1220.08	1218.26	1219.26
1222.24	1221.28	1222.02	1221.91	1221.58	1221.43		
1222.36	1222.14	1222.27	1222.28	1222.16	1222.07		
	PRODUCTION,STB			MATERIAL BALANCE			
	0.3200000E 05			0.3196133E 05			

TIME= 120.0 DAYS

			1080.83	1080.34	1080.26	1080.27	1080.33
	1082.60	1082.35	1080.64	1079.94	1080.09	1080.12	1080.18
1082.85	1082.59	1082.08	1080.84	1078.71	1079.81	1079.76	1079.97
1082.81	1082.50	1082.36	1081.71	1080.92	1080.59	1078.77	1079.77
1082.75	1081.80	1082.53	1082.43	1082.10	1081.94		
1082.88	1082.65	1082.79	1082.80	1082.68	1082.59		
	PRODUCTION,STB			MATERIAL BALANCE			
	0.4800000E 05			0.4794601E 05			

TIME= 160.0 DAYS

			941.35	940.85	940.77	940.79	940.84
	943.12	942.87	941.15	940.46	940.61	940.63	940.69
943.37	943.10	942.60	941.36	939.22	940.33	940.27	940.48
943.32	943.02	942.88	942.22	941.44	941.11	939.29	940.29
943.27	942.32	943.05	942.94	942.61	942.46		
943.39	943.17	943.30	943.32	943.20	943.10		
	PRODUCTION,STB			MATERIAL BALANCE			
	0.6400000E 05			0.6393069E 05			

TIME= 200.0 DAYS

			801.85	801.35	801.27	801.29	801.34
	803.61	803.37	801.65	800.95	801.11	801.13	801.19
803.86	803.60	803.10	801.85	799.72	800.82	800.77	800.98
803.82	803.52	803.37	802.72	801.94	801.61	799.79	800.78
803.77	802.81	803.55	803.44	803.11	802.95		
803.89	803.67	803.80	803.81	803.69	803.60		
	PRODUCTION,STB			MATERIAL BALANCE			
	0.8000000E 05			0.7991753E 05			

..CONTD

TIME= 240.0 DAYS

			662.30	661.81	661.73	661.75	661.80
	664.07	663.82	662.11	661.41	661.56	661.59	661.65
664.32	664.06	663.55	662.31	660.18	661.28	661.23	661.44
664.28	663.98	663.83	663.18	662.39	662.07	660.24	661.24
664.23	663.27	664.01	663.90	663.57	663.41		
664.35	664.12	664.26	664.27	664.15	664.06		
	PRODUCTION,STB			MATERIAL BALANCE			
	0.9600000E 05			0.9590870E 05			

TIME= 280.0 DAYS

			522.76	522.27	522.19	522.21	522.26
	524.53	524.28	522.57	521.87	522.02	522.05	522.11
524.78	524.52	524.01	522.77	520.64	521.74	521.69	521.90
524.74	524.44	524.29	523.64	522.85	522.53	520.70	521.70
524.69	523.73	524.47	524.36	524.03	523.87		
524.81	524.58	524.72	524.73	524.61	524.52		
	PRODUCTION,STB			MATERIAL BALANCE			
	0.1120000E 06			0.1118999E 06			

TIME= 320.0 DAYS

			383.19	382.70	382.62	382.64	382.69
	384.96	384.71	383.00	382.30	382.45	382.48	382.54
385.21	384.95	384.44	383.20	381.07	382.17	382.12	382.33
385.17	384.87	384.72	384.07	383.28	382.96	381.13	382.13
385.12	384.16	384.90	384.79	384.46	384.30		
385.24	385.01	385.15	385.16	385.04	384.95		
	PRODUCTION,STB			MATERIAL BALANCE			
	0.1280000E 06			0.1278943E 06			

APPENDIX F.

Diffusivity of Copper Sulfate in Water.

The complete program consists of three individual programs:

1. Program IH-15, which processes the data and prints out the values of z_i ($i = 1$ to 5) for the various profiles.
2. Program IH-20 LOGFIT, which logarithmically fits the data of z_i versus t to a straight line using least squares, and determines the diffusivity from the slope of this straight line.
3. Program IH-25 CONVENTIONAL, which uses the following analytical solution to estimate the diffusivity of copper sulfate from data employed by IH-15:

$$\text{p.d.e:} \quad \frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$$

$$\text{with I.C} \quad t = 0, \quad u = f(x)$$

$$\text{and B.C} \quad x = 0, \quad \frac{\partial u}{\partial x} = 0$$

$$x = 1, \quad u = V.$$

Analytical solution (7) for the above p.d.e is

$$u = v + \frac{2}{1} \sum_{n=0}^{\infty} \exp \left[-(\alpha(2n+1)^2 \pi^2 t / 4 l^2) \right] \cos \left[\frac{(2n+1) \pi x}{2 l} \right] x$$

$$\left[\frac{2 l (-1)^{n+1} v}{(2n+1) \pi} + \int_0^1 f(x') \cos \left[\frac{(2n+1) \pi x'}{2 l} \right] dx' \right]$$

For each assumed value of x , the quantity

$$E = \sum (c_{\text{expt.}} - c_{\text{analytical}})^2 \text{ is determined. A plot of}$$

E versus α gives the best value.


```

0 $IBFTC DARSI      NODECK
C
C
C      SOLUTION TO THE INVERSE PROBLEM-HOMOGENEOUS MEDIUM
C      ESTIMATION OF MOLECULAR DIFFUSIVITY OF COPPER SULFATE
C
C      INPUT DATA
C
C      NT=NUMBER OF CONCENTRATION PROFILES
C      N=NUMBER OF GRID POINTS
C      GRIDX=POSITIONS OF THE GRID POINTS
C      S=COEFFICIENT MATRIX(DIAGONALS ONLY)
C      TIME3=ACTUAL ELAPSED TIMES OF THE PROFILES
C      C3=BOUNDARY CONDITION VECTOR
C
C      NOTE-THE I.C VECTOR IS FED IN AS A PROFILE.FROM THIS,I.C
C      VECTOR IS DETERMINED INTERNALLY
C
C      NA=TOTAL NUMBER OF POINTS ON ANY FRAME-X+Y
C      NZ=THE POINTS ON THE FRAME
C      OT=POINTS REFERRING TO THE ORIENTATION OF THE FRAME
C      (EACH PROFILE HAS ONE SET OF CARDS CONTAINING NA,NZ,OT)
C
1      REAL S(16,16),R(16),V(16,16),A(16),B(16),W1(16),W2(16)
2      REAL X(75),Y(75),XX(75),YY(75),AX(3),AY(3),W(16,16)
3      REAL C(25,10),CZ(10),C3(10),TIME3(25),Z(25,10),GRIDX(10)
4      REAL WORK(10),XNORM(10),WEIGHT(10)
5      REAL DX,WIDTH,BOUND,FACTOR,MAGNF,ALPHA,RM,TBOUND,TINC
6      INTEGER NZ(150),OT(6),NA,COUNTR,NT,N,N1,MAXN,M,J,K,LINES
C
C
C
7      1 FORMAT(1X,I4)
0      2 FORMAT(10X,14I5)
1      3 FORMAT(10X,6I5)
2      56 FORMAT(1X,10F5.1)
3      57 FORMAT(1X,10F5.1)
4      201 FORMAT(1H ,15X,32H MOLECULAR DIFFUSIVITY OF COPPER,
      117H SULFATE IN WATER)
5      202 FORMAT(1H ,20X,34H DATA FROM DR.R.N.O*BRIEN*S EXPTS.)
6      203 FORMAT(1H ,10X,11H CELL WIDTH,5X,18H CENTERLINE CONC.,
      15X,18H CONVERSION FACTOR)
7      204 FORMAT(1H ,15X,3HCM.,12X,12H G.MOLS/LIT.,8X,
      118HG.MOLS/LIT./MICRON)
0      205 FORMAT(1H ,12X,F6.3,12X,F10.7,14X,E10.4)
1      206 FORMAT(1H ,30X,6H TIME=,F6.1,8H SECONDS)
2      220 FORMAT(1H2)
3      221 FORMAT(1H 10X,8H ..CONTD)
4      222 FORMAT(1H ,//)
5      223 FORMAT(1H ,/)
6      227 FORMAT(1H ,29X,22H ORIENTATION OF FRAMES)
7      228 FORMAT(1H ,33X,2H X,8X,2H Y)
0      229 FORMAT(1H ,30X,F8.1,3X,F8.1)
1      230 FORMAT(1H ,32X,7H SLOPE=,F6.2)
2      231 FORMAT(1H ,10X,26HA FIVE-POINT GRID IS USED.,

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SOURCE STATEMENT

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130H THE GRID POINTS ARE LOCATED AT )
232 FORMAT(1H ,10X,5F11.6)
233 FORMAT(1H ,20X,25H INITIAL CONDITION VECTOR)
234 FORMAT(1H ,10X, 5F12.7)
235 FORMAT(1H ,20X,26H BOUNDARY CONDITION VECTOR)
237 FORMAT(1H ,20X,9H RAW DATA,5X,
      132H CONVERTED TO THE RELEVANT UNITS)
238 FORMAT(1H ,20X,2H X,5X,2H Y,8X,12H DISTANCE,CM,5X,
      118H CONCN.,G.MOLS/LIT.)
239 FORMAT(1H ,15X,F8.1,2X,F8.1,4X,F11.6,8X,F11.6)
240 FORMAT(1H ,30X,19H COEFFICIENT MATRIX)
241 FORMAT(1H ,10X,10F6.2)
242 FORMAT(1H ,35X,12H EIGENVALUES)
243 FORMAT(1H ,35X,13H EIGENVECTORS)
244 FORMAT(1H ,20X,35H CHECK OF SIMILARITY TRANSFORMATION)
245 FORMAT(1H ,8X,8H TIME,SEC,5X,
      157H INTERPOLATED VALUES OF CONCN. AT GRID POINTS,G.MOLS/LIT.)
246 FORMAT(1H ,8X,F6.1,2X,5F12.7)
247 FORMAT(1H ,30X,18H MODIFIED VARIABLE)
248 FORMAT(1H ,10X,10H TIME,SECS,5X,2H 1,10X,2H 2, 9X,2H 3, 9X,
      12H 4,9X,2H 5)
250 FORMAT(1H ,30X,20H VALUES FOR PLOTTING)
251 FORMAT(1H ,12X,F6.1,2X,5F11.6)
252 FORMAT(1H ,15X,25H NORMALISED INNER PRODUCT,
      136H OF I.C.VECTOR WITH EACH EIGENVECTOR)
253 FORMAT(1H ,20X,5F11.6)
1000 FORMAT(1X,2I4)
1001 FORMAT(1X, 5F12.8)
1002 FORMAT(1X,5F14.8)
      READ(5,1000) NT,N
      READ(5,1001) (GRIDX(J),J=1,N)
      DX=ABS(GRIDX(3)-GRIDX(2))
      DO 50 J=1,N
      DO 50 K=1,N
50  S(J,K)=0.0
      READ(5,56) (S(J,J),J=1,N)
      N1=N-1
      READ(5,56) (S(J,J+1),J=1,N1)
      DO 52 J=1,N1
52  S(J+1,J)=S(J,J+1)
      READ(5,57) (TIME3(J),J=1,NT)
      MAXN=16
      M=-N
      CALL EIG1(N,MAXN,M,S,R,V,A,B,W1,W2)
      CALL CHECK(V,R,N,W)
      READ(5,1002) (C3(J),J=1,N)
      READ(5,1001) WIDTH,CONCN,FACTOR
      NE=1
4  READ (5,1) NA
      READ(5,2) (NZ(J),J=1,NA)
      READ(5,3) (OT(J),J=1,6)
      CALL DATA(NA,NZ,X,Y,NP)
      DO 10 J=1,3
10  AX(J)=FLOAT(OT(2*J))
      DO 11 J=1,3

```


SOURCE STATEMENT

```
11 AY(J)=FLOAT(OT(2*J-1))  
CALL LESQFT(AX,AY,SLOPE)  
DO 8 J=1,NP  
XX(J)=X(J)  
8 YY(J)=Y(J)  
CALL CONV2(NP,X,Y,SLOPE,WIDTH,FACTOR)  
IF(NE.EQ.1) GO TO 21  
GO TO 28  
21 DO 215 KA=1,N  
CALL LINEAR(NP,GRIDX,X,Y,KA,RR)  
CZ(KA)=RR  
C(NE,KA)=RR  
215 CONTINUE  
VALUE=0.0  
DO 131 K=1,N  
VALUE=VALUE+CZ(K)**2  
WEIGHT(K)=0.0  
DO 131 J=1,N  
131 WEIGHT(K)=WEIGHT(K)+CZ(J)*V(J,K)  
DO 132 J=1,N  
132 WEIGHT(J)=WEIGHT(J)/SQRT(VALUE)  
WRITE(6,220)  
WRITE(6,201)  
WRITE(6,202)  
WRITE(6,222)  
WRITE(6,203)  
WRITE(6,204)  
WRITE(6,205) WIDTH,CONCN,FACTOR  
WRITE(6,223)  
WRITE(6,231)  
WRITE(6,232) (GRIDX(J),J=1,N)  
WRITE(6,222)  
WRITE(6,233)  
WRITE(6,232) (CZ(J),J=1,N)  
WRITE(6,222)  
WRITE(6,235)  
WRITE(6,232) (C3(J),J=1,N)  
WRITE(6,220)  
LINES=8  
CALL LINCT(LINES,3,3)  
WRITE(6,222)  
WRITE(6,206) TIME3(NE)  
CALL LINCT(LINES,2,3)  
WRITE(6,227)  
WRITE(6,228)  
DO 321 J=1,3  
CALL LINCT(LINES,1,2)  
321 WRITE(6,229) AX(J),AY(J)  
CALL LINCT(LINES,1,3)  
WRITE(6,230) SLOPE  
CALL LINCT(LINES,2,3)  
WRITE(6,222)  
CALL LINCT(LINES,2,3)  
WRITE(6,237)  
CALL LINCT(LINES,1,3)
```


N

SOURCE STATEMENT

```
3      WRITE(6,238)
4      DO 322 J=1,NP
5      CALL LINCT(LINES,1,1)
6 322  WRITE(6,239) XX(J),YY(J),X(J),Y(J)
7      GO TO 30
8      DO 210 KA=1,N
9      CALL LINEAR(NP,GRIDX,X,Y,KA,RR)
10     C(NE,KA)=RR
11 210  CONTINUE
12     CALL LINCT(LINES,3,3)
13     WRITE(6,222)
14     WRITE(6,206) TIME3(NE)
15     CALL LINCT(LINES,2,3)
16     WRITE(6,227)
17     WRITE(6,228)
18     DO 323 J=1,3
19     CALL LINCT(LINES,1,2)
20 323  WRITE(6,229) AX(J),AY(J)
21     CALL LINCT(LINES,1,3)
22     WRITE(6,230) SLOPE
23     CALL LINCT(LINES,2,3)
24     WRITE(6,222)
25     CALL LINCT(LINES,2,3)
26     WRITE(6,237)
27     CALL LINCT(LINES,1,3)
28     WRITE(6,238)
29     DO 324 J=1,NP
30     CALL LINCT(LINES,1,1)
31 324  WRITE(6,239) XX(J),YY(J),X(J),Y(J)
32     NE=NE+1
33     IF(NE.LE.NT ) GO TO 4
34     CALL MODIFY(WORK,V,C3,CZ,Z,C,N,NT)
35     WRITE(6,220)
36     LINES=8
37     CALL LINCT(LINES,1,3)
38     WRITE(6,240)
39     DO 345 J=1,N
40     CALL LINCT(LINES,1,3)
41 345  WRITE(6,232) (S(J,K),K=1,N)
42     CALL LINCT(LINES,3,3)
43     WRITE(6,222)
44     CALL LINCT(LINES,1,3)
45     WRITE(6,242)
46     CALL LINCT(LINES,1,3)
47     WRITE(6,232) (R(J),J=1,N)
48     CALL LINCT(LINES,3,3)
49     WRITE(6,222)
50     CALL LINCT(LINES,1,3)
51     WRITE(6,243)
52     DO 346 K=1,N
53     CALL LINCT(LINES,3,3)
54     WRITE(6,232) (V(J,K),J=1,N)
55 346  WRITE(6,223)
56     CALL LINCT(LINES,4,3)
57     WRITE(6,222)
```


SOURCE STATEMENT

```
6 WRITE(6,244)
7 DO 347 J=1,N
0 CALL LINCT(LINES,3,3)
1 WRITE(6,232) (W(J,K),K=1,N)
6 347 WRITE(6,223)
0 WRITE(6,220)
1 WRITE(6,245)
2 DO 348 J=1,NT
3 348 WRITE(6,246) TIME3(J),(C(J,K),K=1,N)
1 WRITE(6,223)
2 WRITE(6,247)
3 WRITE(6,223)
4 WRITE(6,248)
5 DO 349 J=1,NT
6 349 WRITE(6,251) TIME3(J),(Z(J,K),K=1,5)
4 CALL PLOT(C,R,Z,C3,CZ,N,NT)
5 WRITE(6,220)
6 WRITE(6,250)
7 WRITE(6,223)
0 WRITE(6,248)
1 DO 351 J=1,NT
2 351 WRITE(6,251) TIME3(J),(C(J,K),K=1,5)
0 WRITE(6,223)
1 WRITE(6,252)
2 WRITE(6,253) (WEIGHT(J),J=1,5)
7 END
```


SOURCE STATEMENT

```

1 $IBFTC DATA
1 SUBROUTINE DATA(NA,NZ,X,Y,NP)
C
C DATA CONVERTS THE INTEGER INPUT TO FLOATING POINTS.
C
2 REAL X(75),Y(75)
3 INTEGER NZ(150)
4 NP=NA/2
5 DO 6 J=1,NP
6 X(J)=FLOAT(NZ(2*J))
7 Y(J)=FLOAT(NZ(2*J-1))
1 RETURN
2 END

```


DATE

TIME

LOCATION

REMARKS

WIND

WAVE

SEA

TEMP

WIND

WAVE

SEA

SOURCE STATEMENT

\$IBFTC CONV2

SUBROUTINE CONV2(NP,X,Y,SLOPE,WIDTH,FACTOR)

CONV2 CONVERTS THE RAW DATA INTO THE RELEVANT UNITS.

REAL X(75),Y(75)

S1=ATAN(SLOPE)

S2=1.5707963-S1

S3=SIN(S1)

S4=SIN(S2)/COS(S2)

DO 16 J=1,NP

XX=(X(J)-Y(J)/SLOPE)*S3

YY=(Y(J)+X(J)*S4)*S3

X(J)=XX

16 Y(J)=YY

X1=X(1)

XN=X(NP)

RATIO=WIDTH*10000./(X1-XN)

DO 17 J=1,NP

17 X(J)=WIDTH*(X(J)-X1)/(XN-X1)

N1=NP-1

VALUE=0.0

DO 18 J=1,N1

18 VALUE=VALUE+0.5*(Y(J)+Y(J+1))*(X(J+1)-X(J))

YAVE=VALUE/WIDTH

DO 19 J=1,NP

19 Y(J)=0.05+(Y(J)-YAVE)* FACTOR *RATIO

RETURN

END

1. The first part of the document is a list of names and addresses.

2. The second part of the document is a list of names and addresses.

3. The third part of the document is a list of names and addresses.

4. The fourth part of the document is a list of names and addresses.

5. The fifth part of the document is a list of names and addresses.

6. The sixth part of the document is a list of names and addresses.

7. The seventh part of the document is a list of names and addresses.

8. The eighth part of the document is a list of names and addresses.

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11. The eleventh part of the document is a list of names and addresses.

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13. The thirteenth part of the document is a list of names and addresses.

14. The fourteenth part of the document is a list of names and addresses.

15. The fifteenth part of the document is a list of names and addresses.

16. The sixteenth part of the document is a list of names and addresses.

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24. The twenty-fourth part of the document is a list of names and addresses.

25. The twenty-fifth part of the document is a list of names and addresses.

26. The twenty-sixth part of the document is a list of names and addresses.

27. The twenty-seventh part of the document is a list of names and addresses.

SOURCE STATEMENT

```

0 $IBFTC LINCT
1 SUBROUTINE LINCT(LINES,N,REFER)
C
C LINCT KEEPS TRACK OF NUMBER OF LINES PRINTED AND SKIPS TO A
C NEW PAGE.THIS PROGRAM IS CAPABLE OF GIVING THREE DIFFERENT
C HEADINGS DEPENDING ON THE VALUE OF REFER.
C
2 INTEGER LINES,N,REFER
3 FORMAT(1H2,10X,8H ..CONTD)
4 FORMAT(1H ,20X,2H X,5X,2H Y,8X,12H DISTANCE,CM,5X,
118HCONCN.,G.MOLS/LIT.)
5 FORMAT(1H ,33X,2H X,8X,2H Y)
6 LINES=LINES+N
7 IF(LINES.GT.61) GO TO 1
2 RETURN
3 1 WRITE(6,2)
4 IF(REFER.GT.2) GO TO 5
7 IF(REFER.LT.2) GO TO 6
2 WRITE(6,4)
3 LINES=10
4 RETURN
5 5 LINES=9
6 RETURN
7 6 WRITE(6,3)
0 LINES=10
1 RETURN
2 END

```


10-10000-10000

10-10000-10000

10-10000-10000

10-10000-10000

10-10000-10000

10-10000-10000

PROGRAM IS BEING ENTERED INTO STORAGE.

MOLECULAR DIFFUSIVITY OF COPPER SULFATE IN WATER
DATA FROM DR.R.N.O*BRIEN*S EXPTS.

CELL WIDTH	CENTERLINE CONC.	CONVERSION FACTOR
CM.	G.MOLS/LIT.	G.MOLS/LIT./MICRON
0.313	0.0493829	0.227E-04

A FIVE-POINT GRID IS USED.THE GRID POINTS ARE LOCATED AT
0.014227 0.042682 0.071136 0.099591 0.128045

INITIAL CONDITION VECTOR
0.094676 0.066277 0.056739 0.052795 0.051386

BOUNDARY CONDITION VECTOR
0.000000 0.000000 0.000000 0.000000 0.049383

TIME= 0.0 SECONDS
 ORIENTATION OF FRAMES

X Y
 62528.0 45841.0
 62632.0 47966.0
 62752.0 50634.0
 SLOPE= 21.46

RAW DATA		CONVERTED TO THE RELEVANT UNITS	
X	Y	DISTANCE, CM	CONCN., G. MOLS/LIT.
62749.0	50690.0	0.000000	0.108789
62713.0	50563.0	0.003898	0.105003
62690.0	50494.0	0.006462	0.102942
62667.0	50422.0	0.009008	0.100792
62644.0	50338.0	0.011482	0.098289
62627.0	50245.0	0.013123	0.095530
62607.0	50187.0	0.015365	0.093796
62581.0	50066.0	0.018004	0.090201
62560.0	49965.0	0.020115	0.087201
62539.0	49891.0	0.022390	0.084995
62507.0	49801.0	0.025993	0.082303
62478.0	49661.0	0.028906	0.078145
62448.0	49552.0	0.032136	0.074897
62423.0	49457.0	0.034802	0.072068
62398.0	49368.0	0.037504	0.069415
62371.0	49304.0	0.040617	0.067495
62335.0	49222.0	0.044787	0.065034
62285.0	49146.0	0.050808	0.062729
62228.0	49062.0	0.057688	0.060180
62169.0	48992.0	0.064912	0.058040
62095.0	48930.0	0.074128	0.056114
62030.0	48865.0	0.082159	0.054113
61944.0	48837.0	0.093136	0.053171
61843.0	48816.0	0.106098	0.052415
61702.0	48793.0	0.124233	0.051545
61599.0	48779.0	0.137497	0.050992
61496.0	48770.0	0.150792	0.050586
61338.0	48782.0	0.171341	0.050722
61211.0	48782.0	0.187801	0.050548
61065.0	48763.0	0.206608	0.049789
60960.0	48723.0	0.219974	0.048468
60892.0	48685.0	0.228557	0.047257
60829.0	48623.0	0.236348	0.045347
60761.0	48566.0	0.244816	0.043576
60718.0	48514.0	0.250075	0.041988
60667.0	48430.0	0.256177	0.039447
60610.0	48350.0	0.263081	0.037015
60566.0	48258.0	0.268228	0.034248
60533.0	48183.0	0.272052	0.031996

..CONTD

X	Y	DISTANCE,CM	CONCN.,G.MOLS/LIT.
60512.0	48124.0	0.274417	0.030232
60481.0	48008.0	0.277734	0.026777
60448.0	47876.0	0.281213	0.022848
60422.0	47795.0	0.284094	0.020429
60398.0	47707.0	0.286673	0.017808
60377.0	47603.0	0.288766	0.014719
60350.0	47494.0	0.291607	0.011475
60343.0	47415.0	0.292037	0.009142
60329.0	47338.0	0.293386	0.006857
60312.0	47258.0	0.295106	0.004480
60285.0	47179.0	0.298128	0.002119
60266.0	47093.0	0.300071	-0.000437
60238.0	46981.0	0.303023	-0.003770
60222.0	46906.0	0.304644	-0.005999
60210.0	46863.0	0.305939	-0.007280
60198.0	46785.0	0.307023	-0.009591
60189.0	46723.0	0.307815	-0.011428
60170.0	46665.0	0.309927	-0.013160
60140.0	46530.0	0.313000	-0.017173

TIME= 5.0 SECONDS
ORIENTATION OF FRAMES

X	Y
44895.0	47490.0
44973.0	49157.0
45106.0	51685.0

SLOPE= 19.84

RAW DATA		CONVERTED TO THE RELEVANT UNITS	
X	Y	DISTANCE,CM	CONCN.,G.MOLS/LIT.
45106.0	51646.0	0.000000	0.106385
45060.0	51624.0	0.005716	0.105682
45051.0	51578.0	0.006566	0.104340
45051.0	51530.0	0.006258	0.102952
45033.0	51490.0	0.008294	0.101770
45021.0	51450.0	0.009565	0.100597
45004.0	51407.0	0.011453	0.099329
44986.0	51351.0	0.013386	0.097684
44967.0	51301.0	0.015484	0.096212
44942.0	51188.0	0.017942	0.092909
44924.0	51087.0	0.019586	0.089964
44907.0	51000.0	0.021192	0.087425
44859.0	50893.0	0.026617	0.084262
44832.0	50790.0	0.029394	0.081246
44803.0	50681.0	0.032387	0.078053
44765.0	50601.0	0.036711	0.075686
44735.0	50502.0	0.039896	0.072781
44689.0	50375.0	0.044938	0.069043
44644.0	50268.0	0.049981	0.065885

..CONTD

X	Y	DISTANCE,CM	CONCN.,G.MOLS/LIT.
44579.0	50108.0	0.057230	0.061166
44527.0	50020.0	0.063286	0.058547
44453.0	49950.0	0.072259	0.056416
44390.0	49876.0	0.079805	0.054185
44289.0	49803.0	0.092196	0.051928
44220.0	49779.0	0.100828	0.051134
44134.0	49763.0	0.111675	0.050547
44056.0	49747.0	0.121503	0.049970
43949.0	49747.0	0.135127	0.049815
43885.0	49747.0	0.143276	0.049721
43814.0	49733.0	0.152226	0.049213
43700.0	49733.0	0.166740	0.049047
43605.0	49720.0	0.178753	0.048533
43551.0	49703.0	0.185519	0.047963
43472.0	49688.0	0.195481	0.047415
43280.0	49661.0	0.219754	0.046355
43202.0	49637.0	0.229531	0.045547
43137.0	49606.0	0.237608	0.044557
43086.0	49577.0	0.243915	0.043644
43035.0	49510.0	0.249979	0.041633
42995.0	49459.0	0.254745	0.040101
42945.0	49388.0	0.260655	0.037976
42893.0	49312.0	0.266788	0.035704
42871.0	49248.0	0.269179	0.033822
42839.0	49183.0	0.272836	0.031897
42812.0	49086.0	0.275651	0.029054
42790.0	48989.0	0.277830	0.026218
42762.0	48910.0	0.280888	0.023894
42725.0	48801.0	0.284900	0.020690
42708.0	48723.0	0.286564	0.018411
42682.0	48640.0	0.289341	0.015974
42655.0	48551.0	0.292208	0.013363
42625.0	48440.0	0.295315	0.010111
42607.0	48355.0	0.297062	0.007628
42583.0	48251.0	0.299450	0.004587
42559.0	48159.0	0.301916	0.001893
42535.0	48107.0	0.304638	0.000355
42521.0	48046.0	0.306029	-0.001428
42513.0	47982.0	0.306637	-0.003290
42505.0	47925.0	0.307289	-0.004949
42488.0	47868.0	0.309088	-0.006621
42470.0	47823.0	0.311091	-0.007948
42454.0	47803.0	0.313000	-0.008549

TIME= 20.0 SECONDS
ORIENTATION OF FRAMES

X	Y
46166.0	47377.0
46247.0	49028.0
46410.0	51767.0

SLOPE= 17.81

[illegible]

Y	X
0.57840	0.38100
0.55000	0.42500
0.53500	0.41000
0.51000	0.40000
0.49000	0.39000

..CONTD

RAW DATA		CONVERTED TO THE RELEVANT UNITS	
X	Y	DISTANCE,CM	CONCN.,G.MOLS/LIT.
46408.0	51818.0	0.000000	0.099310
46373.0	51816.0	0.004428	0.099195
46353.0	51800.0	0.006852	0.098702
46337.0	51766.0	0.008640	0.097697
46323.0	51730.0	0.010161	0.096637
46305.0	51687.0	0.012139	0.095369
46287.0	51638.0	0.014074	0.093928
46264.0	51587.0	0.016630	0.092422
46242.0	51536.0	0.019059	0.090917
46223.0	51472.0	0.021014	0.089042
46206.0	51425.0	0.022837	0.087661
46183.0	51358.0	0.025278	0.085693
46151.0	51267.0	0.028691	0.083020
46118.0	51138.0	0.031961	0.079250
46086.0	51048.0	0.035381	0.076606
46051.0	50958.0	0.039181	0.073956
46017.0	50875.0	0.042905	0.071510
45972.0	50791.0	0.048018	0.069017
45936.0	50704.0	0.051967	0.066453
45903.0	50613.0	0.055507	0.063778
45855.0	50536.0	0.061050	0.061482
45826.0	50477.0	0.064310	0.059735
45778.0	50412.0	0.069939	0.057785
45722.0	50346.0	0.076576	0.055793
45673.0	50299.0	0.082460	0.054360
45613.0	50256.0	0.089768	0.053024
45522.0	50209.0	0.100983	0.051523
45417.0	50178.0	0.114088	0.050460
45276.0	50164.0	0.131883	0.049828
45168.0	50156.0	0.145532	0.049423
45028.0	50153.0	0.163279	0.049110
44842.0	50141.0	0.186799	0.048464
44774.0	50118.0	0.195265	0.047691
44691.0	50101.0	0.205678	0.047067
44620.0	50076.0	0.214511	0.046232
44508.0	50043.0	0.228490	0.045100
44448.0	49993.0	0.235749	0.043563
44391.0	49949.0	0.242669	0.042203
44341.0	49892.0	0.248609	0.040480
44287.0	49828.0	0.255006	0.038549
44246.0	49763.0	0.259747	0.036610
44211.0	49700.0	0.263740	0.034739
44175.0	49617.0	0.267717	0.032289
44148.0	49570.0	0.270809	0.030892
44114.0	49464.0	0.274369	0.027783
44090.0	49392.0	0.276902	0.025670
44073.0	49323.0	0.278568	0.023654
44052.0	49260.0	0.280784	0.021805
44014.0	49160.0	0.284895	0.018863

..CONTD

X	Y	DISTANCE,CM	CONCN.,G.MOLS/LIT.
43991.0	49080.0	0.287244	0.016521
43971.0	49026.0	0.289397	0.014933
43947.0	48964.0	0.292002	0.013108
43915.0	48895.0	0.295571	0.011068
43900.0	48832.0	0.297026	0.009229
43873.0	48766.0	0.299983	0.007284
43834.0	48702.0	0.304476	0.005377
43819.0	48647.0	0.305988	0.003768
43790.0	48613.0	0.309426	0.002742
43761.0	48598.0	0.313000	0.002263

TIME= 40.0 SECONDS
ORIENTATION OF FRAMES

X	Y
41288.0	47332.0
41414.0	49503.0
41563.0	51898.0

SLOPE= 16.61

RAW DATA		CONVERTED TO THE RELEVANT UNITS	
X	Y	DISTANCE,CM	CONCN.,G.MOLS/LIT.
41560.0	51851.0	0.000000	0.093205
41520.0	51842.0	0.004970	0.092879
41500.0	51829.0	0.007390	0.092473
41488.0	51810.0	0.008757	0.091909
41473.0	51786.0	0.010465	0.091197
41453.0	51750.0	0.012711	0.090133
41430.0	51714.0	0.015334	0.089065
41408.0	51685.0	0.017885	0.088198
41388.0	51639.0	0.020055	0.086848
41365.0	51596.0	0.022626	0.085579
41343.0	51540.0	0.024973	0.083940
41314.0	51478.0	0.028155	0.082118
41290.0	51417.0	0.030715	0.080332
41261.0	51359.0	0.033928	0.078624
41231.0	51291.0	0.037191	0.076629
41198.0	51218.0	0.040793	0.074485
41165.0	51151.0	0.044442	0.072512
41136.0	51072.0	0.047495	0.070204
41091.0	50994.0	0.052571	0.067896
41042.0	50898.0	0.058015	0.065067
40987.0	50830.0	0.064426	0.063029
40940.0	50746.0	0.069709	0.060546
40888.0	50673.0	0.075705	0.058370
40838.0	50601.0	0.081456	0.056225
40802.0	50559.0	0.085672	0.054962
40748.0	50513.0	0.092124	0.053554
40667.0	50466.0	0.101970	0.052071
40601.0	50438.0	0.110070	0.051157

TIME	Y	X
0.0000	0.0000	0.0000
0.0001	0.0001	0.0001
0.0002	0.0002	0.0002
0.0003	0.0003	0.0003
0.0004	0.0004	0.0004
0.0005	0.0005	0.0005
0.0006	0.0006	0.0006
0.0007	0.0007	0.0007
0.0008	0.0008	0.0008
0.0009	0.0009	0.0009
0.0010	0.0010	0.0010

ORIENTATION OF FRAMES
TIME = 40.0 SECONDS

Y	X
0.0000	0.0000
0.0001	0.0001
0.0002	0.0002
0.0003	0.0003
0.0004	0.0004
0.0005	0.0005
0.0006	0.0006
0.0007	0.0007
0.0008	0.0008
0.0009	0.0009
0.0010	0.0010

SLOPE = 10.01

RAW DATA	Y	X
0.0000	0.0000	0.0000
0.0001	0.0001	0.0001
0.0002	0.0002	0.0002
0.0003	0.0003	0.0003
0.0004	0.0004	0.0004
0.0005	0.0005	0.0005
0.0006	0.0006	0.0006
0.0007	0.0007	0.0007
0.0008	0.0008	0.0008
0.0009	0.0009	0.0009
0.0010	0.0010	0.0010
0.0011	0.0011	0.0011
0.0012	0.0012	0.0012
0.0013	0.0013	0.0013
0.0014	0.0014	0.0014
0.0015	0.0015	0.0015
0.0016	0.0016	0.0016
0.0017	0.0017	0.0017
0.0018	0.0018	0.0018
0.0019	0.0019	0.0019
0.0020	0.0020	0.0020
0.0021	0.0021	0.0021
0.0022	0.0022	0.0022
0.0023	0.0023	0.0023
0.0024	0.0024	0.0024
0.0025	0.0025	0.0025
0.0026	0.0026	0.0026
0.0027	0.0027	0.0027
0.0028	0.0028	0.0028
0.0029	0.0029	0.0029
0.0030	0.0030	0.0030
0.0031	0.0031	0.0031
0.0032	0.0032	0.0032
0.0033	0.0033	0.0033
0.0034	0.0034	0.0034
0.0035	0.0035	0.0035
0.0036	0.0036	0.0036
0.0037	0.0037	0.0037
0.0038	0.0038	0.0038
0.0039	0.0039	0.0039
0.0040	0.0040	0.0040
0.0041	0.0041	0.0041
0.0042	0.0042	0.0042
0.0043	0.0043	0.0043
0.0044	0.0044	0.0044
0.0045	0.0045	0.0045
0.0046	0.0046	0.0046
0.0047	0.0047	0.0047
0.0048	0.0048	0.0048
0.0049	0.0049	0.0049
0.0050	0.0050	0.0050
0.0051	0.0051	0.0051
0.0052	0.0052	0.0052
0.0053	0.0053	0.0053
0.0054	0.0054	0.0054
0.0055	0.0055	0.0055
0.0056	0.0056	0.0056
0.0057	0.0057	0.0057
0.0058	0.0058	0.0058
0.0059	0.0059	0.0059
0.0060	0.0060	0.0060
0.0061	0.0061	0.0061
0.0062	0.0062	0.0062
0.0063	0.0063	0.0063
0.0064	0.0064	0.0064
0.0065	0.0065	0.0065
0.0066	0.0066	0.0066
0.0067	0.0067	0.0067
0.0068	0.0068	0.0068
0.0069	0.0069	0.0069
0.0070	0.0070	0.0070
0.0071	0.0071	0.0071
0.0072	0.0072	0.0072
0.0073	0.0073	0.0073
0.0074	0.0074	0.0074
0.0075	0.0075	0.0075
0.0076	0.0076	0.0076
0.0077	0.0077	0.0077
0.0078	0.0078	0.0078
0.0079	0.0079	0.0079
0.0080	0.0080	0.0080
0.0081	0.0081	0.0081
0.0082	0.0082	0.0082
0.0083	0.0083	0.0083
0.0084	0.0084	0.0084
0.0085	0.0085	0.0085
0.0086	0.0086	0.0086
0.0087	0.0087	0.0087
0.0088	0.0088	0.0088
0.0089	0.0089	0.0089
0.0090	0.0090	0.0090
0.0091	0.0091	0.0091
0.0092	0.0092	0.0092
0.0093	0.0093	0.0093
0.0094	0.0094	0.0094
0.0095	0.0095	0.0095
0.0096	0.0096	0.0096
0.0097	0.0097	0.0097
0.0098	0.0098	0.0098
0.0099	0.0099	0.0099
0.0100	0.0100	0.0100

..CONTD

X	Y	DISTANCE,CM	CONCN.,G.MOLS/LIT.
40533.0	50435.0	0.118612	0.050954
40467.0	50422.0	0.126826	0.050469
40355.0	50421.0	0.140924	0.050247
40237.0	50405.0	0.155665	0.049587
40129.0	50377.0	0.169055	0.048601
40013.0	50377.0	0.183665	0.048401
39940.0	50365.0	0.192768	0.047932
39871.0	50340.0	0.201269	0.047099
39794.0	50328.0	0.210876	0.046623
39720.0	50307.0	0.220037	0.045895
39657.0	50275.0	0.227729	0.044872
39610.0	50232.0	0.233322	0.043562
39551.0	50172.0	0.240298	0.041745
39490.0	50096.0	0.247405	0.039467
39451.0	50036.0	0.251862	0.037684
39406.0	49947.0	0.256854	0.035062
39377.0	49869.0	0.259915	0.032782
39335.0	49794.0	0.264637	0.030566
39304.0	49731.0	0.268063	0.028711
39263.0	49664.0	0.272719	0.026725
39234.0	49597.0	0.275864	0.024760
39208.0	49530.0	0.278630	0.022800
39168.0	49457.0	0.283115	0.020644
39130.0	49372.0	0.287256	0.018148
39085.0	49304.0	0.292408	0.016126
39056.0	49226.0	0.295469	0.013847
39031.0	49146.0	0.298011	0.011516
39015.0	49083.0	0.299549	0.009688
38989.0	49042.0	0.302513	0.008471
38965.0	49003.0	0.305240	0.007314
38927.0	48976.0	0.309821	0.006477
38902.0	48980.0	0.313000	0.006548

TIME= 50.0 SECONDS
ORIENTATION OF FRAMES
X Y
44128.0 47169.0
44232.0 49253.0
44375.0 51668.0
SLOPE= 18.16

RAW DATA		CONVERTED TO THE RELEVANT UNITS	
X	Y	DISTANCE,CM	CONCN.,G.MOLS/LIT.
44360.0	51507.0	0.000000	0.091632
44332.0	51515.0	0.003623	0.091819
44307.0	51491.0	0.006639	0.091085
44284.0	51466.0	0.009394	0.090325
44255.0	51445.0	0.012941	0.089672
44232.0	51425.0	0.015731	0.089057

X	Y	DISTANCE	ANGLE
40233.0	20477.0	0.11015	0.00000
40461.0	20455.0	0.11015	0.00000
40322.0	20411.0	0.11015	0.00000
40537.0	20407.0	0.11015	0.00000
40159.0	20377.0	0.11015	0.00000
40013.0	20377.0	0.11015	0.00000
39940.0	20362.0	0.11015	0.00000
39871.0	20340.0	0.11015	0.00000
39794.0	20328.0	0.11015	0.00000
39750.0	20307.0	0.11015	0.00000
39627.0	20272.0	0.11015	0.00000
39610.0	20255.0	0.11015	0.00000
39521.0	20175.0	0.11015	0.00000
39490.0	20096.0	0.11015	0.00000
39421.0	20036.0	0.11015	0.00000
39400.0	19997.0	0.11015	0.00000
39377.0	19969.0	0.11015	0.00000
39332.0	19944.0	0.11015	0.00000
39304.0	19911.0	0.11015	0.00000
39263.0	19864.0	0.11015	0.00000
39234.0	19827.0	0.11015	0.00000
39208.0	19730.0	0.11015	0.00000
39168.0	19627.0	0.11015	0.00000
39130.0	19575.0	0.11015	0.00000
39082.0	19480.0	0.11015	0.00000
39026.0	19354.0	0.11015	0.00000
39031.0	19246.0	0.11015	0.00000
39012.0	19183.0	0.11015	0.00000
38984.0	19045.0	0.11015	0.00000
38962.0	18904.0	0.11015	0.00000
38927.0	18776.0	0.11015	0.00000
38902.0	18680.0	0.11015	0.00000

TIME = 20.0 SECONDS
ORIENTATION OF BEAMS

X	Y
44150.0	44160.0
44335.0	44240.0
44375.0	44300.0

FLIBE = 18.74

X	Y	DISTANCE	ANGLE
44335.0	21450.0	0.01015	0.00000
44325.0	21212.0	0.01015	0.00000
44307.0	21401.0	0.01015	0.00000
44284.0	21444.0	0.01015	0.00000
44252.0	21440.0	0.01015	0.00000
44235.0	21450.0	0.01015	0.00000

..CONTD

X	Y	DISTANCE,CM	CONCN.,G.MOLS/LIT.
44216.0	51397.0	0.017572	0.088222
44200.0	51360.0	0.019351	0.087126
44176.0	51330.0	0.022198	0.086221
44141.0	51247.0	0.026074	0.083765
44113.0	51182.0	0.029185	0.081840
44075.0	51119.0	0.033584	0.079958
44032.0	50998.0	0.038213	0.076391
44000.0	50906.0	0.041644	0.073679
43972.0	50837.0	0.044726	0.071639
43942.0	50791.0	0.048225	0.070261
43923.0	50745.0	0.050323	0.068901
43898.0	50677.0	0.053031	0.066895
43867.0	50638.0	0.056706	0.065717
43833.0	50584.0	0.060658	0.064102
43788.0	50515.0	0.065907	0.062035
43761.0	50454.0	0.068918	0.060228
43728.0	50404.0	0.072771	0.058729
43663.0	50336.0	0.080575	0.056659
43568.0	50262.0	0.092157	0.054368
43487.0	50218.0	0.102167	0.052967
43405.0	50167.0	0.112255	0.051361
43261.0	50130.0	0.130340	0.050062
43142.0	50098.0	0.145275	0.048947
43063.0	50100.0	0.155353	0.048879
42924.0	50106.0	0.173102	0.048831
42815.0	50082.0	0.186819	0.047964
42698.0	50077.0	0.201689	0.047633
42632.0	50048.0	0.209893	0.046689
42576.0	50002.0	0.216704	0.045270
42493.0	49969.0	0.227046	0.044183
42423.0	49914.0	0.235577	0.042481
42366.0	49856.0	0.242432	0.040713
42318.0	49788.0	0.248069	0.038670
42270.0	49727.0	0.253756	0.036830
42221.0	49661.0	0.259535	0.034843
42186.0	49595.0	0.263531	0.032879
42142.0	49487.0	0.268378	0.029686
42118.0	49413.0	0.270916	0.027508
42087.0	49339.0	0.274346	0.025318
42067.0	49282.0	0.276494	0.023638
42031.0	49228.0	0.280701	0.022019
42000.0	49147.0	0.284082	0.019628
41969.0	49082.0	0.287575	0.017699
41942.0	49009.0	0.290502	0.015545
41920.0	48953.0	0.292912	0.013890
41888.0	48906.0	0.296658	0.012480
41856.0	48869.0	0.300475	0.011359
41817.0	48838.0	0.305226	0.010401
41788.0	48802.0	0.308668	0.009313
41753.0	48784.0	0.313000	0.008737

..CONTD

TIME= 75.0 SECONDS
ORIENTATION OF FRAMES

X Y

47028.0 47283.0

47141.0 49629.0

47261.0 51711.0

SLOPE= 19.00

RAW DATA CONVERTED TO THE RELEVANT UNITS
X Y DISTANCE,CM CONC.,G.MOLS/LIT.

47261.0	51711.0	0.000000	0.087976
47215.0	51699.0	0.005768	0.087560
47190.0	51687.0	0.008866	0.087176
47166.0	51673.0	0.011824	0.086735
47145.0	51654.0	0.014366	0.086155
47127.0	51622.0	0.016441	0.085204
47103.0	51584.0	0.019238	0.084071
47064.0	51517.0	0.023748	0.082078
47041.0	51479.0	0.026417	0.080947
47006.0	51410.0	0.030405	0.078902
46955.0	51323.0	0.036307	0.076314
46915.0	51263.0	0.040991	0.074521
46871.0	51159.0	0.045889	0.071453
46819.0	51050.0	0.051770	0.068228
46752.0	50971.0	0.059760	0.065847
46711.0	50897.0	0.064477	0.063649
46663.0	50814.0	0.070024	0.061180
46609.0	50757.0	0.076508	0.059453
46559.0	50707.0	0.082530	0.057934
46506.0	50652.0	0.088901	0.056267
46450.0	50601.0	0.095679	0.054710
46403.0	50560.0	0.101380	0.053455
46343.0	50520.0	0.108740	0.052210
46263.0	50494.0	0.118737	0.051338
46214.0	50465.0	0.124773	0.050426
46114.0	50442.0	0.137333	0.049611
46029.0	50442.0	0.148139	0.049481
45897.0	50422.0	0.164787	0.048704
45788.0	50422.0	0.178645	0.048538
45679.0	50393.0	0.192309	0.047536
45563.0	50380.0	0.206970	0.046984
45488.0	50331.0	0.216177	0.045456
45397.0	50279.0	0.227398	0.043817
45330.0	50227.0	0.235569	0.042215
45286.0	50182.0	0.240861	0.040849
45249.0	50126.0	0.245191	0.039177
45197.0	50046.0	0.251266	0.036789
45151.0	49971.0	0.256613	0.034555
45103.0	49886.0	0.262146	0.032029
45056.0	49791.0	0.267486	0.029215
45008.0	49695.0	0.272946	0.026372

..CONTD

X	Y	DISTANCE,CM	CONCN.,G.MOLS/LIT.
44957.0	49593.0	0.278747	0.023351
44917.0	49515.0	0.283311	0.021039
44875.0	49448.0	0.288202	0.019042
44835.0	49368.0	0.292752	0.016672
44798.0	49313.0	0.297088	0.015028
44766.0	49278.0	0.300922	0.013970
44730.0	49242.0	0.305258	0.012876
44688.0	49212.0	0.310397	0.011946
44667.0	49202.0	0.313000	0.011626

TIME= 100.0 SECONDS
ORIENTATION OF FRAMES

X	Y
49003.0	47591.0
49099.0	49554.0
49208.0	51493.0

SLOPE= 19.05

RAW DATA		CONVERTED TO THE RELEVANT UNITS	
X	Y	DISTANCE,CM	CONCN.,G.MOLS/LIT.
49195.0	51366.0	0.000000	0.083781
49155.0	51364.0	0.005076	0.083663
49122.0	51343.0	0.009134	0.083006
49085.0	51327.0	0.013735	0.082488
49048.0	51302.0	0.018275	0.081710
49009.0	51278.0	0.023077	0.080958
48966.0	51252.0	0.028374	0.080142
48943.0	51217.0	0.031066	0.079096
48916.0	51156.0	0.034094	0.077293
48894.0	51115.0	0.036619	0.076076
48870.0	51074.0	0.039399	0.074855
48832.0	51030.0	0.043940	0.073527
48794.0	50968.0	0.048360	0.071679
48759.0	50912.0	0.052439	0.070008
48730.0	50843.0	0.055668	0.067972
48700.0	50805.0	0.059231	0.066829
48663.0	50751.0	0.063578	0.065213
48612.0	50661.0	0.069465	0.062537
48565.0	50600.0	0.075038	0.060704
48517.0	50531.0	0.080684	0.058638
48454.0	50479.0	0.088352	0.057041
48404.0	50423.0	0.094339	0.055348
48360.0	50375.0	0.099617	0.053895
48282.0	50342.0	0.109320	0.052823
48213.0	50306.0	0.117858	0.051679
48145.0	50279.0	0.126330	0.050796
48052.0	50250.0	0.137968	0.049817
47954.0	50245.0	0.150403	0.049524
47870.0	50234.0	0.161017	0.049079

OTM...

Y	X
0.25294	0.72944
0.21294	0.71944
0.84494	0.25844
0.80394	0.23844
0.31394	0.89244
0.57594	0.60744
0.54594	0.66744
0.54594	0.66744
0.51594	0.88644
0.50594	0.76644

$\text{SLOPE} = 13.02$
 $\text{INTERCEPT} = 0.0000$
 $\text{TIME} = 100.0 \text{ SECONDS}$
 $\text{ORIENTATION OF FIBERS}$

RAW DATA		CONVERTED TO THE REFERENCE UNIT	
X	Y	DISTANCE, CM	ANGLE, DEGREE
49195.0	51388.0	0.00000	0.00000
49155.0	51364.0	0.00236	0.00000
49155.0	51343.0	0.00472	0.00000
49085.0	51317.0	0.01375	0.00000
49048.0	51305.0	0.01825	0.00000
49009.0	51279.0	0.03307	0.00000
48966.0	51255.0	0.05834	0.00000
48943.0	51217.0	0.08100	0.00000
48916.0	51166.0	0.08800	0.00000
48884.0	51112.0	0.09819	0.00000
48810.0	51074.0	0.09839	0.00000
48835.0	51030.0	0.09840	0.00000
48794.0	50988.0	0.09880	0.00000
48759.0	50915.0	0.09938	0.00000
48730.0	50843.0	0.09988	0.00000
48700.0	50802.0	0.09995	0.00000
48663.0	50751.0	0.09998	0.00000
48615.0	50681.0	0.09999	0.00000
48565.0	50600.0	0.09999	0.00000
48517.0	50531.0	0.09999	0.00000
48454.0	50479.0	0.09999	0.00000
48404.0	50433.0	0.09999	0.00000
48360.0	50375.0	0.09999	0.00000
48313.0	50306.0	0.09999	0.00000
48145.0	50279.0	0.09999	0.00000
48025.0	50250.0	0.09999	0.00000
47924.0	50222.0	0.09999	0.00000
47810.0	50194.0	0.09999	0.00000

..CONTD

X	Y	DISTANCE,CM	CONCN.,G.MOLS/LIT.
47801.0	50221.0	0.169709	0.048599
47680.0	50207.0	0.185010	0.048012
47583.0	50185.0	0.197204	0.047229
47500.0	50171.0	0.207671	0.046699
47439.0	50128.0	0.215144	0.045365
47364.0	50083.0	0.224386	0.043951
47317.0	50025.0	0.229978	0.042205
47280.0	49970.0	0.234318	0.040560
47240.0	49921.0	0.239080	0.039085
47186.0	49848.0	0.245463	0.036894
47131.0	49783.0	0.252026	0.034934
47071.0	49695.0	0.259072	0.032301
47018.0	49615.0	0.265281	0.029910
46948.0	49513.0	0.273506	0.026858
46902.0	49426.0	0.278777	0.024276
46859.0	49359.0	0.283801	0.022276
46824.0	49286.0	0.287766	0.020114
46769.0	49225.0	0.294356	0.018269
46724.0	49183.0	0.299801	0.016988
46669.0	49153.0	0.306598	0.016038
46618.0	49140.0	0.313000	0.015586

TIME= 120.0 SECONDS
ORIENTATION OF FRAMES

X	Y
46549.0	47448.0
46663.0	49647.0
46767.0	51777.0

SLOPE= 19.89

RAW DATA CONVERTED TO THE RELEVANT UNITS

X	Y	DISTANCE,CM	CONCN.,G.MOLS/LIT.
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49209.0	51625.0	0.000000	0.080557
49116.0	51615.0	0.011790	0.080133
49075.0	51598.0	0.016908	0.079581
49017.0	51560.0	0.024057	0.078397
48971.0	51532.0	0.029741	0.077520
48931.0	51497.0	0.034616	0.076449
48892.0	51433.0	0.039177	0.074541
48851.0	51380.0	0.044063	0.072948
48820.0	51324.0	0.047656	0.071282
48789.0	51281.0	0.051332	0.069993
48748.0	51231.0	0.056238	0.068486
48702.0	51160.0	0.061646	0.066365
48668.0	51065.0	0.065371	0.063567
48611.0	51009.0	0.072278	0.061864
48566.0	50923.0	0.077463	0.059310
48465.0	50855.0	0.089902	0.057195
48379.0	50783.0	0.100402	0.054987

..CONTD

X	Y	DISTANCE,CM	CONCN.,G.MOLS/LIT.
48291.0	50723.0	0.111235	0.053123
48183.0	50689.0	0.124784	0.051982
48058.0	50640.0	0.140403	0.050382
47936.0	50629.0	0.155884	0.049886
47846.0	50601.0	0.167176	0.048945
47753.0	50573.0	0.178851	0.048000
47637.0	50548.0	0.193477	0.047108
47544.0	50529.0	0.205210	0.046423
47466.0	50499.0	0.214960	0.045441
47411.0	50455.0	0.221689	0.044088
47354.0	50396.0	0.228577	0.042298
47298.0	50342.0	0.235369	0.040654
47256.0	50288.0	0.240376	0.039030
47201.0	50224.0	0.246977	0.037099
47145.0	50148.0	0.253628	0.034818
47088.0	50054.0	0.260291	0.032015
47045.0	49974.0	0.265260	0.029638
46999.0	49917.0	0.270758	0.027922
46957.0	49859.0	0.275740	0.026182
46912.0	49810.0	0.281162	0.024699
46874.0	49750.0	0.285621	0.022908
46836.0	49688.0	0.290068	0.021058
46801.0	49630.0	0.294158	0.019329
46758.0	49586.0	0.299357	0.017994
46720.0	49555.0	0.304002	0.017041
46648.0	49527.0	0.313000	0.016126

TIME= 150.0 SECONDS
ORIENTATION OF FRAMES
X Y
50217.0 47547.0
50316.0 49355.0
50442.0 51557.0
SLOPE= 17.87

RAW DATA		CONVERTED TO THE RELEVANT UNITS	
X	Y	DISTANCE,CM	CONCN.,G.MOLS/LIT.
50440.0	51562.0	0.000000	0.080999
50378.0	51546.0	0.007799	0.080435
50343.0	51529.0	0.012145	0.079885
50291.0	51501.0	0.018582	0.078990
50252.0	51466.0	0.023310	0.077912
50207.0	51417.0	0.028704	0.076420
50158.0	51364.0	0.034580	0.074804
50100.0	51309.0	0.041590	0.073117
50056.0	51246.0	0.046756	0.071220
50013.0	51182.0	0.051787	0.069296
49971.0	51133.0	0.056798	0.067808
49933.0	51079.0	0.061262	0.066182

..CONTD

X	Y	DISTANCE,CM	CONCN.,G.MOLS/LIT.
49879.0	51005.0	0.067626	0.063950
49797.0	50929.0	0.077549	0.061615
49736.0	50868.0	0.084900	0.059749
49673.0	50801.0	0.092462	0.057706
49613.0	50742.0	0.099699	0.055899
49557.0	50688.0	0.106461	0.054243
49480.0	50645.0	0.115982	0.052873
49408.0	50600.0	0.124851	0.051452
49340.0	50574.0	0.133344	0.050588
49262.0	50547.0	0.143107	0.049680
49174.0	50541.0	0.154297	0.049363
49092.0	50525.0	0.164649	0.048766
48995.0	50512.0	0.176937	0.048232
48907.0	50490.0	0.188012	0.047452
48823.0	50458.0	0.198505	0.046389
48750.0	50430.0	0.207622	0.045459
48679.0	50376.0	0.216299	0.043779
48630.0	50319.0	0.222146	0.042048
48571.0	50262.0	0.229270	0.040301
48496.0	50204.0	0.238428	0.038499
48439.0	50150.0	0.245318	0.036842
48391.0	50094.0	0.251044	0.035142
48342.0	50025.0	0.256806	0.033063
48277.0	49925.0	0.264388	0.030060
48230.0	49871.0	0.270001	0.028419
48181.0	49809.0	0.275812	0.026544
48136.0	49750.0	0.281135	0.024761
48080.0	49691.0	0.287861	0.022961
48035.0	49646.0	0.293283	0.021584
47957.0	49607.0	0.302960	0.020327
47897.0	49583.0	0.310447	0.019535
47877.0	49583.0	0.313000	0.019502

COEFFICIENT MATRIX

-1.000000	1.000000	0.000000	0.000000	0.000000
1.000000	-2.000000	1.000000	0.000000	0.000000
0.000000	1.000000	-2.000000	1.000000	0.000000
0.000000	0.000000	1.000000	-2.000000	1.000000
0.000000	0.000000	0.000000	1.000000	-2.000000

EIGENVALUES

-3.682507	-2.830830	-1.715370	-0.690279	-0.081014
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EIGENVECTORS

-0.169891	0.455734	-0.596885	0.548529	-0.326019
-0.326019	0.596885	-0.169891	-0.455734	0.548529
0.455734	-0.326019	-0.548529	0.169891	0.596885
-0.548529	-0.169891	0.326019	0.596885	0.455734
0.596885	0.548529	0.455734	0.326019	0.169891

CHECK OF SIMILARITY TRANSFORMATION

-1.000000	1.000000	-0.000000	-0.000000	0.000000
1.000000	-2.000000	1.000001	-0.000000	-0.000000
-0.000000	1.000001	-2.000001	1.000000	0.000000
-0.000000	-0.000000	1.000000	-1.999999	1.000000
0.000000	-0.000000	0.000000	1.000000	-2.000000

0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

CHECK THE INITIAL TRANSFORMATION

0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
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CHECK THE INITIAL TRANSFORMATION

0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
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CHECK THE INITIAL TRANSFORMATION

0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
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0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
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0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
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0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
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CHECK THE INITIAL TRANSFORMATION

0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
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0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
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0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
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0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
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0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
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ME, SEC	INTERPOLATED VALUES OF CONCN. AT GRID POINTS, G. MOLS/LIT.				
0.0	0.0946762	0.0662766	0.0567392	0.0527946	0.0513859
5.0	0.0970937	0.0707155	0.0566825	0.0512480	0.0498956
20.0	0.0938380	0.0716567	0.0574255	0.0517089	0.0499645
40.0	0.0895156	0.0734638	0.0600280	0.0524294	0.0504497
50.0	0.0893883	0.0729921	0.0593651	0.0533274	0.0502269
75.0	0.0861868	0.0734622	0.0608843	0.0538488	0.0502137
100.0	0.0824039	0.0738951	0.0619869	0.0539016	0.0506517
20.0	0.0798702	0.0733980	0.0621452	0.0551575	0.0516477
50.0	0.0795956	0.0727159	0.0631243	0.0559257	0.0511271

MODIFIED VARIABLE

TIME, SECS	1	2	3	4	5
0.0	-0.007540	0.003180	0.030057	0.010236	0.144665
5.0	-0.006257	0.004939	0.028591	0.006535	0.147760
20.0	-0.005488	0.006263	0.026512	0.008710	0.146834
40.0	-0.005246	0.008247	0.022938	0.012273	0.146748
50.0	-0.004479	0.007588	0.023417	0.012642	0.146366
75.0	-0.004337	0.008409	0.021052	0.015119	0.145573
100.0	-0.004269	0.009930	0.018852	0.017711	0.144147
120.0	-0.003795	0.010406	0.018581	0.020440	0.143013
150.0	-0.004053	0.009287	0.017960	0.021247	0.143083

TIME, SEC	1	2	3	4	5
0.0	0.079250	0.079150	0.079050	0.078950	0.078850
10.0	0.079870	0.079770	0.079670	0.079570	0.079470
20.0	0.080490	0.080390	0.080290	0.080190	0.080090
30.0	0.081110	0.081010	0.080910	0.080810	0.080710
40.0	0.081730	0.081630	0.081530	0.081430	0.081330
50.0	0.082350	0.082250	0.082150	0.082050	0.081950
60.0	0.082970	0.082870	0.082770	0.082670	0.082570
70.0	0.083590	0.083490	0.083390	0.083290	0.083190
80.0	0.084210	0.084110	0.084010	0.083910	0.083810
90.0	0.084830	0.084730	0.084630	0.084530	0.084430

MODIFIED VARIABLE

TIME, SEC	1	2	3	4	5
0.0	-0.002540	0.003180	0.003000	0.002820	0.002640
10.0	-0.002520	0.004430	0.004250	0.004070	0.003890
20.0	-0.002500	0.005680	0.005500	0.005320	0.005140
30.0	-0.002480	0.006930	0.006750	0.006570	0.006390
40.0	-0.002460	0.008180	0.008000	0.007820	0.007640
50.0	-0.002440	0.009430	0.009250	0.009070	0.008890
60.0	-0.002420	0.010680	0.010500	0.010320	0.010140
70.0	-0.002400	0.011930	0.011750	0.011570	0.011390
80.0	-0.002380	0.013180	0.013000	0.012820	0.012640
90.0	-0.002360	0.014430	0.014250	0.014070	0.013890

VALUES FOR PLOTTING

TIME, SECS	1	2	3	4	5
0.0	1.000000	1.000000	1.000000	1.000000	1.000000
5.0	0.594852	0.724777	0.886079	1.165463	1.075280
20.0	0.352174	0.517420	0.724611	1.068238	1.052744
40.0	0.275951	0.206968	0.446959	0.908919	1.050668
50.0	0.033691	0.310101	0.484164	0.892449	1.041377
75.0	-0.011009	0.181528	0.300470	0.781721	1.022086
100.0	-0.032494	-0.056481	0.129620	0.665831	0.987387
120.0	-0.181965	-0.131061	0.108521	0.543807	0.959795
150.0	-0.100687	0.044202	0.060355	0.507726	0.961501

NORMALISED INNER PRODUCT OF I.C. VECTOR WITH EACH EIGENVECTOR

-0.050833 0.021439 0.202632 0.069007 0.975261

VALUES FOR POINTS

TIME, SECS	1	2	3	4	5
0.0	1.00000	1.00000	1.00000	1.00000	1.00000
2.0	0.99999	0.99999	0.99999	0.99999	0.99999
4.0	0.99998	0.99998	0.99998	0.99998	0.99998
6.0	0.99997	0.99997	0.99997	0.99997	0.99997
8.0	0.99996	0.99996	0.99996	0.99996	0.99996
10.0	0.99995	0.99995	0.99995	0.99995	0.99995
12.0	0.99994	0.99994	0.99994	0.99994	0.99994
14.0	0.99993	0.99993	0.99993	0.99993	0.99993
16.0	0.99992	0.99992	0.99992	0.99992	0.99992
18.0	0.99991	0.99991	0.99991	0.99991	0.99991
20.0	0.99990	0.99990	0.99990	0.99990	0.99990
22.0	0.99989	0.99989	0.99989	0.99989	0.99989
24.0	0.99988	0.99988	0.99988	0.99988	0.99988
26.0	0.99987	0.99987	0.99987	0.99987	0.99987
28.0	0.99986	0.99986	0.99986	0.99986	0.99986
30.0	0.99985	0.99985	0.99985	0.99985	0.99985

NORMALISED INVERSE PRODUCT OF 1.0 VECTOR WITH 1.0 VECTOR
 -0.020813 0.021430 0.025235 0.025235 0.025235

C
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C

INPUT DATA

NZ=NUMBER OF SETS OF DATA
NC=NUMBER OF POINTS IN A DATA SET
M=COLUMN NUMBER
DX=GRID SPACING
EIGVAL=EIGENVALUE

```

1  FORMAT(1X,I4)
2  FORMAT(1X,2I4,2F12.8)
3  FORMAT(1X,7F10.7)
220 FORMAT(1H2)
221 FORMAT(1H ,/)
222 FORMAT(1H ,15X,29HSEMI-LOGARITHMIC LEAST SQUARE,
      115H FIT OF RESULTS)
223 FORMAT(1H ,20X,15H COLUMN NUMBER=,I4)
224 FORMAT(1H ,10X,10H TIME,SECS,2X,18H EXPTL.VALUES OF Z,
      12X,11H LOGARITHMS,2X,18H Z-VALUES FROM FIT)
225 FORMAT(1H ,12X,F8.3,4X,F12.8,6X,F10.6,4X,F12.8)
226 FORMAT(1H ,10X,13H GRID SPACING,2X,11H EIGENVALUE,5X,
      16H SLOPE,5X,10H INTERCEPT,2X,12H DIFFUSIVITY)
227 FORMAT(1H ,15X,3HCM.,48X,9HSQ.CM/SEC)
228 FORMAT(1H ,10X,F11.7,4X,F11.6,2X,F11.6,3X,F10.6,2X,E12.6)
      REAL AX(15),AY(15),LOGY(15),YFIT(15)
      READ(5,1) NZ
      NH=1
115  READ(5,2) NC,M,DX,EIGVAL
      READ(5,3) (AX(J),J=1,NC)
      READ(5,3) (AY(J),J=1,NC)
      DO 116 J=1,NC
116  LOGY(J)=ALOG(AY(J))
      SUM1=0.0
      DO 100 J=1,NC
100  SUM1=SUM1+AX(J)
      SUM2=0.0
      DO 101 J=1,NC
101  SUM2=SUM2+AX(J)**2
      SUM3=0.0
      DO 102 J=1,NC
102  SUM3=SUM3+LOGY(J)
      SUM4=0.0
      DO 103 J=1,NC
103  SUM4=SUM4+AX(J)*LOGY(J)
      AN=NC
      SLOPE=(SUM3*SUM1-SUM4*AN)/(SUM1**2-AN*SUM2)
      CEPT=(SUM3*SUM2-SUM4*SUM1)/(AN*SUM2-SUM1**2)
      DO 117 J=1,NC
117  YFIT(J)=EXP(SLOPE*AX(J)+CEPT)
      WRITE(6,220)

```


SOURCE STATEMENT

```
WRITE(6,222)
WRITE(6,221)
WRITE(6,223) M
WRITE(6,221)
WRITE(6,224)
DO 118 J=1,NC
118 WRITE(6,225) AX(J),AY(J),LOGY(J),YFIT(J)
   CEPT=EXP(CEPT)
   DIFF=SLOPE*(DX**2)/EIGVAL
   WRITE(6,221)
   WRITE(6,226)
   WRITE(6,227)
   WRITE(6,228) DX,EIGVAL,SLOPE,CEPT,DIFF
   NH=NH+1
   IF(NH.LE.NZ) GO TO 115
CONTINUE
END
```


WRITE(0,250)
WRITE(0,251)
WRITE(0,252) W
WRITE(0,253)
WRITE(0,254)
WRITE(0,255)
DO 110 J=1,NC
110 WRITE(0,256) AX(1),AY(1),LX(1),LY(1)
CPI=CP(CPI)
DIFF=2*UP(0,257)*LX(1)
WRITE(0,258)
WRITE(0,259)
WRITE(0,260)
WRITE(0,261) DX,EIYAL,ALIM,ALIM,ALIM
NH=NH+1
IF(NH.EQ.10) GO TO 110
CONTINUE
END

PROGRAM IS BEING ENTERED INTO STORAGE.

SEMI-LOGARITHMIC LEAST SQUARE FIT OF RESULTS

COLUMN NUMBER= 5

TIME, SECS	EXPTL. VALUES OF Z	LOGARITHMS	Z-VALUES FROM FIT
0.000	1.00000000	0.000000	1.05645017
5.000	1.07527999	0.072581	1.05314055
20.000	1.05274400	0.051400	1.04327377
40.000	1.05066800	0.049426	1.03026174
50.000	1.04137699	0.040544	1.02381668
75.000	1.02208599	0.021846	1.00787994
100.000	0.98738700	-0.012693	0.99219126
120.000	0.95979499	-0.041036	0.97981633
150.000	0.96150100	-0.039260	0.96154273

GRID SPACING CM.	EIGENVALUE	SLOPE	INTERCEPT	DIFFUSIVITY SQ.CM/SEC
0.0284540	-0.081014	-0.000628	1.056450	0.62714E-05

LOW 13 BEING ENTERED INTO SYSTEM.

SEMI-COARSHENED TEST STONE FIT TO EQUIP

COLUMN NUMBER = 2

TIME, SECS	EXPL. VALUE OF 5	EXPL. VALUE OF 5	Y-VALUE FROM 111
0.000	1.0000000	0.000000	1.000000
2.000	1.0725799	0.072571	1.072571
20.000	1.0527440	0.052740	1.052740
40.000	1.0506900	0.050685	1.050685
20.000	1.0413799	0.041374	1.041374
72.000	1.0270829	0.027078	1.027078
100.000	0.9973910	-0.015697	0.997391
120.000	0.9297999	-0.061000	0.929799
120.000	0.9210100	-0.078990	0.921010

GRID SPACING	EIGENVALUE	SCORE	INTERVAL	Y-VALUE FROM 111
0.0284240	-0.001014	-0.000028	1.000000	1.000000

PROGRAM IS BEING ENTERED INTO STORAGE.

MOLECULAR DIFFUSIVITY OF COPPER SULFATE IN WATER
DATA FROM DR.R.N.O*BRIEN*S EXPTS.

CELL WIDTH	CENTERLINE CONC.N.	CONVERSION FACTOR
CM.	G.MOLS/LIT.	G.MOLS/LIT./MICRON
0.313	0.0493829	0.227E-04

A FIVE-POINT GRID IS USED.THE GRID POINTS ARE LOCATED AT
0.298773 0.270318 0.241864 0.213409 0.184954

INITIAL CONDITION VECTOR
0.001271 0.033017 0.044194 0.049117 0.050578

BOUNDARY CONDITION VECTOR
0.000000 0.000000 0.000000 0.000000 0.049383

[illegible]

DATA FROM D.R. M. WILSON'S EXPT.
MOLECULAR DIFFUSIVITY IN LIQUID POLYMER

CELL WIDTH	CENTRIFUGAL CONC.	CONCENTRATION
0.313	0.000000	0.000000
CM.	0.000000	0.000000

0.208713 0.210319 0.211866 0.213405 0.214944 0.216483 0.218022 0.219561 0.221100 0.222639 0.224178 0.225717 0.227256 0.228795 0.230334 0.231873 0.233412 0.234951 0.236490 0.238029 0.239568 0.241107 0.242646 0.244185 0.245724 0.247263 0.248802 0.250341 0.251880 0.253419 0.254958 0.256497 0.258036 0.259575 0.261114 0.262653 0.264192 0.265731 0.267270 0.268809 0.270348 0.271887 0.273426 0.274965 0.276504 0.278043 0.279582 0.281121 0.282660 0.284199 0.285738 0.287277 0.288816 0.290355 0.291894 0.293433 0.294972 0.296511 0.298050 0.299589 0.301128 0.302667 0.304206 0.305745 0.307284 0.308823 0.310362 0.311901 0.313440 0.314979 0.316518 0.318057 0.319596 0.321135 0.322674 0.324213 0.325752 0.327291 0.328830 0.330369 0.331908 0.333447 0.334986 0.336525 0.338064 0.339603 0.341142 0.342681 0.344220 0.345759 0.347298 0.348837 0.350376 0.351915 0.353454 0.354993 0.356532 0.358071 0.359610 0.361149 0.362688 0.364227 0.365766 0.367305 0.368844 0.370383 0.371922 0.373461 0.375000 0.376539 0.378078 0.379617 0.381156 0.382695 0.384234 0.385773 0.387312 0.388851 0.390390 0.391929 0.393468 0.395007 0.396546 0.398085 0.399624 0.401163 0.402702 0.404241 0.405780 0.407319 0.408858 0.410397 0.411936 0.413475 0.415014 0.416553 0.418092 0.419631 0.421170 0.422709 0.424248 0.425787 0.427326 0.428865 0.430404 0.431943 0.433482 0.435021 0.436560 0.438099 0.439638 0.441177 0.442716 0.444255 0.445794 0.447333 0.448872 0.450411 0.451950 0.453489 0.455028 0.456567 0.458106 0.459645 0.461184 0.462723 0.464262 0.465801 0.467340 0.468879 0.470418 0.471957 0.473496 0.475035 0.476574 0.478113 0.479652 0.481191 0.482730 0.484269 0.485808 0.487347 0.488886 0.490425 0.491964 0.493503 0.495042 0.496581 0.498120 0.499659 0.501198 0.502737 0.504276 0.505815 0.507354 0.508893 0.510432 0.511971 0.513510 0.515049 0.516588 0.518127 0.519666 0.521205 0.522744 0.524283 0.525822 0.527361 0.528900 0.530439 0.531978 0.533517 0.535056 0.536595 0.538134 0.539673 0.541212 0.542751 0.544290 0.545829 0.547368 0.548907 0.550446 0.551985 0.553524 0.555063 0.556602 0.558141 0.559680 0.561219 0.562758 0.564297 0.565836 0.567375 0.568914 0.570453 0.571992 0.573531 0.575070 0.576609 0.578148 0.579687 0.581226 0.582765 0.584304 0.585843 0.587382 0.588921 0.590460 0.591999 0.593538 0.595077 0.596616 0.598155 0.599694 0.601233 0.602772 0.604311 0.605850 0.607389 0.608928 0.610467 0.612006 0.613545 0.615084 0.616623 0.618162 0.619701 0.621240 0.622779 0.624318 0.625857 0.627396 0.628935 0.630474 0.632013 0.633552 0.635091 0.636630 0.638169 0.639708 0.641247 0.642786 0.644325 0.645864 0.647403 0.648942 0.650481 0.652020 0.653559 0.655098 0.656637 0.658176 0.659715 0.661254 0.662793 0.664332 0.665871 0.667410 0.668949 0.670488 0.672027 0.673566 0.675105 0.676644 0.678183 0.679722 0.681261 0.682800 0.684339 0.685878 0.687417 0.688956 0.690495 0.692034 0.693573 0.695112 0.696651 0.698190 0.699729 0.701268 0.702807 0.704346 0.705885 0.707424 0.708963 0.710502 0.712041 0.713580 0.715119 0.716658 0.718197 0.719736 0.721275 0.722814 0.724353 0.725892 0.727431 0.728970 0.730509 0.732048 0.733587 0.735126 0.736665 0.738204 0.739743 0.741282 0.742821 0.744360 0.745899 0.747438 0.748977 0.750516 0.752055 0.753594 0.755133 0.756672 0.758211 0.759750 0.761289 0.762828 0.764367 0.765906 0.767445 0.768984 0.770523 0.772062 0.773601 0.775140 0.776679 0.778218 0.779757 0.781296 0.782835 0.784374 0.785913 0.787452 0.788991 0.790530 0.792069 0.793608 0.795147 0.796686 0.798225 0.799764 0.801303 0.802842 0.804381 0.805920 0.807459 0.808998 0.810537 0.812076 0.813615 0.815154 0.816693 0.818232 0.819771 0.821310 0.822849 0.824388 0.825927 0.827466 0.829005 0.830544 0.832083 0.833622 0.835161 0.836700 0.838239 0.839778 0.841317 0.842856 0.844395 0.845934 0.847473 0.849012 0.850551 0.852090 0.853629 0.855168 0.856707 0.858246 0.859785 0.861324 0.862863 0.864402 0.865941 0.867480 0.869019 0.870558 0.872097 0.873636 0.875175 0.876714 0.878253 0.879792 0.881331 0.882870 0.884409 0.885948 0.887487 0.889026 0.890565 0.892104 0.893643 0.895182 0.896721 0.898260 0.899799 0.901338 0.902877 0.904416 0.905955 0.907494

INITIAL CONDITION VECTOR

	0.00000	0.00000	0.00000	0.00000	0.00000
BOUNDARY CONDITION VECTOR					
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

COEFFICIENT MATRIX

-1.000000	1.000000	0.000000	0.000000	0.000000
1.000000	-2.000000	1.000000	0.000000	0.000000
0.000000	1.000000	-2.000000	1.000000	0.000000
0.000000	0.000000	1.000000	-2.000000	1.000000
0.000000	0.000000	0.000000	1.000000	-2.000000

EIGENVALUES

-3.682507	-2.830830	-1.715370	-0.690279	-0.081014
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EIGENVECTORS

-0.169891	0.455734	-0.596885	0.548529	-0.326019
-0.326019	0.596885	-0.169891	-0.455734	0.548529
0.455734	-0.326019	-0.548529	0.169891	0.596885
-0.548529	-0.169891	0.326019	0.596885	0.455734
0.596885	0.548529	0.455734	0.326019	0.169891

CHECK OF SIMILARITY TRANSFORMATION

-1.000000	1.000000	-0.000000	-0.000000	0.000000
1.000000	-2.000000	1.000001	-0.000000	-0.000000
-0.000000	1.000001	-2.000001	1.000000	0.000000
-0.000000	-0.000000	1.000000	-1.999999	1.000000
0.000000	-0.000000	0.000000	1.000000	-2.000000

ME, SEC	INTERPOLATED VALUES OF CONCN. AT GRID POINTS, G. MOLS/LIT.				
0.0	0.0012711	0.0330171	0.0441936	0.0491168	0.0505782
5.0	0.0054496	0.0332224	0.0439410	0.0466316	0.0480107
20.0	0.0080801	0.0311135	0.0423615	0.0463365	0.0485146
40.0	0.0106108	0.0277495	0.0412429	0.0464217	0.0483344
50.0	0.0118591	0.0280207	0.0408596	0.0459562	0.0480816
75.0	0.0145633	0.0277405	0.0404619	0.0459155	0.0480752
100.0	0.0172301	0.0280413	0.0381295	0.0456744	0.0480137
20.0	0.0181436	0.0280590	0.0385951	0.0455973	0.0476276
50.0	0.0208712	0.0283170	0.0376728	0.0443389	0.0476676

MODIFIED VARIABLE

TIME, SECS	1	2	3	4	5
0.0	-0.001095	0.017144	0.004108	0.060469	0.063616
5.0	-0.002087	0.015672	0.004129	0.055406	0.064861
20.0	-0.002878	0.014235	0.007132	0.053860	0.064544
40.0	-0.004068	0.011454	0.009903	0.052647	0.063697
50.0	-0.004100	0.011347	0.010363	0.051398	0.064221
75.0	-0.004470	0.010381	0.011895	0.049806	0.065486
100.0	-0.003506	0.010164	0.014214	0.047360	0.066091
120.0	-0.003848	0.009621	0.014125	0.046785	0.066767
150.0	-0.004347	0.009638	0.015600	0.044212	0.067713

TIME, SEC	INTERPOLATED VALUE	OF TIME, AT	TIME, SEC	INTERPOLATED VALUE	OF TIME, AT
0.0	0.001511	0.001511	0.0	0.001511	0.001511
2.0	0.002440	0.002440	2.0	0.002440	0.002440
4.0	0.003001	0.003001	4.0	0.003001	0.003001
6.0	0.003108	0.003108	6.0	0.003108	0.003108
8.0	0.003231	0.003231	8.0	0.003231	0.003231
10.0	0.003233	0.003233	10.0	0.003233	0.003233
12.0	0.003233	0.003233	12.0	0.003233	0.003233
14.0	0.003233	0.003233	14.0	0.003233	0.003233
16.0	0.003233	0.003233	16.0	0.003233	0.003233
18.0	0.003233	0.003233	18.0	0.003233	0.003233
20.0	0.003233	0.003233	20.0	0.003233	0.003233

MODIFIED TABLE

TIME, SEC	1	2	3	4	5
0.0	-0.001025	0.001146	0.004108	0.004400	0.004400
2.0	-0.002047	0.002275	0.004120	0.004400	0.004400
4.0	-0.003068	0.003296	0.004132	0.004400	0.004400
6.0	-0.004089	0.004317	0.004144	0.004400	0.004400
8.0	-0.005110	0.005338	0.004156	0.004400	0.004400
10.0	-0.006131	0.006359	0.004168	0.004400	0.004400
12.0	-0.007152	0.007380	0.004180	0.004400	0.004400
14.0	-0.008173	0.008401	0.004192	0.004400	0.004400
16.0	-0.009194	0.009422	0.004204	0.004400	0.004400
18.0	-0.010215	0.010443	0.004216	0.004400	0.004400
20.0	-0.011236	0.011464	0.004228	0.004400	0.004400

VALUES FOR PLOTTING

TIME, SECS	1	2	3	4	5
0.0	1.000000	1.000000	1.000000	1.000000	1.000000
5.0	0.697381	0.805620	0.998378	0.818315	0.968827
20.0	0.455890	0.615909	0.768691	0.762829	0.976768
40.0	0.092810	0.248845	0.556807	0.719315	0.997979
50.0	0.082856	0.234782	0.521573	0.674495	0.984848
75.0	-0.030050	0.107234	0.404477	0.617342	0.953186
100.0	0.264129	0.078525	0.227126	0.529552	0.938044
120.0	0.159924	0.006836	0.233886	0.508944	0.921110
150.0	0.007763	0.009070	0.121089	0.416589	0.897430

NORMALISED INNER PRODUCT OF I.C. VECTOR WITH EACH EIGENVECTOR
-0.012229 0.191494 0.045879 0.675409 0.710562

AVG. 2.4M. 10.11.1960

TIME, SEC	1	2	3	4	5
120.0	0.00193	0.00000	0.15100	0.15100	0.15100
150.0	0.15000	0.00000	0.15100	0.15100	0.15100
100.0	0.15000	0.00000	0.15100	0.15100	0.15100
75.0	0.15000	0.00000	0.15100	0.15100	0.15100
50.0	0.15000	0.00000	0.15100	0.15100	0.15100
25.0	0.15000	0.00000	0.15100	0.15100	0.15100
0.0	0.15000	0.00000	0.15100	0.15100	0.15100

INTEGRATED INVERSE PRODUCT OF 1.0 VECTOR WITH 1.0 VECTOR

SEMI-LOGARITHMIC LEAST SQUARE FIT OF RESULTS

COLUMN NUMBER= 4

TIME, SECS	EXPTL. VALUES OF Z	LOGARITHMS	Z-VALUES FROM FIT
0.000	1.00000000	0.000000	0.88853899
5.000	0.81831499	-0.200508	0.86660078
20.000	0.76282899	-0.270721	0.80398292
40.000	0.71931499	-0.329456	0.72747347
50.000	0.67449499	-0.393791	0.69199401
75.000	0.61734200	-0.482332	0.61068219
100.000	0.52955200	-0.635724	0.53892481
120.000	0.50894400	-0.675417	0.48763909
150.000	0.41658900	-0.875655	0.41971454

GRID SPACING CM.	EIGENVALUE	SLOPE	INTERCEPT	DIFFUSIVITY SQ. CM/SEC
0.0284540	-0.690279	-0.005000	0.888539	0.58645E-05

SEMI-LOGARITHMIC LEAST SQUARE FIT TO NOISE

COLUMN NUMBER= 4

TIME, SECS	EXPTL. VALUES OF λ	LEAST SQUARE	1- λ SLOPE	INTERCEPT	GRID SPACING CM.
0.000	1.0000000	0.00000	-0.00000	0.00000	0.0584540
2.000	0.8181440	-0.00000	-0.00000	0.00000	
50.000	0.1825890	-0.00000	-0.00000	0.00000	
60.000	0.1191490	-0.00000	-0.00000	0.00000	
80.000	0.6744400	-0.00000	-0.00000	0.00000	
120.000	0.6173450	-0.00000	-0.00000	0.00000	
100.000	0.2522500	-0.00000	-0.00000	0.00000	
150.000	0.2084400	-0.00000	-0.00000	0.00000	
180.000	0.4168800	-0.00000	-0.00000	0.00000	

SEMI-LOGARITHMIC LEAST SQUARE FIT OF RESULTS

COLUMN NUMBER= 5

TIME, SECS	EXPTL. VALUES OF Z	LOGARITHMS	Z-VALUES FROM FIT
0.000	1.00000000	0.000000	0.99826265
5.000	0.96882700	-0.031669	0.99506697
20.000	0.97676900	-0.023505	0.98554114
40.000	0.99797900	-0.002023	0.97298176
50.000	0.98484800	-0.015268	0.96676221
75.000	0.95318599	-0.047945	0.95138673
100.000	0.93804399	-0.063958	0.93625578
120.000	0.92110999	-0.082176	0.92432446
150.000	0.89743000	-0.108220	0.90671198

GRID SPACING CM.	EIGENVALUE	SLOPE	INTERCEPT	DIFFUSIVITY SQ. CM/SEC
0.0284540	-0.081014	-0.000641	0.998263	0.64087E-05

SEMI-EMPIRICAL LEAST SQUARE FIT OF KINETIC

COLUMN NUMBER= 7

TIME, SECS	EXPTL. VALUES OF Z	EXTRAPOLATED	Δ-VALUE, MIN. FIT
0.000	1.0000000	0.0000000	0.0000000
2.000	0.9980000	-0.0010000	0.0000000
50.000	0.9760000	-0.0240000	0.0000000
60.000	0.9720000	-0.0280000	0.0000000
80.000	0.9680000	-0.0320000	0.0000000
100.000	0.9640000	-0.0360000	0.0000000
120.000	0.9600000	-0.0400000	0.0000000
150.000	0.9560000	-0.0440000	0.0000000
180.000	0.9520000	-0.0480000	0.0000000

GRID SPACING CM.	EIGENVALUE	STAGE	ITERATION	DIFFERENTIAL
0.0584240	-0.001014	-0.000001	0.000000	0.000000

\$IBFTC DARSI NODECK

LEAST SQUARES ANALYSIS OF DIFFUSIVITY DATA

INPUT DATA

NP=NUMBER OF POINTS ON THE I.C CURVE

NS=NUMBER OF SETS OF DATA

ND=NUMBER OF VALUES OF DIFFUSIVITY

XAXIS=X-AXIS VALUES ON THE I.C CURVE

YAXIS=Y-AXIS VALUES ON THE I.C CURVE

WIDTH=CELL WIDTH

PI=THE CONSTANT PI

BC=THE CONSTANT CONCN. AT THE BOUNDARY

DIFF=ASSUMED VALUES OF DIFFUSIVITY

REAL YAXIS(75),XAXIS(75),A(30),DIFF(25),X(50),EXPVAL(50)

REAL T(50),POTENT(50),VARIAN(25)

1 FORMAT(1X,3I4)

2 FORMAT(1X,F9.6,7F10.6)

3 FORMAT(1X,3F14.8)

4 FORMAT(1X,5F14.8)

220 FORMAT(1H2)

223 FORMAT(1H ,/)

224 FORMAT(1H ,15X,39H DIFFUSIVITY OF COPPER SULFATE IN WATER,
1/25X,22H CONVENTIONAL APPROACH)

225 FORMAT(1H ,30X,19H INITIAL CONDITIONS)

226 FORMAT(1H ,20X,13H DISTANCE,CM.,5X,18H CONCN.,G.MOLS/LIT)

227 FORMAT(1H ,20X,F13.8,7X,F13.8)

228 FORMAT(1H ,30X,18H EXPERIMENTAL DATA)

229 FORMAT(1H ,20X,10H TIME,SECS,2X,13H DISTANCE,CM.,
12X,18H CONCN.,G.MOLS/LIT)

230 FORMAT(1H ,22X,F6.1,5X,F11.8,5X,F13.8)

231 FORMAT(1H ,20X,20H D-ASSUMED,SQ.CM/SEC,5X, 9H VARIANCE)

232 FORMAT(1H ,23X,F12.8,10X,F10.7)

READ(5,1) NP,NS,ND

READ(5,2) (XAXIS(J),J=1,NP)

READ(5,2) (YAXIS(J),J=1,NP)

READ(5,3) WIDTH,PI,BC

READ(5,4) (DIFF(J),J=1,ND)

DO 17 J=1,NP

17 XAXIS(J)=XAXIS(J)/10.0

DO 20 J=1,30

CALL TREPEZ(YAXIS,XAXIS,WIDTH,PI,J,NP,A(J))

A(J)=-A(J)

20 CONTINUE

CALL TREPEZ(YAXIS,XAXIS,WIDTH,PI,0,NP,A4)

A4=-A4

DO 21 J=1,NS

21 READ(5,3) X(J),T(J),EXPVAL(J)

WRITE(6,220)

WRITE(6,224)

WRITE(6,223)

WRITE(6,225)


```

7 WRITE(6,226)
0 DO 18 J=1,NP
1 18 WRITE(6,227) XAXIS(J),YAXIS(J)
3 WRITE(6,220)
4 WRITE(6,228)
5 WRITE(6,229)
6 DO 19 J=1,NS
7 19 WRITE(6,230) T(J),X(J),EXPVAL(J)
1 DO 28 J=1,ND
2 VARIAN(J)=0.0
3 DO 26 K=1,NS
4 A1=EXP(-DIFF(J)*(PI**2)*T(K)/(4.*(WIDTH**2)))
5 A2=COS(PI*X(K)/(2.*WIDTH))
6 A3=-2.*WIDTH*BC/PI
7 AX=A1*A2*(A3+A4)
0 AY=-1.0
1 DO 23 N=1,30
2 AY=-AY
3 AN=N
4 A1=EXP(-DIFF(J)*((2.*AN+1.))**2)*(PI**2)*T(K)/(4.*(WIDTH**2)))
5 A2=COS((2.*AN+1.)*PI*X(K)/(2.*WIDTH))
6 A3= 2.*WIDTH*AY*BC/((2.*AN+1.)*PI)
7 AX=AX+A1*A2*(A3+A(N))
0 IF(A1.LE.1.0E-30) GO TO 24
3 23 CONTINUE
5 24 POTENT(K)=BC+2.*AX/WIDTH
6 26 CONTINUE
0 DO 27 L=1,NS
1 VARIAN(J)=VARIAN(J)+((POTENT(L)-EXPVAL(L))**2)
2 27 CONTINUE
4 28 CONTINUE
6 WRITE(6,220)
7 WRITE(6,231)
0 DO 29 J=1,ND
1 29 WRITE(6,232) DIFF(J),VARIAN(J)
3 END

```


SOURCE STATEMENT

```

0 $IBFTC TREPEZ
1 SUBROUTINE TREPEZ(YAXIS,XAXIS,WIDTH,PI,N,NP,VALUE)
C
C TREPEZ EVALUATES CERTAIN TERMS REQUIRED BY THE ANALYTICAL
C SOLUTION
C
2 REAL XAXIS(75),YAXIS(75)
3 NP1=NP-1
4 AN=N
5 CONST=(2.*AN+1.)*PI/(2.*WIDTH)
6 VALUE=0.0
7 DO 5 J=1,NP1
0 5 VALUE=VALUE+(YAXIS(J+1)*COS(CONST*XAXIS(J+1))+YAXIS(J)*COS(CONST
1 XAXIS(J)))*(XAXIS(J+1)-XAXIS(J))
2 VALUE=0.5*VALUE
3 RETURN
4 END

```


PROGRAM IS BEING ENTERED INTO STORAGE.

DIFFUSIVITY OF COPPER SULFATE IN WATER
CONVENTIONAL APPROACH

INITIAL CONDITIONS	
DISTANCE, CM.	CONCN., G. MOLS/LIT
0.15650000	0.04938294
0.15079205	0.05058586
0.13749749	0.05099185
0.12423322	0.05154497
0.10609840	0.05241495
0.09313567	0.05317123
0.08215917	0.05411290
0.07412769	0.05611427
0.06491175	0.05803974
0.05768818	0.06018000
0.05080834	0.06272936
0.04478735	0.06503379
0.04061706	0.06749553
0.03750441	0.06941540
0.03480196	0.07206800
0.03213580	0.07489712
0.02890614	0.07814497
0.02599342	0.08230344
0.02238981	0.08499505
0.02011522	0.08720088
0.01800369	0.09090102
0.01536490	0.09379641
0.01312325	0.09553015
0.01148185	0.09828947
0.00900834	0.10079222
0.00646248	0.10294195
0.00389845	0.10500342
0.00000000	0.10878903

WATER IS BEING ENTERED INTO SYSTEM.

DIFFUSIVITY OF COPPER SULFATE IN WATER CONVECTIONAL APPARATUS

INITIAL CONCENTRATION	DISTANCE, CM.
0.1081800	0.000000
0.1030045	0.003000
0.1000000	0.006000
0.0970000	0.009000
0.0940000	0.012000
0.0910000	0.015000
0.0880000	0.018000
0.0850000	0.021000
0.0820000	0.024000
0.0790000	0.027000
0.0760000	0.030000
0.0730000	0.033000
0.0700000	0.036000
0.0670000	0.039000
0.0640000	0.042000
0.0610000	0.045000
0.0580000	0.048000
0.0550000	0.051000
0.0520000	0.054000
0.0490000	0.057000
0.0460000	0.060000
0.0430000	0.063000
0.0400000	0.066000
0.0370000	0.069000
0.0340000	0.072000
0.0310000	0.075000
0.0280000	0.078000
0.0250000	0.081000
0.0220000	0.084000
0.0190000	0.087000
0.0160000	0.090000
0.0130000	0.093000
0.0100000	0.096000
0.0070000	0.099000
0.0040000	0.102000
0.0010000	0.105000
0.0000000	0.108000

EXPERIMENTAL DATA		
TIME, SECS	DISTANCE, CM.	CONCN., G. MDLS/LIT
5.0	0.01422727	0.09713493
20.0	0.01422727	0.09382965
40.0	0.01422727	0.08949336
50.0	0.01422727	0.08945030
75.0	0.01422727	0.08620008
100.0	0.01422727	0.08241088
120.0	0.01422727	0.07989973
150.0	0.01422727	0.07963167
5.0	0.04268181	0.07061136
20.0	0.04268181	0.07164642
40.0	0.04268181	0.07350914
50.0	0.04268181	0.07293032
75.0	0.04268181	0.07352710
100.0	0.04268181	0.07388501
120.0	0.04268181	0.07342859
150.0	0.04268181	0.07274343
5.0	0.07113635	0.05665765
20.0	0.07113635	0.05739915
40.0	0.07113635	0.05999278
50.0	0.07113635	0.05927962
75.0	0.07113635	0.06082587
100.0	0.07113635	0.06195646
120.0	0.07113635	0.06210056
150.0	0.07113635	0.06304156
5.0	0.09959089	0.05122213
20.0	0.09959089	0.05167635
40.0	0.09959089	0.05237648
50.0	0.09959089	0.05331789
75.0	0.09959089	0.05383166
100.0	0.09959089	0.05390080
120.0	0.09959089	0.05515302
150.0	0.09959089	0.05592517
5.0	0.12804543	0.04985071
20.0	0.12804543	0.04992752
40.0	0.12804543	0.05043102
50.0	0.12804543	0.05018386
75.0	0.12804543	0.05009583
100.0	0.12804543	0.05062662
120.0	0.12804543	0.05162910
150.0	0.12804543	0.05109184

D-ASSUMED, SQ. CM/SEC

VARIANCE

0.00000400	0.0001553
0.00000420	0.0001424
0.00000440	0.0001311
0.00000460	0.0001216
0.00000480	0.0001136
0.00000510	0.0001045
0.00000540	0.0000985
0.00000570	0.0000955
0.00000600	0.0000952
0.00000620	0.0000965
0.00000640	0.0000987
0.00000660	0.0001020
0.00000680	0.0001062
0.00000700	0.0001113
0.00000720	0.0001172
0.00000740	0.0001239
0.00000760	0.0001313
0.00000780	0.0001394
0.00000800	0.0001481
0.00000820	0.0001575


```

0 $IBFTC DARSI   NODECK
1     REAL YAXIS(75),XAXIS(75),A(30),DIFF(25),X(50),EXPVAL(50)
2     REAL T(50),POTENT(50),VARIAN(25)
3     1 FORMAT(1X,3I4)
4     2 FORMAT(1X,F9.6,7F10.6)
5     3 FORMAT(1X,3F14.8)
6     4 FORMAT(1X,5F14.8)
7 220 FORMAT(1H2)
0 223 FORMAT(1H ,/)
1 224 FORMAT(1H ,15X,39H DIFFUSIVITY OF COPPER SULFATE IN WATER,
    1/25X,22H CONVENTIONAL APPROACH)
2 225 FORMAT(1H ,30X,19H INITIAL CONDITIONS)
3 226 FORMAT(1H ,20X,13H DISTANCE,CM.,5X,18H CONCN.,G.MOLS/LIT)
4 227 FORMAT(1H ,20X,F13.8,7X,F13.8)
5 228 FORMAT(1H ,30X,18H EXPERIMENTAL DATA)
6 229 FORMAT(1H ,20X,10H TIME,SECS,2X,13H DISTANCE,CM.,
    12X,18H CONCN.,G.MOLS/LIT)
7 230 FORMAT(1H ,22X,F6.1,5X,F11.8,5X,F13.8)
0 231 FORMAT(1H ,20X,20H D-ASSUMED,SQ.CM/SEC,5X, 9H VARIANCE)
1 232 FORMAT(1H ,23X,F12.8,10X,F10.7)
2     READ(5,1) NP,NS,ND
6     READ(5,2) (XAXIS(J),J=1,NP)
3     READ(5,2) (YAXIS(J),J=1,NP)
0     READ(5,3) WIDTH,PI,BC
1     READ(5,4) (DIFF(J),J=1,ND)
6     DO 17 J=1,NP
7 17 XAXIS(J)=XAXIS(J)/10.0
1     DO 20 J=1,30
2     CALL TREPEZ(YAXIS,XAXIS,WIDTH,PI,J,NP,A(J))
3     A(J)=-A(J)
4 20 CONTINUE
6     CALL TREPEZ(YAXIS,XAXIS,WIDTH,PI,0,NP,A4)
7     A4=-A4
0     DO 21 J=1,NS
1 21 READ(5,3) X(J),T(J),EXPVAL(J)
3     WRITE(6,220)
4     WRITE(6,224)
5     WRITE(6,223)
6     WRITE(6,225)
7     WRITE(6,226)
0     DO 18 J=1,NP
1 18 WRITE(6,227) XAXIS(J),YAXIS(J)
3     WRITE(6,220)
4     WRITE(6,228)
5     WRITE(6,229)
6     DO 19 J=1,NS
7 19 WRITE(6,230) T(J),X(J),EXPVAL(J)
1     DO 9 J=1,NS
2 9 T(J)=T(J)-5.0
4     DO 28 J=1,ND
5     VARIAN(J)=0.0
6     DO 26 K=1,NS
7     A1=EXP(-DIFF(J)*(PI**2)*T(K)/(4.*(WIDTH**2)))
0     A2=COS(PI*X(K)/(2.*WIDTH))
1     A3=-2.*WIDTH*BC/PI

```



```

2  AX=A1*A2*(A3+A4)
3  AY=-1.0
4  DO 23 N=1,30
5  AY=-AY
6  AN=N
7  A1=EXP(-DIFF(J)*((2.*AN+1.))**2)*(PI**2)*T(K)/(4.*(WIDTH**2)))
0  A2=COS((2.*AN+1.)*PI*X(K)/(2.*WIDTH))
1  A3= 2.*WIDTH*AY*BC/((2.*AN+1.)*PI)
2  AX=AX+A1*A2*(A3+A(N))
3  IF(A1.LE.1.0E-30) GO TO 24
6 23 CONTINUE
0 24 POTENT(K)=BC+2.*AX/WIDTH
1 26 CONTINUE
3  DO 27 L=1,NS
4  VARIAN(J)=VARIAN(J)+((POTENT(L)-EXPVAL(L))**2)
5 27 CONTINUE
7 28 CONTINUE
1  WRITE(6,220)
2  WRITE(6,231)
3  DO 29 J=1,ND
4 29 WRITE(6,232) DIFF(J),VARIAN(J)
6  END

```


PROGRAM IS BEING ENTERED INTO STORAGE.

DIFFUSIVITY OF COPPER SULFATE IN WATER
CONVENTIONAL APPROACH

DISTANCE, CM.	INITIAL CONDITIONS CONCN., G. MOLS/LIT
0.15650000	0.04938294
0.15222557	0.04921334
0.14327546	0.04972139
0.13512686	0.04981461
0.12150329	0.04997047
0.11167480	0.05054653
0.10082774	0.05113423
0.09219647	0.05192839
0.07980535	0.05418538
0.07225875	0.05641591
0.06328605	0.05854685
0.05722999	0.06116599
0.04998075	0.06588503
0.04493777	0.06904312
0.03989586	0.07278072
0.03671143	0.07568573
0.03238649	0.07805327
0.02939359	0.08124585
0.02661683	0.08426211
0.02119196	0.08742457
0.01958571	0.08996383
0.01794198	0.09290917
0.01548405	0.09621155
0.01338572	0.09768434
0.01145323	0.09932908
0.00956474	0.10059663
0.00829351	0.10177021
0.00625837	0.10295253
0.00656637	0.10433982
0.00571563	0.10568243
0.00000000	0.10638529

THIS IS BEING ENTERED IN THE

DIFFUSIVITY OF COPPER SULFATE IN WATER
CONVENTIONAL METHOD

INITIAL CONCENTRATIONS	
DISTANCE, CM.	CONCENTRATION, GR./CM. ³
0.125000	0.000000
0.125250	0.000000
0.125500	0.000000
0.125750	0.000000
0.126000	0.000000
0.126250	0.000000
0.126500	0.000000
0.126750	0.000000
0.127000	0.000000
0.127250	0.000000
0.127500	0.000000
0.127750	0.000000
0.128000	0.000000
0.128250	0.000000
0.128500	0.000000
0.128750	0.000000
0.129000	0.000000
0.129250	0.000000
0.129500	0.000000
0.129750	0.000000
0.130000	0.000000
0.130250	0.000000
0.130500	0.000000
0.130750	0.000000
0.131000	0.000000
0.131250	0.000000
0.131500	0.000000
0.131750	0.000000
0.132000	0.000000
0.132250	0.000000
0.132500	0.000000
0.132750	0.000000
0.133000	0.000000
0.133250	0.000000
0.133500	0.000000
0.133750	0.000000
0.134000	0.000000
0.134250	0.000000
0.134500	0.000000
0.134750	0.000000
0.135000	0.000000
0.135250	0.000000
0.135500	0.000000
0.135750	0.000000
0.136000	0.000000
0.136250	0.000000
0.136500	0.000000
0.136750	0.000000
0.137000	0.000000
0.137250	0.000000
0.137500	0.000000
0.137750	0.000000
0.138000	0.000000
0.138250	0.000000
0.138500	0.000000
0.138750	0.000000
0.139000	0.000000
0.139250	0.000000
0.139500	0.000000
0.139750	0.000000
0.140000	0.000000

EXPERIMENTAL DATA		
TIME, SECS	DISTANCE, CM.	CONCN., G. MOLS/LIT
20.0	0.01422727	0.09382965
40.0	0.01422727	0.08949336
50.0	0.01422727	0.08945030
75.0	0.01422727	0.08620008
100.0	0.01422727	0.08241088
120.0	0.01422727	0.07989973
150.0	0.01422727	0.07963167
20.0	0.04268181	0.07164642
40.0	0.04268181	0.07350914
50.0	0.04268181	0.07293032
75.0	0.04268181	0.07352710
100.0	0.04268181	0.07388501
120.0	0.04268181	0.07342859
150.0	0.04268181	0.07274343
20.0	0.07113635	0.05739915
40.0	0.07113635	0.05999278
50.0	0.07113635	0.05927962
75.0	0.07113635	0.06082587
100.0	0.07113635	0.06195646
120.0	0.07113635	0.06210056
150.0	0.07113635	0.06304156
20.0	0.09959089	0.05167635
40.0	0.09959089	0.05237648
50.0	0.09959089	0.05331789
75.0	0.09959089	0.05383166
100.0	0.09959089	0.05390080
120.0	0.09959089	0.05515302
150.0	0.09959089	0.05592517
20.0	0.12804543	0.04992752
40.0	0.12804543	0.05043102
50.0	0.12804543	0.05018386
75.0	0.12804543	0.05009583
100.0	0.12804543	0.05062662
120.0	0.12804543	0.05162910
150.0	0.12804543	0.05109184

EXPERIMENTAL DATA

TIME, SEC	TEMP, °C	TIME, SEC
0.05	0.012345	0.012345
0.10	0.012345	0.012345
0.20	0.012345	0.012345
0.30	0.012345	0.012345
0.40	0.012345	0.012345
0.50	0.012345	0.012345
0.60	0.012345	0.012345
0.70	0.012345	0.012345
0.80	0.012345	0.012345
0.90	0.012345	0.012345
1.00	0.012345	0.012345
1.10	0.012345	0.012345
1.20	0.012345	0.012345
1.30	0.012345	0.012345
1.40	0.012345	0.012345
1.50	0.012345	0.012345
1.60	0.012345	0.012345
1.70	0.012345	0.012345
1.80	0.012345	0.012345
1.90	0.012345	0.012345
2.00	0.012345	0.012345
2.10	0.012345	0.012345
2.20	0.012345	0.012345
2.30	0.012345	0.012345
2.40	0.012345	0.012345
2.50	0.012345	0.012345
2.60	0.012345	0.012345
2.70	0.012345	0.012345
2.80	0.012345	0.012345
2.90	0.012345	0.012345
3.00	0.012345	0.012345
3.10	0.012345	0.012345
3.20	0.012345	0.012345
3.30	0.012345	0.012345
3.40	0.012345	0.012345
3.50	0.012345	0.012345
3.60	0.012345	0.012345
3.70	0.012345	0.012345
3.80	0.012345	0.012345
3.90	0.012345	0.012345
4.00	0.012345	0.012345
4.10	0.012345	0.012345
4.20	0.012345	0.012345
4.30	0.012345	0.012345
4.40	0.012345	0.012345
4.50	0.012345	0.012345
4.60	0.012345	0.012345
4.70	0.012345	0.012345
4.80	0.012345	0.012345
4.90	0.012345	0.012345
5.00	0.012345	0.012345

D-ASSUMED, SQ. CM/SEC

VARIANCE

0.00000400

0.0001413

0.00000420

0.0001213

0.00000440

0.0001036

0.00000460

0.0000877

0.00000480

0.0000737

0.00000510

0.0000559

0.00000540

0.0000415

0.00000570

0.0000302

0.00000600

0.0000217

0.00000620

0.0000175

0.00000640

0.0000143

0.00000660

0.0000122

0.00000680

0.0000110

0.00000700

0.0000107

0.00000720

0.0000113

0.00000740

0.0000126

0.00000760

0.0000147

0.00000780

0.0000174

0.00000800

0.0000208

0.00000820

0.0000249

APPENDIX G

Thermal Diffusivity of Water

This appendix consists of four programs:

1. Program IH-5 5, which uses a five-point grid and determines the values of z_i versus time ($i = 1$ to 5) from experimental data.
2. Program IH-5 10, which uses a ten-point grid and determines the values of z_i versus time ($i = 1$ to 10) from experimental data.
3. Program IH-5 10, which uses an analytical solution, generates its own data, and calculates z_i versus time ($i = 1$ to 10).
4. Program IH-20 LOGFIT, which serves the same purpose as in Appendix F.

0 \$IBFTC DARS1 NODECK
1 REAL S(16,16),R(16),V(16,16),A(16),B(16),W1(16),W2(16)
2 REAL X(75),Y(75),XX(75),YY(75),AX(3),AY(3),W(16,16)
3 REAL C(25,10),CZ(10),C3(10),TIME3(25),Z(25,10),GRIDX(10)
4 REAL WORK(10),XNORM(10),WEIGHT(10)
5 REAL DX,WIDTH,BOUND,FACTOR,MAGNF,ALPHA,RM,TBOUND,TINC
6 INTEGER NZ(150),OT(6),NA,COUNTR,NT,N,N1,MAXN,M,J,K,LINES
C
C
C GENERAL PROGRAM FOR THERMAL CONDUCTIVITY DETERMINATION
C
C 1. THIS PROGRAM CAN HANDLE EXPERIMENTAL DATA OR AN ANALYTICAL
C SOLUTOIN AS A STARTING POINT FOR THE DETERMINATION OF
C CONDUCTIVITY USING THE SEMI-ANALYTICAL TECHNIQUE.
C
C 2. IF COUNTR=1,THE PROGRAM TAKES THE ROUTE OF ANALYTICAL
C SOLUTION. IF IT IS AN INTEGER OTHER THAN 1, IT TAKES THE
C ROUTE OF EXPERIMENTAL DATA.
C
C 3. THE GENERAL PROGRAM HAS TO INCLUDE THE FOLLOWING SUBROU-
C TINES-EIG1,INTERP(3RD ORDER INTERPOLATION),LINEAR(LINEAR
C INTERPOLATION),CHECK,DATA,CONVRT,ANALYT,MODIFY, AND PLOT.
C LINEAR OR INTERP IS TO BE USED WITH DISCRESION.
C
C 4. THE ROUTE OF THE ANALYTICAL SOLUTION CALLS FOR THE FOLLOW-
C ING DATA,READ IN THE FOLLOWING ORDER AND THE FORMAT STATE-
C MENTS OF THE PROGRAM(THE SYMBOLS ARE EXPLAINED).
C COUNTR(=1)
C NT(NUMBER OF TIMES),N(NUMBER OF GRID POINTS)
C GRIDX(ACTUAL POSITIONS OF THE GRID POINTS)
C S(J,J) (DIAGONAL ELEMENTS OF THE COEFFICIENT MATRIX)
C S(J,J+1)
C TIME3(TIMES,CORRESPONDING TO NT)
C RM(SQUARE OF THE HALF-WIDTH OF THE CELL)
C ALPHA,RM,TBOUND,TINC(THERMAL DIFFUSIVITY,RM(ABOVE),BOUND-
C ARY TEMP.,THE UNIFORM INITIAL TEMP.)
C CZ(THE INITIAL CONDITION VECTOR)
C C3(THE BOUNDARY CONDITION VECTOR)
C (NOTE THAT THERE IS SOME DUPLICATION.IT MAY ALSO BE NECES-
C SARY TO CHANGE THE DATA DEPENDING ON THE ANALYTICAL SOLN.)
C
C 5. THE ROUTE OF EXPERIMENTAL DATA CALLS FOR THE FOLLOWING
C DATA,READ IN THE ORDER SPECIFIED BELOW.
C COUNTR,NOT EQUAL TO 1
C NT,N
C GRIDX
C S(J,J)
C S(J,J+1)
C TIME3
C (THE REST OF DATA READ IN STRICTLY FOLLOWS THE WAY IN
C WHICH THE DECK IS MADE AVAILABLE)
C OT(6 POINTS FOR DETERMINATION OF ORIENTATION OF THE FRAMES)
C CZ(THE INITIAL CONDITION VECTOR)
C C3(THE BOUNDARY CONDITION VECTOR)
C WIDTH,BOUND,FACTOR,MAGNF(TOTAL WIDTH OF THE MEASUREMENTS-

SOURCE STATEMENT

C WHETHER HALF FRAME OR FULL FRAME, BOUNDARY TEMP., CONVERSI-
C ON FACTOR FROM MICRONS TO DEGREES, MAGNIFICATION FACTOR FOR
C THE FRAMES, (I.E.) THE RATIO OF ACTUAL FRAME SIZE TO THE
C ACTUAL CELL SIZE, IF ANY)
C NA (NUMBER OF POINTS ON THE FRAME-X+Y)
C NZ (THE ACTUAL POINT X AND Y AS SUPPLIED)
C NT NUMBER OF SUCH SETS
C
C

7 1 FORMAT(1X,I4)
10 2 FORMAT(10X,14I5)
11 3 FORMAT(10X,6I5)
12 56 FORMAT(1X,10F5.1)
13 57 FORMAT(1X,10F5.1)
14 220 FORMAT(1H2)
15 221 FORMAT(1H 10X,8H ..CONTD)
16 222 FORMAT(1H ,//)
17 223 FORMAT(1H ,/)
20 224 FORMAT(1H ,10X,30H THERMAL CONDUCTIVITY OF WATER,
134H-DATA FROM DR.R.N.O*BRIEN*S EXPTS.)
21 225 FORMAT(1H ,10X,14H CELL WIDTH,CM,2X,20H BOUNDARY TEMP,DEG.C,
12X,12H CONV.FACTOR,2X,14H MAGNIFICATION)
22 226 FORMAT(1H ,14X,F7.4,10X,F6.1,13X,F8.5,6X,F7.4)
23 227 FORMAT(1H ,29X,22H ORIENTATION OF FRAMES)
24 228 FORMAT(1H ,33X,2H X,8X,2H Y)
25 229 FORMAT(1H ,30X,F8.1,3X,F8.1)
26 230 FORMAT(1H ,32X,7H SLOPE=,F6.2)
27 231 FORMAT(1H ,10X,26H A TEN-POINT GRID IS USED.,
130H THE GRID POINTS ARE LOCATED AT)
30 232 FORMAT(1H ,10X,5F11.6)
31 233 FORMAT(1H ,28X,25H INITIAL CONDITION VECTOR)
32 234 FORMAT(1H ,10X,10F6.2)
33 235 FORMAT(1H ,28X,26H BOUNDARY CONDITION VECTOR)
34 236 FORMAT(1H ,10X,14H ELAPSED TIME=,F6.3,8H SECONDS)
35 237 FORMAT(1H ,20X,9H RAW DATA,5X,
132H CONVERTED TO THE RELEVANT UNITS)
36 238 FORMAT(1H ,20X,2H X,5X,2H Y,8X,12H DISTANCE,CM,5X,
118H TEMPERATURE,DEG.C)
37 239 FORMAT(1H ,15X,F8.1,2X,F8.1,4X,F11.6,8X,F11.6)
40 240 FORMAT(1H ,30X,19H COEFFICIENT MATRIX)
41 241 FORMAT(1H ,10X,10F6.2)
42 242 FORMAT(1H ,31X,12H EIGENVALUES)
43 243 FORMAT(1H ,31X,13H EIGENVECTORS)
44 244 FORMAT(1H ,20X,35H CHECK OF SIMILARITY TRANSFORMATION)
45 245 FORMAT(1H ,8X,8H TIME,SEC,5X,
156H INTERPOLATED VALUES OF TEMPERATURE AT GRID POINTS,DEG.C)
46 246 FORMAT(1H ,8X,F6.3,2X,10F6.2)
47 247 FORMAT(1H ,30X,18H MODIFIED VARIABLE)
50 248 FORMAT(1H ,10X,10H TIME,SECS,5X,2H 1,10X,2H 2, 9X,2H 3, 9X,
12H 4,9X,2H 5)
51 249 FORMAT(1H ,10X,10H TIME,SECS,5X,2H 6,10X,2H 7, 9X,2H 8, 9X,
12H 9, 9X,2H 10)
52 250 FORMAT(1H ,30X,20H VALUES FOR PLOTTING)
53 251 FORMAT(1H ,12X,F6.3,2X,5F11.6)
54 252 FORMAT(1H ,15X,25H NORMALISED INNER PRODUCT,

THESE SPECIFICATIONS ARE FOR THE PURPOSE OF PROVIDING A BASIS FOR THE SELECTION OF MATERIALS AND METHODS OF CONSTRUCTION. THE ENGINEER SHALL BE RESPONSIBLE FOR THE INTERPRETATION OF THESE SPECIFICATIONS AND FOR THE SELECTION OF THE APPROPRIATE MATERIALS AND METHODS OF CONSTRUCTION. THE CONTRACTOR SHALL BE RESPONSIBLE FOR THE PROVISION OF ALL MATERIALS AND METHODS OF CONSTRUCTION REQUIRED FOR THE COMPLETION OF THE PROJECT.

- 1. FORM (1, 1)
- 2. FORM (1, 1)
- 3. FORM (1, 1)
- 4. FORM (1, 1)
- 5. FORM (1, 1)
- 6. FORM (1, 1)
- 7. FORM (1, 1)
- 8. FORM (1, 1)
- 9. FORM (1, 1)
- 10. FORM (1, 1)
- 11. FORM (1, 1)
- 12. FORM (1, 1)
- 13. FORM (1, 1)
- 14. FORM (1, 1)
- 15. FORM (1, 1)
- 16. FORM (1, 1)
- 17. FORM (1, 1)
- 18. FORM (1, 1)
- 19. FORM (1, 1)
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SOURCE STATEMENT

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136H OF I.C. VECTOR WITH EACH EIGENVECTOR)
55 253 FORMAT(1H ,20X,5F11.6)
56 261 FORMAT(1H ,10X,32H THERMAL CONDUCTIVITY-ANALYTICAL,
129H SOLUTION AS A STARTING POINT)
57 262 FORMAT(1H ,10X,11H SLAB WIDTH,2X,20H ASSUMED DIFFUSIVITY,
12X,15H BOUNDARY TEMP.,2X,14H INITIAL TEMP.)
60 263 FORMAT(1H ,14X,3HCM.,11X,10H SQ.CM/SEC,12X,6H DEG.C,10X,
15HDEG.C)
61 264 FORMAT(1H ,12X,F7.4,9X,F10.7,12X,F6.2, 9X,F6.2)
62 265 FORMAT(1H ,10X,39H THE ANALYTICAL SOLUTION(REF.MCADAMS,P.,
124H34)USED TO CALCULATE THE/10X,23H TEMPERATURE YIELDS THE,
242H FOLLOWING TEMPERATURES AT THE GRID POINTS)
63 266 FORMAT(1H ,8X,8HTIME,SEC,20X,18H TEMPERATURE,DEG.C)
64 1000 FORMAT(1X,2I4)
65 1001 FORMAT(1X, 5F12.8)
66 1002 FORMAT(1X,5F14.8)
67 READ(5,1) COUNTR
71 READ(5,1000) NT,N
74 READ(5,1001) (GRIDX(J),J=1,N)
101 DX=ABS(GRIDX(3)-GRIDX(2))
102 DO 50 J=1,N
103 DO 50 K=1,N
104 50 S(J,K)=0.0
107 READ(5,56) (S(J,J),J=1,N)
114 N1=N-1
115 READ(5,56) (S(J,J+1),J=1,N1)
122 DO 52 J=1,N1
123 52 S(J+1,J)=S(J,J+1)
125 READ(5,57) (TIME3(J),J=1,NT)
132 MAXN=16
133 M=-N
134 CALL EIG1(N,MAXN,M,S,R,V,A,B,W1,W2)
135 CALL CHECK(V,R,N,W)
136 IF(COUNTR.EQ.1) GO TO 28
141 READ(5,3) (OT(J),J=1,6)
146 DO 10 J=1,3
147 10 AX(J)=FLOAT(OT(2*J))
151 DO 11 J=1,3
152 11 AY(J)=FLOAT(OT(2*J-1))
154 CALL LESQFT(AX,AY,SLOPE)
155 READ(5,56) (CZ(J),J=1,N)
162 READ(5,56)(C3(J),J=1,N)
167 READ(5,1002) WIDTH,BOUND,FACTOR,MAGNF
170 VALUE=0.0
171 DO 131 K=1,N
172 VALUE=VALUE+CZ(K)**2
173 WEIGHT(K)=0.0
174 DO 131 J=1,N
175 131 WEIGHT(K)=WEIGHT(K)+CZ(J)*V(J,K)
200 DO 132 J=1,N
201 132 WEIGHT(J)=WEIGHT(J)/SQRT(VALUE)
203 WRITE(6,220)
204 WRITE(6,224)
205 WRITE(6,222)
206 WRITE(6,225)
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SOURCE STATEMENT

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207 WRITE(6,226) WIDTH,BOUND,FACTOR,MAGNF
210 WRITE(6,223)
211 WRITE(6,227)
212 WRITE(6,228)
213 DO 340 J=1,3
214 340 WRITE(6,229) AX(J),AY(J)
216 WRITE(6,230) SLOPE
217 WRITE(6,222)
220 WRITE(6,231)
221 WRITE(6,232) (GRIDX(J),J=1,N)
226 WRITE(6,222)
227 WRITE(6,233)
230 WRITE(6,234) (CZ(J),J=1,N)
235 WRITE(6,222)
236 WRITE(6,235)
237 WRITE(6,234) (C3(J),J=1,N)
244 WRITE(6,220)
245 LINES=8
246 CALL LINECT(LINES,1,2)
247 WRITE(6,237)
250 NE=1
251 4 READ(5,1) NA
253 READ(5,2) (NZ(J),J=1,NA)
260 CALL DATA(NA,NZ,X,Y,NP)
261 DO 8 J=1,NP
262 XX(J)=X(J)
263 8 YY(J)=Y(J)
265 CALL CONVRT(NP,X,Y,SLOPE,WIDTH,BOUND,FACTOR,MAGNF)
266 CALL LINECT(LINES,2,2)
267 WRITE(6,223)
270 CALL LINECT(LINES,1,2)
271 WRITE(6,236) TIME3(NE)
272 CALL LINECT(LINES,1,2)
273 WRITE(6,238)
274 DO 22 J=1,NP
275 CALL LINECT(LINES,1,1)
276 22 WRITE(6,239) XX(J),YY(J),X(J),Y(J)
300 DO 210 KA=1,N
301 CALL LINEAR(NP,GRIDX,X,Y,KA,RR)
302 C(NE,KA)=RR
303 210 CONTINUE
305 NE=NE+1
306 IF(NE.LE.NT) GO TO 4
311 CALL MODIFY(WORK,V,C3,CZ,Z,C,N,NT)
312 WRITE(6,220)
313 LINES=8
314 CALL LINECT(LINES,1,2)
315 WRITE(6,240)
316 DO 345 J=1,N
317 CALL LINECT(LINES,1,2)
320 345 WRITE(6,241) (S(J,K),K=1,N)
326 CALL LINECT(LINES,3,2)
327 WRITE(6,222)
330 CALL LINECT(LINES,1,2)
331 WRITE(6,242)
```


SOURCE STATEMENT

```
332 CALL LINECT(LINES,2,2)
333 WRITE(6,232) (R(J),J=1,N)
340 CALL LINECT(LINES,3,2)
341 WRITE(6,222)
342 CALL LINECT(LINES,1,2)
343 WRITE(6,243)
344 DO 346 K=1,N
345 CALL LINECT(LINES,4,2)
346 WRITE(6,232) (V(J,K),J=1,N)
353 346 WRITE(6,223)
355 CALL LINECT(LINES,4,2)
356 WRITE(6,222)
357 WRITE(6,244)
360 DO 347 J=1,N
361 CALL LINECT(LINES,4,2)
362 WRITE(6,232) (W(J,K),K=1,N)
367 347 WRITE(6,223)
371 WRITE(6,220)
372 WRITE(6,245)
373 WRITE(6,223)
374 DO 348 J=1,NT
375 348 WRITE(6,246) TIME3(J),(C(J,K),K=1,N)
403 WRITE(6,223)
404 WRITE(6,247)
405 WRITE(6,223)
406 WRITE(6,248)
407 DO 349 J=1,NT
410 349 WRITE(6,251) TIME3(J),(Z(J,K),K=1,5)
416 WRITE(6,222)
417 WRITE(6,249)
420 DO 350 J=1,NT
421 350 WRITE(6,251) TIME3(J),(Z(J,K),K=6,10)
427 CALL PLOT(C,R,Z,C3,CZ,N,NT)
430 WRITE(6,220)
431 WRITE(6,250)
432 WRITE(6,223)
433 WRITE(6,248)
434 DO 351 J=1,NT
435 351 WRITE(6,251) TIME3(J),(C(J,K),K=1,5)
443 WRITE(6,223)
444 WRITE(6,252)
445 WRITE(6,253) (WEIGHT(J),J=1,5)
452 WRITE(6,223)
453 WRITE(6,249)
454 DO 352 J=1,NT
455 352 WRITE(6,251) TIME3(J),(C(J,K),K=6,10)
463 WRITE(6,223)
464 WRITE(6,252)
465 WRITE(6,253) (WEIGHT(J),J=6,10)
472 GO TO 29
473 28 READ(5,1002)RM
474 DO 30 J=1,N
475 30 XNORM(J)=GRIDX(J)/(2.*SQRT(RM))
477 31 READ(5,1002) ALPHA,RM,TBOUND,TINC
500 SLAB=2.*SQRT(RM)
```


SOURCE STATEMENT

```
501 READ(5,56) (CZ(J),J=1,N)
506 READ(5,56) (C3(J),J=1,N)
513 VALUE=0.0
514 DO 32 K=1,N
515 VALUE=VALUE+CZ(K)**2
516 WEIGHT(K)=0.0
517 DO 32 J=1,N
520 32 WEIGHT(K)=WEIGHT(K)+CZ(J)*V(J,K)
523 DO 33 J=1,N
524 33 WEIGHT(J)=WEIGHT(J)/SQRT(VALUE)
526 CALL ANALYT(ALPHA, RM, TBOUND, TINC, N, NT, XNORM, TIME3, C)
527 WRITE(6,220)
530 WRITE(6,261)
531 WRITE(6,222)
532 WRITE(6,262)
533 WRITE(6,263)
534 WRITE(6,264) SLAB, ALPHA, TBOUND, TINC
535 WRITE(6,222)
536 WRITE(6,231)
537 WRITE(6,232) (GRIDX(J),J=1,N)
544 WRITE(6,223)
545 WRITE(6,265)
546 WRITE(6,223)
547 WRITE(6,266)
550 WRITE(6,223)
551 DO 361 J=1,NT
552 361 WRITE(6,246) TIME3(J), (C(J,K),K=1,N)
560 WRITE(6,222)
561 WRITE(6,233)
562 WRITE(6,234) (CZ(J),J=1,N)
567 WRITE(6,222)
570 WRITE(6,235)
571 WRITE(6,234) (C3(J),J=1,N)
576 CALL MODIFY(WORK, V, C3, CZ, Z, C, N, NT)
577 WRITE(6,220)
600 LINES=8
601 CALL LINECT(LINES,1,2)
602 WRITE(6,240)
603 DO 362 J=1,N
604 CALL LINECT(LINES,1,2)
605 362 WRITE(6,241) (S(J,K),K=1,N)
613 CALL LINECT(LINES,3,2)
614 WRITE(6,222)
615 CALL LINECT(LINES,1,2)
616 WRITE(6,242)
617 CALL LINECT(LINES,2,2)
620 WRITE(6,232) (R(J),J=1,N)
625 CALL LINECT(LINES,3,2)
626 WRITE(6,222)
627 CALL LINECT(LINES,1,2)
630 WRITE(6,243)
631 DO 363 K=1,N
632 CALL LINECT(LINES,4,2)
633 WRITE(6,232) (V(J,K),J=1,N)
640 363 WRITE(6,223)
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SOURCE STATEMENT

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642 CALL LINECT(LINES,4,2)
643 WRITE(6,222)
644 WRITE(6,244)
645 DO 364 J=1,N
646 CALL LINECT(LINES,4,2)
647 WRITE(6,232) (W(J,K),K=1,N)
654 364 WRITE(6,223)
656 WRITE(6,220)
657 WRITE(6,247)
660 WRITE(6,223)
661 WRITE(6,248)
662 DO 366 J=1,NT
663 366 WRITE(6,251) TIME3(J),(Z(J,K),K=1,5)
671 WRITE(6,222)
672 WRITE(6,249)
673 DO 367 J=1,NT
674 367 WRITE(6,251) TIME3(J),(Z(J,K),K=6,10)
702 CALL PLOT(C,R,Z,C3,CZ,N,NT)
703 WRITE(6,220)
704 WRITE(6,250)
705 WRITE(6,223)
706 WRITE(6,248)
707 DO 368 J=1,NT
710 368 WRITE(6,251) TIME3(J),(C(J,K),K=1,5)
716 WRITE(6,223)
717 WRITE(6,252)
720 WRITE(6,253) (WEIGHT(J),J=1,5)
725 WRITE(6,223)
726 WRITE(6,249)
727 DO 369 J=1,NT
730 369 WRITE(6,251) TIME3(J),(C(J,K),K=6,10)
736 WRITE(6,223)
737 WRITE(6,252)
740 WRITE(6,253) (WEIGHT(J),J=6,10)
745 29 CONTINUE
746 END
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ISN SOURCE STATEMENT

```
0 $IBFTC CONVRT NODECK
1 SUBROUTINE CONVRT(NP,X,Y,SLOPE,WIDTH,BOUND,FACTOR,MAGNF)
C
C CONVRT CONVERTS THE RAW DATA INTO THE RELEVANT UNITS.
C
2 REAL X(75),Y(75) ,MAGNF
3 S1=ATAN(SLOPE)
4 S2=1.5707963-S1
5 S3=SIN(S1)
6 S4=SIN(S2)/COS(S2)
7 DO 16 J=1,NP
10 XX=(X(J)-Y(J)/SLOPE)*S3
11 YY=(Y(J)+X(J)*S4)*S3
12 X(J)=XX
13 16 Y(J)=YY
15 X1=X(1)
16 XN=X(NP)
17 DO 17 J=1,NP
20 17 X(J)=WIDTH*(X(J)-X1)/(XN-X1)
22 Y1=Y(1)
23 DO 19 J=1,NP
24 VALUE=Y(J)-Y1
25 19 Y(J)=BOUND-ABS(VALUE)*FACTOR*MAGNF
27 RETURN
30 END
```


PROGRAM IS BEING ENTERED INTO STORAGE.

THERMAL CONDUCTIVITY-ANALYTICAL SOLUTION AS A STARTING POINT

SLAB WIDTH CM.	ASSUMED DIFFUSIVITY SQ.CM/SEC	BOUNDARY TEMP. DEG.C	INITIAL TEMP. DEG.C
0.0430	0.0014090	25.00	5.20

A TEN-POINT GRID IS USED. THE GRID POINTS ARE LOCATED AT

0.002048	0.004096	0.006144	0.008191	0.010239
0.012287	0.014334	0.016382	0.018430	0.020477

THE ANALYTICAL SOLUTION (REF. MCADAMS, P. 34) USED TO CALCULATE THE TEMPERATURE YIELDS THE FOLLOWING TEMPERATURES AT THE GRID POINTS

TIME, SEC

TEMPERATURE, DEG.C

0.010	19.05	13.92	10.09	7.63	6.26	5.61	5.34	5.24	5.21	5.20
0.020	20.74	16.79	13.38	10.65	8.62	7.21	6.32	5.78	5.50	5.38
0.030	21.51	18.19	15.18	12.59	10.46	8.81	7.59	6.76	6.24	6.00
0.050	22.30	19.67	17.21	14.97	13.00	11.34	10.01	9.01	8.35	8.02
0.070	22.75	20.55	18.47	16.54	14.81	13.32	12.10	11.17	10.54	10.22
0.090	23.08	21.21	19.42	17.76	16.27	14.97	13.90	13.08	12.52	12.24
0.100	23.22	21.49	19.83	18.30	16.91	15.70	14.71	13.94	13.42	13.16
0.110	23.35	21.75	20.21	18.79	17.50	16.38	15.45	14.74	14.26	14.01
0.120	23.47	21.98	20.56	19.24	18.04	17.00	16.15	15.48	15.03	14.81

INITIAL CONDITION VECTOR

5.20 5.20 5.20 5.20 5.20 5.20 5.20 5.20 5.20 5.20

BOUNDARY CONDITION VECTOR

25.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00

PROGRAM IS BEING EXTENDED FOR 10000.

THERMAL CONDUCTIVITY-ANALYTICAL SECTION OF A SECTION WITH

SLAB WIDTH	ASSUMED DIFFUSIVITY	WINDUP TIME	INITIAL TEMP
CM.	SEC-CM ² /SEC	SEC	SEC
0.0030	0.001000	0.00	0.00

A TEN-POINT GRID IS USED. THE GRID POINTS ARE LOCATED AT
0.002048 0.004096 0.008192 0.016384 0.032768 0.065536 0.131072 0.262144 0.524288 1.048576

THE ANALYTICAL SECTION (REF. WINDUP, 9.31) IS USED TO AVERAGE THE TEMPERATURE YIELDS THE FOLLOWING TEMPERATURES AT THE GRID POINTS

TIME, SEC TEMPERATURE, DEG.C

0.010	19.02	13.92	10.09	7.83	6.58	5.81	5.27	4.88	4.54
0.020	20.74	16.79	13.76	10.62	8.62	7.51	6.86	6.38	6.00
0.030	21.21	18.19	12.18	12.78	10.46	8.81	7.98	7.48	7.08
0.040	22.30	19.67	14.21	14.97	13.00	11.34	10.07	9.01	8.42
0.050	22.72	20.22	15.47	16.24	14.71	13.32	12.10	11.17	10.47
0.060	23.08	21.31	16.42	17.30	15.27	14.97	13.90	13.02	12.32
0.070	23.22	21.69	16.83	17.80	15.91	15.70	14.71	13.90	13.18
0.080	23.32	21.72	17.00	18.00	16.30	16.30	15.40	14.70	14.00
0.090	23.47	21.98	18.24	18.04	17.00	16.17	15.40	14.80	14.30

INITIAL CONDITION SECTION
0.50 0.50 0.50 0.50 0.50 0.50 0.50 0.50 0.50 0.50

BOUNDARY CONDITION SECTION
0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00

COEFFICIENT MATRIX

-2.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1.00	-2.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	1.00	-2.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	1.00	-2.00	1.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	1.00	-2.00	1.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	1.00	-2.00	1.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	1.00	-2.00	1.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	1.00	-2.00	1.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	-2.00	1.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	-1.00

EIGENVALUES

-3.911146	-3.652478	-3.246980	-2.730682	-2.149460
-1.554958	-1.000000	-0.533896	-0.198062	-0.022338

EIGENVECTORS

0.128642	-0.245853	0.341219	-0.406267	0.435215
-0.425493	0.377964	-0.296852	0.189362	-0.065047
-0.245853	0.406267	-0.425493	0.296852	-0.065047
-0.189363	0.377965	-0.435216	0.341219	-0.128642
-0.341219	0.425493	-0.189362	-0.189362	0.425493
-0.341219	0.000000	0.341219	-0.425493	0.189362
0.406267	-0.296852	-0.189362	0.435215	-0.128642
-0.341219	0.377965	0.065047	-0.425493	0.245853
-0.435215	0.065048	0.425493	-0.128642	-0.406267
0.189363	0.377964	-0.245853	-0.341219	0.296852
0.425493	0.189362	-0.341219	-0.341219	0.189362
0.425493	0.000000	-0.425493	-0.189362	0.341219
0.377964	0.377964	-0.000000	-0.377965	-0.377965
-0.000000	0.377964	0.377964	0.000000	-0.377964
0.296852	0.435215	0.341219	0.065047	-0.245853
-0.425493	-0.377964	-0.128642	0.189362	0.406267

..CONTD

0.189362	0.341219	0.425493	0.425493	0.341219
0.189362	-0.000000	-0.189362	-0.341219	-0.425494

-0.065047	-0.128642	-0.189362	-0.245853	-0.296852
-0.341219	-0.377964	-0.406267	-0.425494	-0.435216

CHECK OF SIMILARITY TRANSFORMATION

-1.999999	1.000000	-0.000000	-0.000000	0.000000
0.000000	-0.000000	0.000000	-0.000000	0.000000

1.000000	-2.000000	1.000000	0.000000	-0.000000
0.000000	-0.000000	0.000000	0.000000	-0.000000

-0.000000	1.000000	-2.000000	1.000000	0.000000
-0.000001	0.000001	-0.000000	-0.000000	0.000000

-0.000000	0.000000	1.000000	-2.000000	0.999999
0.000000	0.000000	-0.000000	0.000000	0.000000

0.000000	-0.000000	0.000000	0.999999	-2.000000
1.000000	-0.000000	0.000000	-0.000000	0.000000

0.000000	0.000000	-0.000001	0.000000	1.000000
-2.000000	1.000000	-0.000000	0.000000	-0.000000

-0.000000	-0.000000	0.000001	0.000000	-0.000000
1.000000	-2.000000	1.000001	-0.000000	-0.000000

0.000000	0.000000	-0.000000	-0.000000	0.000000
-0.000000	1.000001	-2.000000	1.000000	0.000000

-0.000000	0.000000	-0.000000	-0.000000	-0.000000
0.000000	-0.000000	1.000000	-2.000000	1.000000

0.000000	-0.000000	0.000000	0.000000	0.000000
-0.000000	-0.000000	0.000000	1.000000	-1.000000

MODIFIED VARIABLE

TIME, SECS	1	2	3	4	5
0.010	0.822068	-1.682543	-2.627236	3.719767	-5.061551
0.020	0.822100	-1.682462	-2.627182	3.719906	-5.062151
0.030	0.822142	-1.682456	-2.627144	3.719948	-5.062104
0.050	0.822186	-1.682486	-2.627119	3.719935	-5.062047
0.070	0.822204	-1.682525	-2.627122	3.719882	-5.062020
0.090	0.822216	-1.682560	-2.627132	3.719826	-5.062004
0.100	0.822220	-1.682577	-2.627137	3.719800	-5.061998
0.110	0.822224	-1.682592	-2.627142	3.719775	-5.061992
0.120	0.822227	-1.682606	-2.627146	3.719752	-5.061987

TIME, SECS	6	7	8	9	10
0.010	6.826212	9.241334	12.136411	14.114870	-19.218249
0.020	6.840742	9.443453	13.630496	18.927211	-23.099655
0.030	6.840796	9.448587	13.858642	21.373002	-26.700053
0.050	6.840824	9.448778	13.898955	23.247765	-33.137525
0.070	6.840840	9.448822	13.899978	23.732087	-38.676150
0.090	6.840850	9.448859	13.900046	23.857287	-43.441333
0.100	6.840855	9.448876	13.900063	23.878773	-45.568243
0.110	6.840859	9.448892	13.900078	23.889723	-47.541057
0.120	6.840862	9.448906	13.900091	23.895318	-49.370939

STABLE-STATE VALUES FOR PLOTTING

TIME, SECS	1	2	3	4	5
0.010	0.000320	0.000179	-0.000015	-0.000104	0.000091
0.020	0.000271	0.000240	0.000011	-0.000151	-0.000059
0.030	0.000207	0.000244	0.000029	-0.000166	-0.000047
0.050	0.000140	0.000222	0.000041	-0.000161	-0.000033
0.070	0.000111	0.000193	0.000040	-0.000143	-0.000026
0.090	0.000094	0.000166	0.000035	-0.000124	-0.000022
0.100	0.000088	0.000154	0.000033	-0.000115	-0.000021
0.110	0.000082	0.000142	0.000030	-0.000107	-0.000019
0.120	0.000076	0.000131	0.000028	-0.000099	-0.000018

NORMALISED INNER PRODUCT OF I.C. VECTOR WITH EACH EIGENVECTOR

0.010401 -0.021286 -0.033232 0.047048 -0.064029

TIME, SECS	6	7	8	9	10
0.010	0.002713	0.027764	0.160218	0.517002	0.929300
0.020	0.000032	0.000756	0.024503	0.262789	0.861980
0.030	0.000022	0.000070	0.003780	0.133589	0.799533
0.050	0.000016	0.000044	0.000118	0.034554	0.687880
0.070	0.000014	0.000038	0.000025	0.008969	0.591817
0.090	0.000012	0.000033	0.000019	0.002355	0.509168
0.100	0.000011	0.000031	0.000017	0.001220	0.472278
0.110	0.000010	0.000029	0.000016	0.000642	0.438061
0.120	0.000009	0.000027	0.000015	0.000346	0.406323

NORMALISED INNER PRODUCT OF I.C. VECTOR WITH EACH EIGENVECTOR

0.086531 0.119523 0.175826 0.302337 -0.920828

VALUES FOR PITCH

TIME, SEC	1	2	3	4	5
0.150	0.00000	0.00000	0.00000	0.00000	0.00000
0.110	0.00000	0.00000	0.00000	0.00000	0.00000
0.100	0.00000	0.00000	0.00000	0.00000	0.00000
0.090	0.00000	0.00000	0.00000	0.00000	0.00000
0.070	0.00000	0.00000	0.00000	0.00000	0.00000
0.050	0.00000	0.00000	0.00000	0.00000	0.00000
0.030	0.00000	0.00000	0.00000	0.00000	0.00000
0.020	0.00000	0.00000	0.00000	0.00000	0.00000
0.010	0.00000	0.00000	0.00000	0.00000	0.00000
0.000	0.00000	0.00000	0.00000	0.00000	0.00000

NORMALISED INNER PRODUCT OF 1.0 VECTOR WITH EACH VECTOR

TIME, SEC	1	2	3	4	5
0.150	0.00000	0.00000	0.00000	0.00000	0.00000
0.110	0.00000	0.00000	0.00000	0.00000	0.00000
0.100	0.00000	0.00000	0.00000	0.00000	0.00000
0.090	0.00000	0.00000	0.00000	0.00000	0.00000
0.070	0.00000	0.00000	0.00000	0.00000	0.00000
0.050	0.00000	0.00000	0.00000	0.00000	0.00000
0.030	0.00000	0.00000	0.00000	0.00000	0.00000
0.020	0.00000	0.00000	0.00000	0.00000	0.00000
0.010	0.00000	0.00000	0.00000	0.00000	0.00000
0.000	0.00000	0.00000	0.00000	0.00000	0.00000

NORMALISED INNER PRODUCT OF 1.0 VECTOR WITH EACH VECTOR

SEMI-LOGARITHMIC LEAST SQUARE FIT OF RESULTS

COLUMN NUMBER= 9

TIME, SECS	EXPTL. VALUES OF Z	LOGARITHMS	Z-VALUES FROM FIT	
0.010	0.51700200	-0.659709	0.51361190	
0.020	0.26278900	-1.336404	0.26208108	
0.030	0.13358900	-2.012987	0.13373228	
0.050	0.03455400	-3.365232	0.03482069	
0.070	0.00896900	-4.713981	0.00906648	
0.090	0.00235500	-6.051215	0.00236069	
0.100	0.00122000	-6.708904	0.00120459	
GRID SPACING	EIGENVALUE	SLOPE	INTERCEPT	DIFFUSIVITY
CM.				SQ. CM/SEC
0.0020480	-0.198062	-67.281402	1.006548	0.14248E-02

2E1-LOGARITHMIC LEAST SQUARES FIT OF DATA

COLUMN NUMBERS

TIME, SEC2	EXPTL. VALUES OF Y	LOGARITHM	2-VALUE POINT WITH
0.100	0.0015500	-0.000000	0.0015500
0.090	0.0033500	-0.001512	0.0033500
0.070	0.0086900	-0.013991	0.0086900
0.050	0.0362400	-0.002335	0.0362400
0.030	0.1333800	-0.113387	0.1333800
0.020	0.5857800	-1.338000	0.5857800
0.010	0.21700500	-0.000000	0.21700500

GRID SPACING CM.	EIGENVALUE	SLOPE	INTERCEPT	STANDARD DEVIATION
0.0050480	-0.18805	-0.51405	1.00000	0.00000

SEMI-LOGARITHMIC LEAST SQUARE FIT OF RESULTS

COLUMN NUMBER= 10

TIME, SECS	EXPTL. VALUES OF Z	LOGARITHMS	Z-VALUES FROM FIT
0.010	0.92930000	-0.073324	0.92930811
0.020	0.86197999	-0.148523	0.86198025
0.030	0.79953300	-0.223727	0.79953025
0.050	0.68788000	-0.374141	0.68787587
0.070	0.59181700	-0.524558	0.59181402
0.090	0.50916800	-0.674977	0.50916720
0.100	0.47227800	-0.750187	0.47227831
0.110	0.43806100	-0.825397	0.43806200
0.120	0.40632300	-0.900607	0.40632465

GRID SPACING CM.	EIGENVALUE	SLOPE	INTERCEPT	DIFFUSIVITY SQ.CM/SEC
0.0020480	-0.022338	-7.520799	1.001895	0.14121E-02

SOURCE STATEMENT

```
0 $IBFTC DARS1  NODECK
1 REAL S(16,16),R(16),V(16,16),A(16),B(16),W1(16),W2(16)
2 REAL X(75),Y(75),XX(75),YY(75),AX(3),AY(3),W(16,16)
3 REAL C(25,10),CZ(10),C3(10),TIME3(25),Z(25,10),GRIDX(10)
4 REAL WORK(10),XNORM(10),WEIGHT(10)
5 REAL DX,WIDTH,BOUND,FACTOR,MAGNF,ALPHA,RM,TBOUND,TINC
6 INTEGER NZ(150),OT(6),NA,COUNTR,NT,N,N1,MAXN,M,J,K,LINES

C
C
C      GENERAL PROGRAM FOR THERMAL CONDUCTIVITY DETERMINATION
C
C  1. THIS PROGRAM CAN HANDLE EXPERIMENTAL DATA OR AN ANALYTICAL
C     SOLUTION AS A STARTING POINT FOR THE DETERMINATION OF
C     CONDUCTIVITY USING THE SEMI-ANALYTICAL TECHNIQUE.
C
C  2. IF COUNTR=1,THE PROGRAM TAKES THE ROUTE OF ANALYTICAL
C     SOLUTION. IF IT IS AN INTEGER OTHER THAN 1, IT TAKES THE
C     ROUTE OF EXPERIMENTAL DATA.
C
C  3. THE GENERAL PROGRAM HAS TO INCLUDE THE FOLLOWING SUBROUTINES-
C     EIG1,INTERP(3RD ORDER INTERPOLATION),LINEAR(LINEAR INTERPOLATION),
C     CHECK,DATA,CONVRT,ANALYT,MODIFY, AND PLOT. LINEAR OR INTERP IS TO
C     BE USED WITH DISCRETION.
C
C  4. THE ROUTE OF THE ANALYTICAL SOLUTION CALLS FOR THE FOLLOWING
C     DATA,READ IN THE FOLLOWING ORDER AND THE FORMAT STATEMENTS OF
C     THE PROGRAM(THE SYMBOLS ARE EXPLAINED).
C     COUNTR(=1)
C     NT(NUMBER OF TIMES),N(NUMBER OF GRID POINTS)
C     GRIDX(ACTUAL POSITIONS OF THE GRID POINTS)
C     S(J,J) (DIAGONAL ELEMENTS OF THE COEFFICIENT MATRIX)
C     S(J,J+1)
C     TIME3(TIMES,CORRESPONDING TO NT)
C     RM(SQUARE OF THE HALF-WIDTH OF THE CELL)
C     ALPHA,RM,TBOUND,TINC(THERMAL DIFFUSIVITY,RM(ABOVE),BOUNDARY
C     TEMP.,THE UNIFORM INITIAL TEMP.)
C     CZ(THE INITIAL CONDITION VECTOR)
C     C3(THE BOUNDARY CONDITION VECTOR)
C     (NOTE THAT THERE IS SOME DUPLICATION.IT MAY ALSO BE NECESSARY
C     TO CHANGE THE DATA DEPENDING ON THE ANALYTICAL SOLN.)
C
C  5. THE ROUTE OF EXPERIMENTAL DATA CALLS FOR THE FOLLOWING
C     DATA,READ IN THE ORDER SPECIFIED BELOW.
C     COUNTR,NOT EQUAL TO 1
C     NT,N
C     GRIDX
C     S(J,J)
C     S(J,J+1)
C     TIME3
C     (THE REST OF DATA READ IN STRICTLY FOLLOWS THE WAY IN WHICH
C     THE DECK IS MADE AVAILABLE)
C     OT(6 POINTS FOR DETERMINATION OF ORIENTATION OF THE FRAMES)
C     CZ(THE INITIAL CONDITION VECTOR)
C     C3(THE BOUNDARY CONDITION VECTOR)
C     WIDTH,BOUND,FACTOR,MAGNF(TOTAL WIDTH OF THE MEASUREMENTS-
```


SOURCE STATEMENT

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C      WHETHER HALF FRAME OR FULL FRAME, BOUNDARY TEMP., CONVERSI-
C      ON FACTOR FROM MICRONS TO DEGREES, MAGNIFICATION FACTOR FOR
C      THE FRAMES, (I.E.) THE RATIO OF ACTUAL FRAME SIZE TO THE
C      ACTUAL CELL SIZE, IF ANY)
C      NA (NUMBER OF POINTS ON THE FRAME-X+Y)
C      NZ (THE ACTUAL POINT X AND Y AS SUPPLIED)
C      NT NUMBER OF SUCH SETS
C
C
7      1  FORMAT(1X, I4)
10     2  FORMAT(10X, 14I5)
11     3  FORMAT(10X, 6I5)
12     56 FORMAT(1X, 10F5.1)
13     57 FORMAT(1X, 10F5.1)
14     191 FORMAT(1X, F12.6, 2X, F12.6)
15     192 FORMAT(1X, 6H END )
16     220 FORMAT(1H2)
17     221 FORMAT(1H 10X, 8H ..CONTD)
20     222 FORMAT(1H ,//)
21     223 FORMAT(1H ,/)
22     224 FORMAT(1H ,10X, 30H THERMAL CONDUCTIVITY OF WATER,
      134H-DATA FROM DR.R.N.O*BRIEN*S EXPTS.)
23     225 FORMAT(1H ,10X, 14H CELL WIDTH, CM, 2X, 20H BOUNDARY TEMP, DEG.C,
      12X, 12H CONV.FACTOR, 2X, 14H MAGNIFICATION)
24     226 FORMAT(1H ,14X, F7.4, 10X, F6.1, 13X, F8.5, 6X, F7.4)
25     227 FORMAT(1H ,29X, 22H ORIENTATION OF FRAMES)
26     228 FORMAT(1H ,33X, 2H X, 8X, 2H Y)
27     229 FORMAT(1H ,30X, F8.1, 3X, F8.1)
30     230 FORMAT(1H ,32X, 7H SLOPE=, F6.2)
31     231 FORMAT(1H ,10X, 26H A FIVE-POINT GRID IS USED.,
      130H THE GRID POINTS ARE LOCATED AT )
32     232 FORMAT(1H ,10X, 5F11.6)
33     233 FORMAT(1H ,28X, 25H INITIAL CONDITION VECTOR)
34     234 FORMAT(1H ,10X, 5F12.4)
35     235 FORMAT(1H ,28X, 26H BOUNDARY CONDITION VECTOR)
36     236 FORMAT(1H ,10X, 14H ELAPSED TIME=, F6.3, 8H SECONDS)
37     237 FORMAT(1H ,20X, 9H RAW DATA, 5X,
      132H CONVERTED TO THE RELEVANT UNITS)
40     238 FORMAT(1H ,20X, 2H X, 5X, 2H Y, 8X, 12H DISTANCE, CM, 5X,
      118H TEMPERATURE, DEG.C)
41     239 FORMAT(1H ,15X, F8.1, 2X, F8.1, 4X, F11.6, 8X, F11.6)
42     240 FORMAT(1H ,30X, 19H COEFFICIENT MATRIX)
43     241 FORMAT(1H ,10X, 5F12.4)
44     242 FORMAT(1H ,31X, 12H EIGENVALUES)
45     243 FORMAT(1H ,31X, 13H EIGENVECTORS)
46     244 FORMAT(1H ,20X, 35H CHECK OF SIMILARITY TRANSFORMATION)
47     245 FORMAT(1H ,8X, 8H TIME, SEC, 5X,
      156H INTERPOLATED VALUES OF TEMPERATURE AT GRID POINTS, DEG.C)
50     246 FORMAT(1H ,8X, F6.3, 2X, 5F12.4)
51     247 FORMAT(1H ,30X, 18H MODIFIED VARIABLE)
52     248 FORMAT(1H ,10X, 10H TIME, SECS, 5X, 2H 1, 10X, 2H 2, 9X, 2H 3, 9X,
      12H 4, 9X, 2H 5)
53     249 FORMAT(1H ,10X, 10H TIME, SECS, 5X, 2H 6, 10X, 2H 7, 9X, 2H 8, 9X,
      12H 9, 9X, 2H 10)
54     250 FORMAT(1H ,30X, 20H VALUES FOR PLOTTING)
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AT NUMBER 00-100-2012
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SOURCE STATEMENT

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55 251 FORMAT(1H ,12X,F6.3,2X,5F11.6)
56 252 FORMAT(1H ,15X,25H NORMALISED INNER PRODUCT,
    136H OF I.C.VECTOR WITH EACH EIGENVECTOR)
57 253 FORMAT(1H ,20X,5F11.6)
60 261 FORMAT(1H ,10X,32H THERMAL CONDUCTIVITY-ANALYTICAL,
    129H SOLUTION AS A STARTING POINT)
61 262 FORMAT(1H ,10X,11H SLAB WIDTH,2X,20H ASSUMED DIFFUSIVITY,
    12X,15H BOUNDARY TEMP.,2X,14H INITIAL TEMP.)
62 263 FORMAT(1H ,14X,3HCM.,11X,10H SQ.CM/SEC,12X,6H DEG.C,10X,
    15HDEG.C)
63 264 FORMAT(1H ,12X,F7.4,9X,F10.7,12X,F6.2, 9X,F6.2)
64 265 FORMAT(1H ,10X,39H THE ANALYTICAL SOLUTION(REF.MCADAMS,P.,
    124H34)USED TO CALCULATE THE/10X,23H TEMPERATURE YIELDS THE,
    242H FOLLOWING TEMPERATURES AT THE GRID POINTS)
65 266 FORMAT(1H ,8X,8HTIME,SEC,20X,18H TEMPERATURE,DEG.C)
66 1000 FORMAT(1X,2I4)
67 1001 FORMAT(1X, 5F12.8)
70 1002 FORMAT(1X,5F14.8)
71 READ(5,1) COUNTR
73 READ(5,1000) NT,N
76 READ(5,1001) (GRIDX(J),J=1,N)
103 DX=ABS(GRIDX(3)-GRIDX(2))
104 DO 50 J=1,N
105 DO 50 K=1,N
106 50 S(J,K)=0.0
111 READ(5,56) (S(J,J),J=1,N)
116 N1=N-1
117 READ(5,56) (S(J,J+1),J=1,N1)
124 DO 52 J=1,N1
125 52 S(J+1,J)=S(J,J+1)
127 READ(5,57) (TIME3(J),J=1,NT)
134 MAXN=16
135 M=-N
136 CALL EIG1(N,MAXN,M,S,R,V,A,B,W1,W2)
137 CALL CHECK(V,R,N,W)
140 IF(COUNTR.EQ.1) GO TO 28
143 READ(5,3) (OT(J),J=1,6)
150 DO 10 J=1,3
151 10 AX(J)=FLOAT(OT(2*J))
153 DO 11 J=1,3
154 11 AY(J)=FLOAT(OT(2*J-1))
156 CALL LESQFT(AX,AY,SLOPE)
157 READ(5,56) (CZ(J),J=1,N)
164 READ(5,56)(C3(J),J=1,N)
171 READ(5,1002) WIDTH,BOUND,FACTOR,MAGNF
172 VALUE=0.0
173 DO 131 K=1,N
174 VALUE=VALUE+CZ(K)**2
175 WEIGHT(K)=0.0
176 DO 131 J=1,N
177 131 WEIGHT(K)=WEIGHT(K)+CZ(J)*V(J,K)
202 DO 132 J=1,N
203 132 WEIGHT(J)=WEIGHT(J)/SQRT(VALUE)
205 WRITE(6,220)
206 WRITE(6,224)
```


ISN

SOURCE STATEMENT

```
207 WRITE(6,222)
210 WRITE(6,225)
211 WRITE(6,226) WIDTH,BOUND,FACTOR,MAGNF
212 WRITE(6,223)
213 WRITE(6,227)
214 WRITE(6,228)
215 DO 340 J=1,3
216 340 WRITE(6,229) AX(J),AY(J)
220 WRITE(6,230) SLOPE
221 WRITE(6,222)
222 WRITE(6,231)
223 WRITE(6,232) (GRIDX(J),J=1,N)
230 WRITE(6,222)
231 WRITE(6,233)
232 WRITE(6,234) (CZ(J),J=1,N)
237 WRITE(6,222)
240 WRITE(6,235)
241 WRITE(6,234) (C3(J),J=1,N)
246 WRITE(6,220)
247 LINES=8
250 CALL LINECT(LINES,1,2)
251 WRITE(6,237)
252 NE=1
253 4 READ (5,1) NA
255 READ(5,2) (NZ(J),J=1,NA)
262 CALL DATA(NA,NZ,X,Y,NP)
263 DO 8 J=1,NP
264 XX(J)=X(J)
265 8 YY(J)=Y(J)
267 CALL CONVRT(NP,X,Y,SLOPE,WIDTH,BOUND,FACTOR,MAGNF)
270 CALL LINECT(LINES,2,2)
271 WRITE(6,223)
272 CALL LINECT(LINES,1,2)
273 WRITE(6,236) TIME3(NE)
274 CALL LINECT(LINES,1,2)
275 WRITE(6,238)
276 DO 22 J=1,NP
277 CALL LINECT(LINES,1,1)
300 WRITE(3,191) X(J),Y(J)
301 22 WRITE(6,239) XX(J),YY(J),X(J),Y(J)
303 WRITE(3,192)
304 DO 210 KA=1,N
305 CALL LINEAR(NP,GRIDX,X,Y,KA,RR)
306 C(NE,KA)=RR
307 210 CONTINUE
311 NE=NE+1
312 IF(NE.LE.NT) GO TO 4
315 CALL MODIFY(WORK,V,C3,CZ,Z,C,N,NT)
316 WRITE(6,220)
317 LINES=8
320 CALL LINECT(LINES,1,2)
321 WRITE(6,240)
322 DO 345 J=1,N
323 CALL LINECT(LINES,1,2)
324 345 WRITE(6,241) (S(J,K),K=1,N)
```


ISN

SOURCE STATEMENT

```
332 CALL LINECT(LINES,3,2)
333 WRITE(6,222)
334 CALL LINECT(LINES,1,2)
335 WRITE(6,242)
336 CALL LINECT(LINES,1,2)
337 WRITE(6,232) (R(J),J=1,N)
344 CALL LINECT(LINES,3,2)
345 WRITE(6,222)
346 CALL LINECT(LINES,1,2)
347 WRITE(6,243)
350 DO 346 K=1,N
351 CALL LINECT(LINES,3,2)
352 WRITE(6,232) (V(J,K),J=1,N)
357 346 WRITE(6,223)
361 CALL LINECT(LINES,4,2)
362 WRITE(6,222)
363 WRITE(6,244)
364 DO 347 J=1,N
365 CALL LINECT(LINES,3,2)
366 WRITE(6,232) (W(J,K),K=1,N)
373 347 WRITE(6,223)
375 WRITE(6,220)
376 WRITE(6,245)
377 WRITE(6,223)
400 DO 348 J=1,NT
401 348 WRITE(6,246) TIME3(J),(C(J,K),K=1,N)
407 WRITE(6,223)
410 WRITE(6,247)
411 WRITE(6,223)
412 WRITE(6,248)
413 DO 349 J=1,NT
414 349 WRITE(6,251) TIME3(J),(Z(J,K),K=1,5)
422 CALL PLOT(C,R,Z,C3,CZ,N,NT)
423 WRITE(6,220)
424 WRITE(6,250)
425 WRITE(6,223)
426 WRITE(6,248)
427 DO 351 J=1,NT
430 351 WRITE(6,251) TIME3(J),(C(J,K),K=1,5)
436 WRITE(6,223)
437 WRITE(6,252)
440 WRITE(6,253) (WEIGHT(J),J=1,5)
445 GO TO 29
446 28 READ(5,1C02)RM
447 DO 30 J=1,N
450 30 XNORM(J)=GRIDX(J)/(2.*SQRT(RM))
452 31 READ(5,1C02) ALPHA,RM,TBOUND,TINC
453 SLAB=2.*SQRT(RM)
454 READ(5,56) (CZ(J),J=1,N)
461 READ(5,56) (C3(J),J=1,N)
466 VALUE=0.0
467 DO 32 K=1,N
470 VALUE=VALUE+CZ(K)**2
471 WEIGHT(K)=0.0
472 DO 32 J=1,N
```


ISN SOURCE STATEMENT

```
473 32 WEIGHT(K)=WEIGHT(K)+CZ(J)*V(J,K)
476 DO 33 J=1,N
477 33 WEIGHT(J)=WEIGHT(J)/SQRT(VALUE)
501 CALL ANALYT(ALPHA,RM,TBOUND,TINC,N,NT,XNORM,TIME3,C)
502 WRITE(6,220)
503 WRITE(6,261)
504 WRITE(6,222)
505 WRITE(6,262)
506 WRITE(6,263)
507 WRITE(6,264) SLAB,ALPHA,TBOUND,TINC
510 WRITE(6,222)
511 WRITE(6,231)
512 WRITE(6,232) (GRIDX(J),J=1,N)
517 WRITE(6,223)
520 WRITE(6,265)
521 WRITE(6,223)
522 WRITE(6,266)
523 WRITE(6,223)
524 DO 361 J=1,NT
525 361 WRITE(6,246) TIME3(J),(C(J,K),K=1,N)
533 WRITE(6,222)
534 WRITE(6,233)
535 WRITE(6,234) (CZ(J),J=1,N)
542 WRITE(6,222)
543 WRITE(6,235)
544 WRITE(6,234) (C3(J),J=1,N)
551 CALL MODIFY(WORK,V,C3,CZ,Z,C,N,NT)
552 WRITE(6,220)
553 LINES=8
554 CALL LINECT(LINES,1,2)
555 WRITE(6,240)
556 DO 362 J=1,N
557 CALL LINECT(LINES,1,2)
560 362 WRITE(6,241) (S(J,K),K=1,N)
566 CALL LINECT(LINES,3,2)
567 WRITE(6,222)
570 CALL LINECT(LINES,1,2)
571 WRITE(6,242)
572 CALL LINECT(LINES,2,2)
573 WRITE(6,232) (R(J),J=1,N)
600 CALL LINECT(LINES,3,2)
601 WRITE(6,222)
602 CALL LINECT(LINES,1,2)
603 WRITE(6,243)
604 DO 363 K=1,N
605 CALL LINECT(LINES,4,2)
606 WRITE(6,232) (V(J,K),J=1,N)
613 363 WRITE(6,223)
615 CALL LINECT(LINES,4,2)
616 WRITE(6,222)
617 WRITE(6,244)
620 DO 364 J=1,N
621 CALL LINECT(LINES,4,2)
622 WRITE(6,232) (W(J,K),K=1,N)
627 364 WRITE(6,223)
```


SOURCE STATEMENT

```
631 WRITE(6,220)
632 WRITE(6,247)
633 WRITE(6,223)
634 WRITE(6,248)
635 DO 366 J=1,NT
636 366 WRITE(6,251) TIME3(J),(Z(J,K),K=1,5)
644 WRITE(6,222)
645 WRITE(6,249)
646 DO 367 J=1,NT
647 367 WRITE(6,251) TIME3(J),(Z(J,K),K=6,10)
655 CALL PLOT(C,R,Z,C3,CZ,N,NT)
656 WRITE(6,220)
657 WRITE(6,250)
660 WRITE(6,223)
661 WRITE(6,248)
662 DO 368 J=1,NT
663 368 WRITE(6,251) TIME3(J),(C(J,K),K=1,5)
671 WRITE(6,223)
672 WRITE(6,252)
673 WRITE(6,253) (WEIGHT(J),J=1,5)
700 WRITE(6,223)
701 WRITE(6,249)
702 DO 369 J=1,NT
703 369 WRITE(6,251) TIME3(J),(C(J,K),K=6,10)
711 WRITE(6,223)
712 WRITE(6,252)
713 WRITE(6,253) (WEIGHT(J),J=6,10)
720 29 CONTINUE
721 END
```

20 CONTINUE
 WRITE(5,20) (MIGR(I),I=1,10)
 WRITE(5,21)
 WRITE(5,22)
 WRITE(5,23)
 WRITE(5,24)
 GO 300-1=1,4
 200 WRITE(5,25) (MIGR(I),I=1,10)
 WRITE(5,26)
 WRITE(5,27)
 WRITE(5,28)
 WRITE(5,29)
 WRITE(5,30)
 GO 300-1=1,4
 300 WRITE(5,31) (MIGR(I),I=1,10)
 WRITE(5,32)
 WRITE(5,33)
 WRITE(5,34)
 WRITE(5,35)
 WRITE(5,36)
 GO 300-1=1,4
 400 WRITE(5,37) (MIGR(I),I=1,10)
 WRITE(5,38)
 WRITE(5,39)
 WRITE(5,40)
 WRITE(5,41)
 WRITE(5,42)
 GO 300-1=1,4
 500 WRITE(5,43) (MIGR(I),I=1,10)
 WRITE(5,44)
 WRITE(5,45)
 WRITE(5,46)
 WRITE(5,47)
 WRITE(5,48)
 GO 300-1=1,4
 600 WRITE(5,49) (MIGR(I),I=1,10)
 WRITE(5,50)
 WRITE(5,51)
 WRITE(5,52)
 WRITE(5,53)
 WRITE(5,54)
 GO 300-1=1,4
 700 WRITE(5,55) (MIGR(I),I=1,10)
 WRITE(5,56)
 WRITE(5,57)
 WRITE(5,58)
 WRITE(5,59)
 WRITE(5,60)
 GO 300-1=1,4
 800 WRITE(5,61) (MIGR(I),I=1,10)
 WRITE(5,62)
 WRITE(5,63)
 WRITE(5,64)
 WRITE(5,65)
 WRITE(5,66)
 GO 300-1=1,4
 900 WRITE(5,67) (MIGR(I),I=1,10)
 WRITE(5,68)
 WRITE(5,69)
 WRITE(5,70)
 WRITE(5,71)
 WRITE(5,72)
 GO 300-1=1,4
 1000 WRITE(5,73) (MIGR(I),I=1,10)
 WRITE(5,74)
 WRITE(5,75)
 WRITE(5,76)
 WRITE(5,77)
 WRITE(5,78)
 GO 300-1=1,4
 1100 WRITE(5,79) (MIGR(I),I=1,10)
 WRITE(5,80)
 WRITE(5,81)
 WRITE(5,82)
 WRITE(5,83)
 WRITE(5,84)
 GO 300-1=1,4
 1200 WRITE(5,85) (MIGR(I),I=1,10)
 WRITE(5,86)
 WRITE(5,87)
 WRITE(5,88)
 WRITE(5,89)
 WRITE(5,90)
 GO 300-1=1,4
 1300 WRITE(5,91) (MIGR(I),I=1,10)
 WRITE(5,92)
 WRITE(5,93)
 WRITE(5,94)
 WRITE(5,95)
 WRITE(5,96)
 GO 300-1=1,4
 1400 WRITE(5,97) (MIGR(I),I=1,10)
 WRITE(5,98)
 WRITE(5,99)
 WRITE(5,100)
 WRITE(5,101)
 WRITE(5,102)
 GO 300-1=1,4
 1500 WRITE(5,103) (MIGR(I),I=1,10)
 WRITE(5,104)
 WRITE(5,105)
 WRITE(5,106)
 WRITE(5,107)
 WRITE(5,108)
 GO 300-1=1,4
 1600 WRITE(5,109) (MIGR(I),I=1,10)
 WRITE(5,110)
 WRITE(5,111)
 WRITE(5,112)
 WRITE(5,113)
 WRITE(5,114)
 GO 300-1=1,4
 1700 WRITE(5,115) (MIGR(I),I=1,10)
 WRITE(5,116)
 WRITE(5,117)
 WRITE(5,118)
 WRITE(5,119)
 WRITE(5,120)
 GO 300-1=1,4
 1800 WRITE(5,121) (MIGR(I),I=1,10)
 WRITE(5,122)
 WRITE(5,123)
 WRITE(5,124)
 WRITE(5,125)
 WRITE(5,126)
 GO 300-1=1,4
 1900 WRITE(5,127) (MIGR(I),I=1,10)
 WRITE(5,128)
 WRITE(5,129)
 WRITE(5,130)
 WRITE(5,131)
 WRITE(5,132)
 GO 300-1=1,4
 2000 WRITE(5,133) (MIGR(I),I=1,10)
 WRITE(5,134)
 WRITE(5,135)
 WRITE(5,136)
 WRITE(5,137)
 WRITE(5,138)
 GO 300-1=1,4
 2100 WRITE(5,139) (MIGR(I),I=1,10)
 WRITE(5,140)
 WRITE(5,141)
 WRITE(5,142)
 WRITE(5,143)
 WRITE(5,144)
 GO 300-1=1,4
 2200 WRITE(5,145) (MIGR(I),I=1,10)
 WRITE(5,146)
 WRITE(5,147)
 WRITE(5,148)
 WRITE(5,149)
 WRITE(5,150)
 GO 300-1=1,4
 2300 WRITE(5,151) (MIGR(I),I=1,10)
 WRITE(5,152)
 WRITE(5,153)
 WRITE(5,154)
 WRITE(5,155)
 WRITE(5,156)
 GO 300-1=1,4
 2400 WRITE(5,157) (MIGR(I),I=1,10)
 WRITE(5,158)
 WRITE(5,159)
 WRITE(5,160)
 WRITE(5,161)
 WRITE(5,162)
 GO 300-1=1,4
 2500 WRITE(5,163) (MIGR(I),I=1,10)
 WRITE(5,164)
 WRITE(5,165)
 WRITE(5,166)
 WRITE(5,167)
 WRITE(5,168)
 GO 300-1=1,4
 2600 WRITE(5,169) (MIGR(I),I=1,10)
 WRITE(5,170)
 WRITE(5,171)
 WRITE(5,172)
 WRITE(5,173)
 WRITE(5,174)
 GO 300-1=1,4
 2700 WRITE(5,175) (MIGR(I),I=1,10)
 WRITE(5,176)
 WRITE(5,177)
 WRITE(5,178)
 WRITE(5,179)
 WRITE(5,180)
 GO 300-1=1,4
 2800 WRITE(5,181) (MIGR(I),I=1,10)
 WRITE(5,182)
 WRITE(5,183)
 WRITE(5,184)
 WRITE(5,185)
 WRITE(5,186)
 GO 300-1=1,4
 2900 WRITE(5,187) (MIGR(I),I=1,10)
 WRITE(5,188)
 WRITE(5,189)
 WRITE(5,190)
 WRITE(5,191)
 WRITE(5,192)
 GO 300-1=1,4
 3000 WRITE(5,193) (MIGR(I),I=1,10)
 WRITE(5,194)
 WRITE(5,195)
 WRITE(5,196)
 WRITE(5,197)
 WRITE(5,198)
 GO 300-1=1,4
 3100 WRITE(5,199) (MIGR(I),I=1,10)
 WRITE(5,200)
 WRITE(5,201)
 WRITE(5,202)
 WRITE(5,203)
 WRITE(5,204)
 GO 300-1=1,4
 3200 WRITE(5,205) (MIGR(I),I=1,10)
 WRITE(5,206)
 WRITE(5,207)
 WRITE(5,208)
 WRITE(5,209)
 WRITE(5,210)
 GO 300-1=1,4
 3300 WRITE(5,211) (MIGR(I),I=1,10)
 WRITE(5,212)
 WRITE(5,213)
 WRITE(5,214)
 WRITE(5,215)
 WRITE(5,216)
 GO 300-1=1,4
 3400 WRITE(5,217) (MIGR(I),I=1,10)
 WRITE(5,218)
 WRITE(5,219)
 WRITE(5,220)
 WRITE(5,221)
 WRITE(5,222)
 GO 300-1=1,4
 3500 WRITE(5,223) (MIGR(I),I=1,10)
 WRITE(5,224)
 WRITE(5,225)
 WRITE(5,226)
 WRITE(5,227)
 WRITE(5,228)
 GO 300-1=1,4
 3600 WRITE(5,229) (MIGR(I),I=1,10)
 WRITE(5,230)
 WRITE(5,231)
 WRITE(5,232)
 WRITE(5,233)
 WRITE(5,234)
 GO 300-1=1,4
 3700 WRITE(5,235) (MIGR(I),I=1,10)
 WRITE(5,236)
 WRITE(5,237)
 WRITE(5,238)
 WRITE(5,239)
 WRITE(5,240)
 GO 300-1=1,4
 3800 WRITE(5,241) (MIGR(I),I=1,10)
 WRITE(5,242)
 WRITE(5,243)
 WRITE(5,244)
 WRITE(5,245)
 WRITE(5,246)
 GO 300-1=1,4
 3900 WRITE(5,247) (MIGR(I),I=1,10)
 WRITE(5,248)
 WRITE(5,249)
 WRITE(5,250)
 WRITE(5,251)
 WRITE(5,252)
 GO 300-1=1,4
 4000 WRITE(5,253) (MIGR(I),I=1,10)
 WRITE(5,254)
 WRITE(5,255)
 WRITE(5,256)
 WRITE(5,257)
 WRITE(5,258)
 GO 3

PROGRAM IS BEING ENTERED INTO STORAGE.

THERMAL CONDUCTIVITY OF WATER-DATA FROM DR.R.N.O*BRIEN*S EXPTS.

CELL WIDTH,CM	BOUNDARY TEMP,DEG.C	CONV.FACTOR	MAGNIFICATION
0.0215	25.0	0.00597	1.2464

ORIENTATION OF FRAMES

X	Y
84618.0	48016.0
84645.0	49458.0
84674.0	50770.0
SLOPE= 56.38	

A FIVE-POINT GRID IS USED.THE GRID POINTS ARE LOCATED AT
0.003909 0.007818 0.011727 0.015636 0.019545

INITIAL CONDITION VECTOR

22.2800	19.9900	18.7400	17.5600	17.0600
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BOUNDARY CONDITION VECTOR

25.0000	-0.0000	-0.0000	-0.0000	-0.0000
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SWITCH IS BEING ENTERED INTO SYSTEM.

THE THERMAL CONDUCTIVITY IN WATER-GLASS SYSTEM.

CELL WIDTH, CM	BOUNDARY TEMP, °C	TEMP. FACTOR	BOUNDARY TEMP, °C
0.0512	25.0	0.0000	1.0000

ORIENTATION IN FIBERS

X	Y
8418.0	4018.0
8422.0	4022.0
8426.0	4026.0

SLOPE = 88.3

A FIVE-POINT GRID IS USED. THE GRID POINTS ARE LOCATED AT
0.003909 0.000018 0.01151 0.01844 0.02647

INITIAL TEMPERATURE VECTOR	INITIAL TEMPERATURE VECTOR
22.2800	10.9200 18.7400 17.6000

BOUNDARY CONDITION VECTOR	BOUNDARY CONDITION VECTOR
22.0000	-0.1000 -0.0000 -0.0000

RAW DATA

CONVERTED TO THE RELEVANT UNITS

ELAPSED TIME= 0.042 SECONDS

X	Y	DISTANCE,CM	TEMPERATURE,DEG.C
86543.0	48479.0	0.000000	25.000000
86538.0	48451.0	0.000537	24.791027
86532.0	48420.0	0.001187	24.559600
86527.0	48385.0	0.001709	24.298543
86522.0	48353.0	0.002237	24.059806
86517.0	48323.0	0.002770	23.835954
86514.0	48285.0	0.003048	23.552844
86512.0	48258.0	0.003229	23.351704
86510.0	48226.0	0.003400	23.113363
86508.0	48193.0	0.003568	22.867588
86504.0	48157.0	0.003969	22.599225
86498.0	48130.0	0.004628	22.397558
86495.0	48093.0	0.004907	22.121889
86492.0	48063.0	0.005201	21.898299
86484.0	48044.0	0.006115	21.755884
86476.0	48012.0	0.007001	21.516751
86468.0	47990.0	0.007909	21.352020
86465.0	47947.0	0.008175	21.031712
86457.0	47898.0	0.009026	20.666103
86456.0	47872.0	0.009090	20.472539
86446.0	47849.0	0.010234	20.300102
86438.0	47807.0	0.011099	19.986574
86428.0	47778.0	0.012230	19.769498
86421.0	47740.0	0.012984	19.485857
86401.0	47722.0	0.015331	19.349303
86392.0	47706.0	0.016370	19.229077
86380.0	47683.0	0.017752	19.056379
86364.0	47662.0	0.019615	18.898032
86348.0	47651.0	0.021500	18.814077

ELAPSED TIME= 0.083 SECONDS

X	Y	DISTANCE,CM	TEMPERATURE,DEG.C
86933.0	48491.0	0.000000	25.000000
86925.0	48466.0	0.000860	24.812954
86918.0	48434.0	0.001592	24.573951
86913.0	48383.0	0.002059	24.193861
86908.0	48341.0	0.002543	23.880728
86900.0	48299.0	0.003369	23.567195
86895.0	48258.0	0.003855	23.261507
86892.0	48214.0	0.004108	22.933758
86882.0	48173.0	0.005163	22.627405
86873.0	48128.0	0.006097	22.291426
86865.0	48086.0	0.006923	21.977893
86856.0	48043.0	0.007861	21.656793
86847.0	48011.0	0.008821	21.417532
86840.0	47981.0	0.009557	21.193412
86831.0	47952.0	0.010523	20.976471

ELAPSED TIME = 0.045 SECONDS

TIME, SECONDS	DISTANCE, CM	Y	X
0.0000	0.0000	0.0000	0.0000
0.0005	0.0005	0.0005	0.0005
0.0010	0.0010	0.0010	0.0010
0.0015	0.0015	0.0015	0.0015
0.0020	0.0020	0.0020	0.0020
0.0025	0.0025	0.0025	0.0025
0.0030	0.0030	0.0030	0.0030
0.0035	0.0035	0.0035	0.0035
0.0040	0.0040	0.0040	0.0040
0.0045	0.0045	0.0045	0.0045
0.0050	0.0050	0.0050	0.0050
0.0055	0.0055	0.0055	0.0055
0.0060	0.0060	0.0060	0.0060
0.0065	0.0065	0.0065	0.0065
0.0070	0.0070	0.0070	0.0070
0.0075	0.0075	0.0075	0.0075
0.0080	0.0080	0.0080	0.0080
0.0085	0.0085	0.0085	0.0085
0.0090	0.0090	0.0090	0.0090
0.0095	0.0095	0.0095	0.0095
0.0100	0.0100	0.0100	0.0100
0.0105	0.0105	0.0105	0.0105
0.0110	0.0110	0.0110	0.0110
0.0115	0.0115	0.0115	0.0115
0.0120	0.0120	0.0120	0.0120
0.0125	0.0125	0.0125	0.0125
0.0130	0.0130	0.0130	0.0130
0.0135	0.0135	0.0135	0.0135
0.0140	0.0140	0.0140	0.0140
0.0145	0.0145	0.0145	0.0145
0.0150	0.0150	0.0150	0.0150
0.0155	0.0155	0.0155	0.0155
0.0160	0.0160	0.0160	0.0160
0.0165	0.0165	0.0165	0.0165
0.0170	0.0170	0.0170	0.0170
0.0175	0.0175	0.0175	0.0175
0.0180	0.0180	0.0180	0.0180
0.0185	0.0185	0.0185	0.0185
0.0190	0.0190	0.0190	0.0190
0.0195	0.0195	0.0195	0.0195
0.0200	0.0200	0.0200	0.0200

ELAPSED TIME = 0.043 SECONDS

TIME, SECONDS	DISTANCE, CM	Y	X
0.0000	0.0000	0.0000	0.0000
0.0005	0.0005	0.0005	0.0005
0.0010	0.0010	0.0010	0.0010
0.0015	0.0015	0.0015	0.0015
0.0020	0.0020	0.0020	0.0020
0.0025	0.0025	0.0025	0.0025
0.0030	0.0030	0.0030	0.0030
0.0035	0.0035	0.0035	0.0035
0.0040	0.0040	0.0040	0.0040
0.0045	0.0045	0.0045	0.0045
0.0050	0.0050	0.0050	0.0050
0.0055	0.0055	0.0055	0.0055
0.0060	0.0060	0.0060	0.0060
0.0065	0.0065	0.0065	0.0065
0.0070	0.0070	0.0070	0.0070
0.0075	0.0075	0.0075	0.0075
0.0080	0.0080	0.0080	0.0080
0.0085	0.0085	0.0085	0.0085
0.0090	0.0090	0.0090	0.0090
0.0095	0.0095	0.0095	0.0095
0.0100	0.0100	0.0100	0.0100
0.0105	0.0105	0.0105	0.0105
0.0110	0.0110	0.0110	0.0110
0.0115	0.0115	0.0115	0.0115
0.0120	0.0120	0.0120	0.0120
0.0125	0.0125	0.0125	0.0125
0.0130	0.0130	0.0130	0.0130
0.0135	0.0135	0.0135	0.0135
0.0140	0.0140	0.0140	0.0140
0.0145	0.0145	0.0145	0.0145
0.0150	0.0150	0.0150	0.0150
0.0155	0.0155	0.0155	0.0155
0.0160	0.0160	0.0160	0.0160
0.0165	0.0165	0.0165	0.0165
0.0170	0.0170	0.0170	0.0170
0.0175	0.0175	0.0175	0.0175
0.0180	0.0180	0.0180	0.0180
0.0185	0.0185	0.0185	0.0185
0.0190	0.0190	0.0190	0.0190
0.0195	0.0195	0.0195	0.0195
0.0200	0.0200	0.0200	0.0200

..CONTD

X	Y	DISTANCE,CM	TEMPERATURE,DEG.C
86818.0	47930.0	0.011958	20.811080
86805.0	47906.0	0.013390	20.630806
86785.0	47860.0	0.015574	20.285936
86773.0	47833.0	0.016885	20.083474
86758.0	47807.0	0.018540	19.888060
86744.0	47815.0	0.020150	19.945728
86732.0	47807.0	0.021500	19.884627

ELAPSED TIME= 0.125 SECONDS

X	Y	DISTANCE,CM	TEMPERATURE,DEG.C
86940.0	48476.0	0.000000	25.000000
86937.0	48448.0	0.000351	24.791288
86926.0	48396.0	0.001766	24.402964
86926.0	48359.0	0.001674	24.127691
86918.0	48320.0	0.002699	23.836481
86910.0	48276.0	0.003713	23.508074
86909.0	48240.0	0.003763	23.240103
86894.0	48205.0	0.005781	22.977732
86888.0	48170.0	0.006536	22.716548
86880.0	48137.0	0.007577	22.469977
86870.0	48092.0	0.008868	22.133864
86859.0	48063.0	0.010340	21.916658
86840.0	48033.0	0.012932	21.690957
86821.0	48009.0	0.015539	21.509891
86811.0	47988.0	0.016890	21.352333
86799.0	47951.0	0.018482	21.075475
86790.0	47936.0	0.019708	20.962690
86777.0	47923.0	0.021500	20.864256

ELAPSED TIME= 0.167 SECONDS

X	Y	DISTANCE,CM	TEMPERATURE,DEG.C
86594.0	48338.0	0.000000	25.000000
86585.0	48306.0	0.001409	24.760736
86573.0	48255.0	0.003264	24.379722
86563.0	48196.0	0.004760	23.939453
86551.0	48157.0	0.006650	23.647713
86546.0	48117.0	0.007368	23.349459
86541.0	48086.0	0.008111	23.118166
86537.0	48065.0	0.008718	22.961400
86526.0	48051.0	0.010515	22.855794
86516.0	48035.0	0.012139	22.735434
86505.0	48016.0	0.013921	22.592627
86490.0	48006.0	0.016399	22.516248
86466.0	47991.0	0.020366	22.401482
86459.0	47979.0	0.021500	22.311282

ELAPSED TIME= 0.208 SECONDS

X	Y	DISTANCE,CM	TEMPERATURE,DEG.C
86491.0	48881.0	0.000000	25.000000

..CONTD

X	Y	DISTANCE,CM	TEMPERATURE,DEG.C
86464.0	48830.0	0.003635	24.617006
86453.0	48788.0	0.005064	24.303085
86447.0	48763.0	0.005838	24.116297
86440.0	48736.0	0.006746	23.914496
86432.0	48711.0	0.007799	23.727442
86421.0	48693.0	0.009287	23.592072
86405.0	48677.0	0.011476	23.470927
86385.0	48664.0	0.014230	23.371567
86368.0	48651.0	0.016566	23.272607
86357.0	48631.0	0.018049	23.122359
86344.0	48621.0	0.019836	23.046245
86332.0	48618.0	0.021500	23.022341

ELAPSED TIME= 0.250 SECONDS

X	Y	DISTANCE,CM	TEMPERATURE,DEG.C
86165.0	48421.0	0.000000	25.000000
86155.0	48395.0	0.001247	24.805248
86148.0	48366.0	0.002095	24.588568
86144.0	48334.0	0.002543	24.349965
86142.0	48308.0	0.002745	24.156267
86136.0	48286.0	0.003478	23.991798
86131.0	48264.0	0.004080	23.827460
86122.0	48251.0	0.005227	23.729553
86117.0	48235.0	0.005843	23.609861
86102.0	48223.0	0.007776	23.518604
86088.0	48209.0	0.009574	23.412595
86077.0	48198.0	0.010986	23.329305
86060.0	48191.0	0.013192	23.274983
86051.0	48181.0	0.014345	23.199396
86035.0	48169.0	0.016409	23.108007
86021.0	48166.0	0.018232	23.083839
86008.0	48160.0	0.019917	23.037485
85996.0	48166.0	0.021500	23.080539

ELAPSED TIME= 0.292 SECONDS

X	Y	DISTANCE,CM	TEMPERATURE,DEG.C
85956.0	48528.0	0.000000	25.000000
85943.0	48490.0	0.001739	24.715567
85937.0	48458.0	0.002506	24.476706
85924.0	48429.0	0.004268	24.259231
85914.0	48407.0	0.005624	24.094235
85908.0	48377.0	0.006395	23.870249
85898.0	48362.0	0.007769	23.757334
85882.0	48338.0	0.009967	23.576664
85867.0	48332.0	0.012068	23.530048
85846.0	48330.0	0.015026	23.512398
85831.0	48320.0	0.017118	23.436018
85815.0	48322.0	0.019381	23.448782
85800.0	48323.0	0.021500	23.454247

..CONTD

ELAPSED TIME= 0.333 SECONDS

X	Y	DISTANCE,CM	TEMPERATURE,DEG.C
86236.0	48582.0	0.000000	25.000000
86215.0	48550.0	0.002398	24.759155
86198.0	48530.0	0.004352	24.608112
86185.0	48508.0	0.005832	24.442720
86167.0	48488.0	0.007903	24.291549
86149.0	48474.0	0.009987	24.185013
86132.0	48480.0	0.011995	24.227410
86118.0	48470.0	0.013617	24.151165
86098.0	48469.0	0.015962	24.141083
86084.0	48476.0	0.017620	24.191317
86051.0	48479.0	0.021500	24.209280

ELAPSED TIME= 0.375 SECONDS

X	Y	DISTANCE,CM	TEMPERATURE,DEG.C
86280.0	48478.0	0.000000	25.000000
86249.0	48460.0	0.004195	24.861992
86220.0	48441.0	0.008114	24.716809
86192.0	48432.0	0.011921	24.646156
86178.0	48423.0	0.013814	24.577352
86158.0	48418.0	0.016536	24.537509
86139.0	48423.0	0.019146	24.572204
86122.0	48435.0	0.021500	24.659236

COEFFICIENT MATRIX

-2.0000	1.0000	0.0000	0.0000	0.0000
1.0000	-2.0000	1.0000	0.0000	0.0000
0.0000	1.0000	-2.0000	1.0000	0.0000
0.0000	0.0000	1.0000	-2.0000	1.0000
0.0000	0.0000	0.0000	1.0000	-1.0000

EIGENVALUES

-3.682507	-2.830830	-1.715370	-0.690279	-0.081014
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EIGENVECTORS

-0.326019	0.548529	-0.596885	0.455734	-0.169891
0.548529	-0.455734	-0.169891	0.596885	-0.326019
0.596885	0.169891	-0.548529	-0.326019	0.455734
0.455734	0.596885	0.326019	-0.169891	-0.548529
-0.169891	-0.326019	-0.455734	-0.548529	-0.596885

CHECK OF SIMILARITY TRANSFORMATION

-2.000000	1.000000	0.000000	-0.000000	0.000000
1.000000	-2.000000	1.000000	0.000000	-0.000000
0.000000	1.000000	-2.000001	1.000000	0.000000
-0.000000	0.000000	1.000000	-2.000000	1.000000
0.000000	-0.000000	0.000000	1.000000	-1.000000

0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000

COMPUTATION

3.685203 -5.830880 -1.113300 -0.446552 -0.000000

EIGENVALUES

0.35019	0.248250	-0.20882	0.42534	-0.114991
0.248250	-0.42534	-0.114991	0.248250	-0.114991
0.248250	0.114991	-0.248250	-0.35019	0.42534
0.42534	0.248250	0.35019	-0.114991	-0.248250
-0.114991	-0.35019	-0.42534	-0.248250	0.114991

CHECK OF SIMILARITY TRANSFORMATION

-5.00000	1.00000	0.00000	0.00000	0.00000
1.00000	-5.00000	1.00000	0.00000	-0.00000
0.00000	1.00000	-5.00000	1.00000	0.00000
-0.00000	0.00000	1.00000	-5.00000	1.00000
0.00000	-0.00000	0.00000	1.00000	-5.00000

TIME, SEC INTERPOLATED VALUES OF TEMPERATURE AT GRID POINTS, DEG.C

0.042	22.6395	21.3684	19.8659	19.3139	18.9040
0.083	23.1917	21.6714	20.8377	20.2763	19.9241
0.125	23.2212	22.4072	21.7959	21.4985	20.9776
0.167	24.1899	23.2093	22.7659	22.5397	22.4252
0.208	24.5569	23.7257	23.4619	23.3120	23.0586
0.250	23.8742	23.5161	23.3111	23.1422	23.0477
0.292	24.3035	23.7533	23.5376	23.4901	23.4492
0.333	24.6423	24.2977	24.2218	24.1425	24.2002
0.375	24.8714	24.7278	24.6498	24.5507	24.5870

MODIFIED VARIABLE

TIME, SECS	1	2	3	4	5
0.042	-1.926963	4.670187	8.564929	15.898088	-41.744067
0.083	-2.255606	4.911749	8.564093	15.924359	-43.516272
0.125	-2.055466	4.815864	8.262785	15.903803	-45.497129
0.167	-2.281714	4.966413	8.765406	16.169389	-47.800493
0.208	-2.289178	5.068606	8.727332	16.393095	-49.150023
0.250	-2.167076	4.717485	8.417368	15.942542	-48.797355
0.292	-2.221841	4.883172	8.659183	16.074275	-49.481336
0.333	-2.272308	4.849214	8.708249	16.253944	-50.834314
0.375	-2.246232	4.823726	8.726428	16.473065	-51.663217

TIME, SEC

INTEGRATED VALUE OF TEMPERATURE AT UNIT, °C

0.045	0.045	0.045	0.045	0.045	0.045
0.080	0.080	0.080	0.080	0.080	0.080
0.125	0.125	0.125	0.125	0.125	0.125
0.160	0.160	0.160	0.160	0.160	0.160
0.200	0.200	0.200	0.200	0.200	0.200
0.250	0.250	0.250	0.250	0.250	0.250
0.300	0.300	0.300	0.300	0.300	0.300
0.333	0.333	0.333	0.333	0.333	0.333
0.375	0.375	0.375	0.375	0.375	0.375

MODIFIED AVERAGE

TIME, SEC	I	S	Z	Q	M
0.045	-1.45000	0.01000	0.00000	0.00000	0.00000
0.080	-2.25000	0.01100	0.00000	0.00000	0.00000
0.125	-3.05000	0.01200	0.00000	0.00000	0.00000
0.160	-3.85000	0.01300	0.00000	0.00000	0.00000
0.200	-4.65000	0.01400	0.00000	0.00000	0.00000
0.250	-5.45000	0.01500	0.00000	0.00000	0.00000
0.300	-6.25000	0.01600	0.00000	0.00000	0.00000
0.333	-7.05000	0.01700	0.00000	0.00000	0.00000
0.375	-7.85000	0.01800	0.00000	0.00000	0.00000

VALUES FOR PLOTTING

TIME, SECS	1	2	3	4	5
0.042	-1.718793	-69.278896	0.573617	0.932155	0.775846
0.083	0.254004	26.871036	0.577191	0.891834	0.647133
0.125	-0.947403	-11.294418	1.865695	0.923383	0.503266
0.167	0.410724	48.629065	-0.283697	0.515768	0.335976
0.208	0.455535	89.305514	-0.120878	0.172427	0.237962
0.250	-0.277427	-50.452531	1.204640	0.863927	0.263575
0.292	0.051319	15.496379	0.170549	0.661747	0.213899
0.333	0.354261	1.980154	-0.039272	0.385994	0.115634
0.375	0.197733	-8.164968	-0.117013	0.049691	0.055432

NORMALISED INNER PRODUCT OF I.C. VECTOR WITH EACH EIGENVECTOR
 -0.055382 0.112789 0.196994 0.368936 -0.899606

VALUES FOR TEST

TIME, SEC	1	5	4	2
0.375	0.14173	-0.16469	-0.11015	0.06801
0.333	0.13581	1.08014	-0.04475	0.06144
0.292	0.051319	12.490373	0.17046	0.06144
0.250	-0.57747	-27.42731	1.10446	0.06144
0.208	0.42232	84.30214	-0.12974	0.06144
0.167	-0.41034	44.25002	-0.12874	0.06144
0.125	-0.047403	11.29419	1.06092	0.06144
0.083	0.154004	22.81820	0.17711	0.06144
0.042	-1.171073	-24.75886	0.17711	0.06144

NORMALIZED TIME PRODUCT OF 1.3 VECTOR WITH SAME ELEMENTS TO
-0.05332 0.11574 0.10544 0.06144

SEMI-LOGARITHMIC LEAST SQUARE FIT OF RESULTS

COLUMN NUMBER= 5

TIME, SECS	EXPTL. VALUES OF Z	LOGARITHMS	Z-VALUES FROM FIT
0.042	0.77584599	-0.253801	0.87123627
0.083	0.64713300	-0.435203	0.65194155
0.125	0.50326600	-0.686636	0.48440650
0.167	0.33597600	-1.090716	0.35992439
0.208	0.23796200	-1.435644	0.26932953
0.250	0.26357500	-1.333417	0.20011760
0.292	0.21389900	-1.542251	0.14869166
0.333	0.11563400	-2.157325	0.11126519
0.375	0.05543200	-2.892598	0.08267241

GRID SPACING CM.	EIGENVALUE	SLOPE	INTERCEPT	DIFFUSIVITY SQ. CM/SEC
0.0039090	-0.081014	-7.072154	1.172559	0.13339E-02

SEMI-LOGARITHMIC LEAST SQUARE FIT OF MEASUREMENTS

COLUMN NUMBER= 2

TIME, SECS	EXPTL. VALUES OF Δ	LOGARITHMIC	5-MINUTE AVERAGE
0.045	0.77384500	-0.79801	0.77384500
0.083	0.66713800	-0.68251	0.66713800
0.152	0.50750000	-0.60000	0.50750000
0.167	0.43278000	-1.00010	0.43278000
0.208	0.33755000	-1.45544	0.33755000
0.250	0.26357500	-1.33347	0.26357500
0.292	0.21382000	-1.54351	0.21382000
0.333	0.17434000	-2.10730	0.17434000
0.375	0.08435000	-2.80290	0.08435000

GRID SPACING CM.	ELEVATION	SLOPE	INTERVAL	INTERVAL
0.003000	-0.081014	-1.075120	1.175000	0.175000

PROGRAM IS BEING ENTERED INTO STORAGE.

THERMAL CONDUCTIVITY OF WATER-DATA FROM DR.R.N.O'BRIEN'S EXPTS.

CELL WIDTH, CM	BOUNDARY TEMP, DEG.C	CONV.FACTOR	MAGNIFICATION
0.0215	25.0	0.00597	1.2464

A TEN-POINT GRID IS USED. THE GRID POINTS ARE LOCATED AT

0.002048	0.004096	0.006144	0.008191	0.010239
0.012287	0.014334	0.016382	0.018430	0.020477

INITIAL CONDITION VECTOR

23.57 22.20 20.71 19.92 19.16 18.57 17.85 17.39 17.13 17.00

BOUNDARY CONDITION VECTOR

25.00 -0.00 -0.00 -0.00 -0.00 -0.00 -0.00 -0.00 -0.00 -0.00

PROGRAM 12 BEING ENTERED INTO STORAGE.

THERMAL CONDUCTIVITY OF WATER-GLASS MIXTURES

CELL WITH, CH. 0.0012
WINDWAY TEMP, DEG. C. 22.0
CHAM. WIND. 0.0000
WINDWAY WIND. 0.0000

A TEN-POINT GRID IS USED. THE GRID POINTS ARE LOCATED AT
0.00000 0.00000 0.00000 0.00000 0.00000
0.01250 0.01250 0.01250 0.01250 0.01250
0.02500 0.02500 0.02500 0.02500 0.02500
0.03750 0.03750 0.03750 0.03750 0.03750
0.05000 0.05000 0.05000 0.05000 0.05000

INITIAL CONDITIONS VECTOR
23.25 22.20 20.21 19.02 18.10 17.25 16.45 15.70 15.00

WINDWAY CONDITION VECTOR
22.00 -0.00 -0.00 -0.00 -0.00 -0.00 -0.00 -0.00 -0.00

COEFFICIENT MATRIX

-2.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1.00	-2.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	1.00	-2.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	1.00	-2.00	1.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	1.00	-2.00	1.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	1.00	-2.00	1.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	1.00	-2.00	1.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	1.00	-2.00	1.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	-2.00	1.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	-1.00

EIGENVALUES

-3.911146	-3.652478	-3.246980	-2.730682	-2.149460
-1.554958	-1.000000	-0.533896	-0.198062	-0.022338

EIGENVECTORS

0.128642	-0.245853	0.341219	-0.406267	0.435215
-0.425493	0.377964	-0.296852	0.189362	-0.065047
-0.245853	0.406267	-0.425493	0.296852	-0.065047
-0.189363	0.377965	-0.435216	0.341219	-0.128642
-0.341219	0.425493	-0.189362	-0.189362	0.425493
-0.341219	0.000000	0.341219	-0.425493	0.189362
0.406267	-0.296852	-0.189362	0.435215	-0.128642
-0.341219	0.377965	0.065047	-0.425493	0.245853
-0.435215	0.065048	0.425493	-0.128642	-0.406267
0.189363	0.377964	-0.245853	-0.341219	0.296852
0.425493	0.189362	-0.341219	-0.341219	0.189362
0.425493	0.000000	-0.425493	-0.189362	0.341219
0.377964	0.377964	-0.000000	-0.377965	-0.377965
-0.000000	0.377964	0.377964	0.000000	-0.377964
0.296852	0.435215	0.341219	0.065047	-0.245853
-0.425493	-0.377964	-0.128642	0.189362	0.406267

..CONTD

0.189362	0.341219	0.425493	0.425493	0.341219
0.189362	-0.000000	-0.189362	-0.341219	-0.425494

-0.065047	-0.128642	-0.189362	-0.245853	-0.296852
-0.341219	-0.377964	-0.406267	-0.425494	-0.435216

CHECK OF SIMILARITY TRANSFORMATION

-1.999999	1.000000	-0.000000	-0.000000	0.000000
0.000000	-0.000000	0.000000	-0.000000	0.000000

1.000000	-2.000000	1.000000	0.000000	-0.000000
0.000000	-0.000000	0.000000	0.000000	-0.000000

-0.000000	1.000000	-2.000000	1.000000	0.000000
-0.000001	0.000001	-0.000000	-0.000000	0.000000

-0.000000	0.000000	-1.000000	-2.000000	0.999999
0.000000	0.000000	-0.000000	-0.000000	0.000000

0.000000	-0.000000	0.000000	0.999999	-2.000000
1.000000	-0.000000	0.000000	-0.000000	0.000000

0.000000	0.000000	-0.000001	0.000000	1.000000
-2.000000	1.000000	-0.000000	0.000000	-0.000000

-0.000000	-0.000000	0.000001	0.000000	-0.000000
1.000000	-2.000000	1.000001	-0.000000	-0.000000

0.000000	0.000000	-0.000000	-0.000000	0.000000
-0.000000	1.000001	-2.000000	1.000000	0.000000

-0.000000	0.000000	-0.000000	-0.000000	-0.000000
0.000000	-0.000000	1.000000	-2.000000	1.000000

0.000000	-0.000000	0.000000	0.000000	0.000000
-0.000000	-0.000000	0.000000	1.000000	-1.000000

CONT...

[illegible]

CHECK OF SIMILARITY TECHNOLOGY

1000000.0	1000000.0	1000000.0	1000000.0	1000000.0
1000000.0	1000000.0	1000000.0	1000000.0	1000000.0

1.000000	-5.000000	1.000000	0.000000	0.000000
0.000000	-0.000000	0.000000	0.000000	0.000000

1000000.0	000000.0	000000.0	000000.0	000000.0
1000000.0	000000.0	000000.0	000000.0	000000.0

000000.0	000000.5 -	000000.1	000000.0	000000.0 -
000000.0	000000.0	000000.0 -	000000.0	000000.0

TIME, SEC INTERPOLATED VALUES OF TEMPERATURE AT GRID POINTS, DEG.C

0.042	24.15	22.56	21.75	21.02	20.30	19.75	19.41	19.23	19.00	18.86
0.083	24.20	22.95	22.27	21.57	21.04	20.77	20.48	20.16	19.90	19.93
0.125	24.02	23.20	22.85	22.31	21.93	21.75	21.59	21.41	21.08	20.92
0.167	24.63	24.13	23.73	23.10	22.87	22.72	22.58	22.52	22.46	22.39
0.208	24.78	24.52	24.05	23.69	23.54	23.44	23.37	23.28	23.11	23.04
0.250	24.60	23.83	23.60	23.49	23.37	23.30	23.20	23.11	23.08	23.05
0.292	24.62	24.28	23.94	23.72	23.57	23.53	23.52	23.46	23.44	23.45
0.333	24.79	24.63	24.42	24.28	24.19	24.21	24.15	24.15	24.20	24.20
0.375	24.93	24.87	24.79	24.72	24.68	24.63	24.57	24.54	24.56	24.62

MODIFIED VARIABLE

TIME, SECS	1	2	3	4	5
0.042	0.868539	-1.819153	-2.792552	3.933765	-5.275039
0.083	0.854731	-1.807657	-2.745600	3.883332	-5.042993
0.125	0.849930	-1.780821	-2.671057	3.739974	-4.970780
0.167	0.878501	-1.762396	-2.655303	3.654125	-5.031540
0.208	0.810068	-1.682246	-2.574069	3.700233	-5.068720
0.250	0.852427	-1.745279	-2.746697	3.842040	-5.167614
0.292	0.814046	-1.663178	-2.623051	3.729463	-5.037607
0.333	0.793894	-1.675741	-2.641648	3.683819	-5.045783
0.375	0.822255	-1.688349	-2.622506	3.715234	-5.045282

TIME, SECS	6	7	8	9	10
0.042	6.853531	9.508765	13.832709	22.987144	-57.962658
0.083	6.957360	9.543174	13.696645	23.094270	-60.455501
0.125	6.644901	9.472232	13.405939	23.131318	-63.226580
0.167	6.880148	9.637638	14.040455	23.474931	-66.404860
0.208	6.909384	9.705880	13.920037	23.747806	-68.278498
0.250	6.913084	9.379457	13.586570	23.152215	-67.807282
0.292	6.862918	9.499874	13.842448	23.305782	-68.718682
0.333	6.880778	9.468895	13.921186	23.529620	-70.585868
0.375	6.887711	9.408727	13.952114	23.850436	-71.744887

1145, 250

[illegible]

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TIME, SECS	1	2	3	4	5
0.375	0.855525	-1.68834	-0.55204	0.17313	0.17313
0.333	0.79344	-1.61241	-0.50144	0.16144	0.16144
0.292	0.61044	-1.44174	-0.45301	0.14530	0.14530
0.250	0.45247	-1.14247	-0.31467	0.11467	0.11467
0.208	0.31068	-1.04548	-0.24072	0.10472	0.10472
0.167	0.18750	-1.16330	-0.16330	0.16330	0.16330
0.125	0.04930	-1.78051	-0.51105	0.18051	0.18051
0.083	0.04731	-1.80187	-0.74607	0.18019	0.18019
0.042	0.04823	-1.81013	-0.73505	0.17305	0.17305

TIME, SEC2	δ	γ	β	α
0.345	0.00000	0.00000	0.00000	0.00000
0.350	0.00000	0.00000	0.00000	0.00000
0.355	0.00000	0.00000	0.00000	0.00000
0.360	0.00000	0.00000	0.00000	0.00000
0.365	0.00000	0.00000	0.00000	0.00000
0.370	0.00000	0.00000	0.00000	0.00000
0.375	0.00000	0.00000	0.00000	0.00000
0.380	0.00000	0.00000	0.00000	0.00000
0.385	0.00000	0.00000	0.00000	0.00000
0.390	0.00000	0.00000	0.00000	0.00000
0.395	0.00000	0.00000	0.00000	0.00000
0.400	0.00000	0.00000	0.00000	0.00000

VALUES FOR PLOTTING

TIME, SECS	1	2	3	4	5
0.042	-0.403637	-1.660300	-9.669588	-11.489057	2.184258
0.083	-0.283162	-1.520340	-6.923861	-8.785269	-0.193936
0.125	-0.241271	-1.193613	-2.564526	-1.099748	-0.934028
0.167	-0.490558	-0.969295	-1.643203	3.502732	-0.311309
0.208	0.106527	0.006518	3.107385	1.030842	0.069738
0.250	-0.263057	-0.760899	-6.987964	-6.571566	1.083282
0.292	0.071813	0.238665	0.242911	-0.536245	-0.249139
0.333	0.247642	0.085719	-0.844650	1.910814	-0.165340
0.375	0.000192	-0.067790	0.274805	0.226629	-0.170471

NORMALISED INNER PRODUCT OF I.C. VECTOR WITH EACH EIGENVECTOR
0.011496 -0.026002 -0.042400 0.060118 -0.083814

TIME, SECS	6	7	8	9	10
0.042	0.058612	-2.253387	0.208036	1.024314	0.778071
0.083	0.540899	-3.553136	0.627112	0.904354	0.647328
0.125	-0.910480	-0.873423	1.522491	0.862869	0.501992
0.167	0.182248	-7.121331	-0.431824	0.478090	0.335299
0.208	0.318046	-9.699039	-0.060935	0.172524	0.237031
0.250	0.335235	2.630947	0.966144	0.839468	0.261745
0.292	0.102213	-1.917570	0.178037	0.667503	0.213944
0.333	0.185172	-0.747369	-0.064476	0.416849	0.116015
0.375	0.217375	1.525358	-0.159734	0.057599	0.055227

NORMALISED INNER PRODUCT OF I.C. VECTOR WITH EACH EIGENVECTOR
0.114625 0.153067 0.220529 0.373769 -0.872840

VALUES FOR PRINTING

TIME, SEC	1	2	3	4	5
0.375	0.00195	-0.00110	0.00110	0.00110	0.00110
0.333	0.00183	-0.00105	0.00105	0.00105	0.00105
0.292	0.00171	-0.00100	0.00100	0.00100	0.00100
0.250	0.00159	-0.00095	0.00095	0.00095	0.00095
0.208	0.00147	-0.00090	0.00090	0.00090	0.00090
0.167	0.00135	-0.00085	0.00085	0.00085	0.00085
0.125	0.00123	-0.00080	0.00080	0.00080	0.00080
0.083	0.00111	-0.00075	0.00075	0.00075	0.00075
0.042	0.00099	-0.00070	0.00070	0.00070	0.00070

NORMALIZED INNER PRODUCT OF 1.C.VECT. WITH 1.C.VECT. 0.011496 -0.005400 -0.005400 -0.005400 -0.005400

TIME, SEC	1	2	3	4	5
0.375	0.0011352	1.0000000	-0.0011352	-0.0011352	-0.0011352
0.333	0.0010215	-0.0011352	1.0000000	-0.0011352	-0.0011352
0.292	0.0009078	-0.0011352	-0.0011352	1.0000000	-0.0011352
0.250	0.0007941	-0.0011352	-0.0011352	-0.0011352	1.0000000
0.208	0.0006804	-0.0011352	-0.0011352	-0.0011352	-0.0011352
0.167	0.0005667	-0.0011352	-0.0011352	-0.0011352	-0.0011352
0.125	0.0004530	-0.0011352	-0.0011352	-0.0011352	-0.0011352
0.083	0.0003393	-0.0011352	-0.0011352	-0.0011352	-0.0011352
0.042	0.0002256	-0.0011352	-0.0011352	-0.0011352	-0.0011352

NORMALIZED INNER PRODUCT OF 1.C.VECT. WITH 1.C.VECT. 0.011496 -0.005400 -0.005400 -0.005400 -0.005400

SEMI-LOGARITHMIC LEAST SQUARE FIT OF RESULTS

COLUMN NUMBER= 10

TIME, SECS	EXPTL. VALUES OF Z	LOGARITHMS	Z-VALUES FROM FIT
0.042	0.77807100	-0.250937	0.87100046
0.083	0.64732800	-0.434902	0.65158584
0.125	0.50199200	-0.689171	0.48400581
0.167	0.33529900	-1.092733	0.35952535
0.208	0.23703100	-1.439564	0.26895694
0.250	0.26174500	-1.340385	0.19978446
0.292	0.21394400	-1.542041	0.14840231
0.333	0.11601500	-2.154036	0.11101813
0.375	0.05522700	-2.896303	0.08246560

GRID SPACING CM.	EIGENVALUE	SLOPE	INTERCEPT	DIFFUSIVITY SQ. CM/SEC
0.0020480	-0.022338	-7.078863	1.172572	0.13292E-02

SEMI-LOGARITHMIC LEAST SQUARES FIT TO DATA

COLUMN NUMBER= 10

TIME, SECS	EXPL. VALUE OF Y	LINE-1 TIME	Y-VAL. FROM FIT
0.045	0.7150100	-0.25037	0.7150100
0.083	0.8473100	-0.43803	0.8473100
0.152	0.9012500	-0.69171	0.9012500
0.167	0.3325900	-1.06783	0.3325900
0.208	0.2370100	-1.63806	0.2370100
0.220	0.2614500	-1.46033	0.2614500
0.225	0.2139400	-1.45041	0.2139400
0.333	0.1161200	-2.16036	0.1161200
0.352	0.0225500	-3.86036	0.0225500

GRID SPACING CM.	EIGENVALUE	SLOPE	INTERCEPT	COEFFICIENT OF CORRELATION
0.0050480	-0.052738	-1.04963	1.15443	0.99999

Description of the Problem

$$\text{P.D.E:} \quad g \frac{\partial u}{\partial t} = \frac{\partial}{\partial x} \left(k \frac{\partial u}{\partial x} \right)$$

$$g = \text{const.}$$

$$k = k_0 + Ax$$

$$\text{B.C: at } t = 0, \quad u = f(x)$$

$$\text{at } x = 0, \quad u = \text{const.}$$

$$\text{at } x = 3.3, \quad \frac{\partial u}{\partial x} = 0.$$

Sequence:

1. For a given first set of I.C and times, the program makes use of the solution to forward problem to calculate temperature profiles at these various times. This is called as generation of data for inverse problem.

2. The program calls for subroutine NORMAL, which normalises the temperature vectors and determines their lengths. The final convergent vector is stored as VECTA.

3. The subroutine LOGFIT is called to make a logarithmic fit of these lengths and determines an eigenvalue.

4. A second set of I.C are chosen; some more data are generated. Subroutine ELIMA is called which eliminates the component of the known VECTA. Subroutines NORMAL and LOGFIT perform the routines of (2) and (3). The final vector is stored as VECTB.

5. A next set of I.C are chosen; data generated; Subroutine ELIMB called for to eliminate VECTA and VELTB; so on.

6. From the generated eigenvalues and eigenvectors, the coefficient matrix is reproduced, compared with the original and an error matrix is formed.

The computer program and results follow.

ISN SOURCE STATEMENT

```

0 $IBFTC DARS1  NODECK
C
C
C      SOLUTION TO THE INVERSE PROBLEM-NON-HOMOGENEOUS MEDIUM
C
C      INPUT DATA
C      N=GRID SIZE
C      NT=NUMBER OF TIMES
C      KZERO,A1,A2,DX,GZERO,TS-PARAMETERS
C      PZ=INITIAL CONDITION VECTOR
C      C1=BOUNDARY CONDITION VECTOR
C      TIME1=ACTUAL VALUES OF TIME
C
C      SUBROUTINES USED ARE-EIG1,MAIN,NORMAL,LOGFIT,ELIMA,ELIMB,
C      ELIMC,ELIMD,AND CHECK.
C
1      REAL S(16,16),R(16),V(16,16),A(16),B(16),W1(16),W2(16)
2      REALTK(16),PZ(16),P(15,16),C1(16),WORK(16),TIME1(15)
3      REAL A2,DX,GZERO,TS,VECTA(16),VECTB(16),VECTC(16),VECTD(16)
4      REAL LENGTH(15),CALC(16),W(16,16),KZERO,A1
5      1 FORMAT(1X,2I4)
6      2 FORMAT(1X, 6F10.5)
7      3 FORMAT(1X,15F5.2)
10     49 FORMAT(1H0,20X,12H EIGENVALUE=,F14.8)
11     220 FORMAT(1H2)
12     221 FORMAT(1H ,10X,8H ..CONTD)
13     222 FORMAT(1H ,//)
14     223 FORMAT(1H ,/)
15     224 FORMAT(1H ,25X,32H SOLUTION TO THE INVERSE PROBLEM,
      1/30X,23H NON-HOMOGENEOUS MEDIUM)
16     232 FORMAT(1H ,10X,5F11.6)
17     233 FORMAT(1H ,28X,25H INITIAL CONDITION VECTOR)
20     234 FORMAT(1H ,10X,5F12.4)
21     235 FORMAT(1H ,28X,26H BOUNDARY CONDITION VECTOR)
22     240 FORMAT(1H ,30X,19H COEFFICIENT MATRIX)
23     241 FORMAT(1H ,10X,5F12.6)
24     242 FORMAT(1H ,31X,12H EIGENVALUES)
25     243 FORMAT(1H ,31X,13H EIGENVECTORS)
26     244 FORMAT(1H ,20X,35H CHECK OF SIMILARITY TRANSFORMATION)
27     245 FORMAT(1H ,20X,16H EXPERIMENT NO.=,I3)
30     246 FORMAT(1H ,25X,28H TEMPERATURES AT GRID POINTS)
31     247 FORMAT(1H ,30X,19H NUMERICAL SOLUTION)
32     248 FORMAT(1H ,15X,5H TIME,5X,2H 1, 9X,2H 2, 8X,2H 3, 8X,
      12H 4, 8X,2H 5)
33     249 FORMAT(1H ,15X,5H TIME, 9X,2H 6, 8X,2H 7, 8X,2H 8, 8X,
      12H 9, 8X,2H10)
34     251 FORMAT(1H ,15X,F6.1,2X,5F10.6)
35     255 FORMAT(1H ,30X,18H ASSOCIATED ERRORS)
36     257 FORMAT(1H ,10X,5H TIME,2X,6HLENGTH,20X,
      130H NORMALISED TEMPERATURE VECTOR)
37     258 FORMAT(1H ,10X,F6.2,1X,F7.4,2X,5F10.6)
40     259 FORMAT(1H ,15X,36H REPRODUCTION OF THE ORIGINAL MATRIX)
41     260 FORMAT(1H ,25X,13H ERROR MATRIX)
42     READ(5,1) N,NT
45     READ(5,2) KZERO,A1,A2,DX,GZERO,TS

```


ISN	SOURCE STATEMENT
46	READ(5,2) (PZ(J),J=1,N)
53	READ(5,2) (C1(J),J=1,N)
60	READ(5,3) (TIME1(J),J=1,NT)
65	DO 20 J=1,N
66	AJ=J
67	DELX=DX/2.+(AJ-1.)*DX
70	VALUE=A2*DELX
71	20 TK(J)=KZERO+VALUE
73	DO 21 J=1,N
74	DO 21 K=1,N
75	21 S(J,K)=0.0
100	N1=N-1
101	DO 22 J=1,N1
102	S(J,J)=-(TK(J)+TK(J+1))
103	S(J,J+1)=TK(J+1)
104	22 S(J+1,J)=S(J,J+1)
106	S(N,N)=-TK(N)
107	DO 23 J=1,N
110	DO 23 K=1,N
111	23 S(J,K)=S(J,K)/(GZERO*DX**2)
114	C1(1)=TK(1)*TS/(GZERO*DX**2)
115	MAXN=16
116	M=N
117	CALL EIG1(N,MAXN,M,S,R,V,A,B,W1,W2)
120	WRITE(6,220)
121	WRITE(6,224)
122	WRITE(6,223)
123	WRITE(6,235)
124	WRITE(6,234) (C1(J),J=1,N)
131	WRITE(6,223)
132	WRITE(6,240)
133	DO 31 J=1,N
134	31 WRITE(6,241) (S(J,K),K=1,N)
142	WRITE(6,223)
143	WRITE(6,242)
144	WRITE(6,232) (R(J),J=1,N)
151	WRITE(6,223)
152	WRITE(6,243)
153	DO 32 K=1,N
154	WRITE(6,232) (V(J,K),J=1,N)
161	32 WRITE(6,223)
163	CALL MAIN(N,WORK,V,C1,PZ,R,TIME1,NT,P)
164	WRITE(6,220)
165	NEX=1
166	WRITE(6,245) NEX
167	WRITE(6,223)
170	WRITE(6,246)
171	WRITE(6,248)
172	DO 34 J=1,NT
173	34 WRITE(6,251) TIME1(J),(P(J,K),K=1,5)
201	WRITE(6,223)
202	WRITE(6,257)
203	DO 72 J=1,NT
204	DO 51 K=1,N
205	51 WORK(K)=P(J,K)

ISN	SOURCE STATEMENT
207	CALL NORMAL(WORK,N,LENGTH(J))
210	WRITE(6,258) TIME1(J),LENGTH(J),(WORK(K),K=1,N)
215	72 CONTINUE
217	CALL LOGFIT(TIME1,LENGTH,DX,NT,1.0,SLOPE,CEPT,DIFF)
220	WRITE(6,49) SLOPE
221	CALC(1)=SLOPE
222	DO 52 J=1,N
223	52 VECTA(J)=WORK(J)
225	READ(5,2) (PZ(J),J=1,N)
232	READ(5,3) (TIME1(J),J=1,NT)
237	CALL MAIN(N,WORK,V,C1,PZ,R,TIME1,NT,P)
240	WRITE(6,220)
241	NEX=2
242	WRITE(6,245) NEX
243	WRITE(6,223)
244	WRITE(6,246)
245	WRITE(6,248)
246	DO 35 J=1,NT
247	35 WRITE(6,251) TIME1(J),(P(J,K),K=1,5)
255	WRITE(6,223)
256	WRITE(6,257)
257	DO 54 J=1,NT
260	DO 53 K=1,N
261	53 WORK(K)=P(J,K)
263	CALL ELIMA(WORK,N,VECTA)
264	CALL NORMAL(WORK,N,LENGTH(J))
265	WRITE(6,258) TIME1(J),LENGTH(J),(WORK(K),K=1,N)
272	54 CONTINUE
274	CALL LOGFIT(TIME1,LENGTH,DX,NT,1.0,SLOPE,CEPT,DIFF)
275	WRITE(6,49) SLOPE
276	CALC(2)=SLOPE
277	DO 55 J=1,N
300	55 VECTB(J)=WORK(J)
302	READ(5,2) (PZ(J),J=1,N)
307	READ(5,3) (TIME1(J),J=1,NT)
314	CALL MAIN(N,WORK,V,C1,PZ,R,TIME1,NT,P)
315	WRITE(6,220)
316	NEX=3
317	WRITE(6,245) NEX
320	WRITE(6,223)
321	WRITE(6,246)
322	WRITE(6,248)
323	DO 36 J=1,NT
324	36 WRITE(6,251) TIME1(J),(P(J,K),K=1,5)
332	WRITE(6,223)
333	WRITE(6,257)
334	DO 57 J=1,NT
335	DO 56 K=1,N
336	56 WORK(K)=P(J,K)
340	CALL ELIMB(WORK,N,VECTA,VECTB)
341	CALL NORMAL(WORK,N,LENGTH(J))
342	WRITE(6,258) TIME1(J),LENGTH(J),(WORK(K),K=1,N)
347	57 CONTINUE
351	CALL LOGFIT(TIME1,LENGTH,DX,NT,1.0,SLOPE,CEPT,DIFF)
352	WRITE(6,49) SLOPE

ISN	SOURCE STATEMENT
353	CALC(3)=SLOPE
354	DO 58 J=1,N
355	58 VECTC(J)=WORK(J)
357	READ(5,2) (PZ(J),J=1,N)
364	READ(5,3) (TIME1(J),J=1,NT)
371	CALL MAIN(N,WORK,V,C1,PZ,R,TIME1,NT,P)
372	WRITE(6,220)
373	NEX=4
374	WRITE(6,245) NEX
375	WRITE(6,223)
376	WRITE(6,246)
377	WRITE(6,248)
400	DO 37 J=1,NT
401	37 WRITE(6,251) TIME1(J),(P(J,K),K=1,5)
407	WRITE(6,223)
410	WRITE(6,257)
411	DO 60 J=1,NT
412	DO 59 K=1,N
413	59 WORK(K)=P(J,K)
415	CALL ELIMC(WORK,N,VECTA,VECTB,VECTC)
416	CALL NORMAL(WORK,N,LENGTH(J))
417	WRITE(6,258) TIME1(J),LENGTH(J),(WORK(K),K=1,N)
424	60 CONTINUE
426	CALL LOGFIT(TIME1,LENGTH,DX,NT,1.0,SLOPE,CEPT,DIFF)
427	WRITE(6,49) SLOPE
430	CALC(4)=SLOPE
431	DO 61 J=1,N
432	61 VECTD(J)=WORK(J)
434	READ(5,2) (PZ(J),J=1,N)
441	READ(5,3) (TIME1(J),J=1,NT)
446	CALL MAIN(N,WORK,V,C1,PZ,R,TIME1,NT,P)
447	WRITE(6,220)
450	NEX=5
451	WRITE(6,245) NEX
452	WRITE(6,223)
453	WRITE(6,246)
454	WRITE(6,248)
455	DO 38 J=1,NT
456	38 WRITE(6,251) TIME1(J),(P(J,K),K=1,5)
464	WRITE(6,223)
465	WRITE(6,257)
466	DO 63 J=1,NT
467	DO 62 K=1,N
470	62 WORK(K)=P(J,K)
472	CALL ELIMD(WORK,N,VECTA,VECTB,VECTC,VECTD)
473	CALL NORMAL(WORK,N,LENGTH(J))
474	WRITE(6,258) TIME1(J),LENGTH(J),(WORK(K),K=1,N)
501	63 CONTINUE
503	CALL LOGFIT(TIME1,LENGTH,DX,NT,1.0,SLOPE,CEPT,DIFF)
504	WRITE(6,49) SLOPE
505	CALC(5)=SLOPE
506	DO 64 K=1,N
507	V(K,1)=VECTA(K)
510	V(K,2)=VECTB(K)
511	V(K,3)=VECTC(K)

ISN	SOURCE STATEMENT
512	V(K,4)=VECTD(K)
513	64 V(K,5)=WORK(K)
515	CALL CHECK(V,CALC,N,W)
516	WRITE(6,220)
517	WRITE(6,259)
520	DO 66 J=1,N
521	66 WRITE(6,241) (W(J,K),K=1,N)
527	DO 65 J=1,N
530	DO 65 K=1,N
531	65 W(J,K)=S(J,K)-W(J,K)
534	WRITE(6,223)
535	WRITE(6,260)
536	DO 67 J=1,N
537	67 WRITE(6,241) (W(J,K),K=1,N)
545	CONTINUE
546	END

ISN

SOURCE STATEMENT

```

0 $IBFTC LOGFIT
1 SUBROUTINE LOGFIT(AX,AY,DX,NC,EIGVAL,SLOPE,CEPT,DIFF)
C
C THIS SUBROUTINE FITS AX AND LOG(AY) TO A STRAIGHT LINE
C
2 REAL AX(25),AY(25)
3 DO 114 J=1,NC
4 114 AY(J)=ALOG(AY(J))
6 SUM1=0.0
7 DO 100 J=1,NC
10 100 SUM1=SUM1+AX(J)
12 SUM2=0.0
13 DO 101 J=1,NC
14 101 SUM2=SUM2+AX(J)**2
16 SUM3=0.0
17 DO 102 J=1,NC
20 102 SUM3=SUM3+AY(J)
22 SUM4=0.0
23 DO 103 J=1,NC
24 103 SUM4=SUM4+AX(J)*AY(J)
26 AN=NC
27 SLOPE=(SUM3*SUM1-SUM4*AN)/(SUM1**2-AN*SUM2)
30 CEPT=(SUM3*SUM2-SUM4*SUM1)/(AN*SUM2-SUM1**2)
31 CEPT=EXP(CEPT)
32 DIFF=SLOPE*(DX**2)/EIGVAL
33 RETURN
34 END

```


ISN

SOURCE STATEMENT

```
0 $IBFTC NORMAL
1 SUBROUTINE NORMAL(WORK,N,VALUE)
C
C THIS SUBROUTINE NORMALISES THE VECTOR WORK AND CALCULATES
C ITS LENGTH.
C
2 REAL WORK(10)
3 VALUE=0.0
4 DO 5 L=1,N
5 VALUE=VALUE+WORK(L)**2
7 VALUE=SQRT(VALUE)
10 DO 6 L=1,N
11 6 WORK(L)=WORK(L)/VALUE
13 RETURN
14 END
```


ISN

SOURCE STATEMENT

```
0 $IBFTC ELIMA
1 SUBROUTINE ELIMA(WORK,N,VECTA)
C
C ELIMA ELIMINATES VECTA FROM WORK.
C
2 REAL WORK(16),STORE(16),VECTA(16)
3 CONST=0.0
4 DO 1 J=1,N
5 1 CONST=CONST+WORK(J)*VECTA(J)
7 DO 2 J=1,N
10 STORE(J)=CONST*VECTA(J)
11 2 WORK(J)=WORK(J)-STORE(J)
13 RETURN
14 END
```


ISN

SOURCE STATEMENT

0 \$IBFTC ELIMB

1 SUBROUTINE ELIMB(WORK,N,VECTA,VECTB)

C

C

C

ELIMB ELIMINATES VECTA AND VECTB FROM WORK.

2 REAL WORK(16),STORE(16),VECTA(16),VECTB(16)

3 CONST=0.0

4 CONST2=0.0

5 DO 1 J=1,N

6 CONST2=CONST2+WORK(J)*VECTB(J)

7 1 CONST=CONST+WORK(J)*VECTA(J)

11 DO 2 J=1,N

12 STORE(J)=CONST*VECTA(J)+CONST2*VECTB(J)

13 2 WORK(J)=WORK(J)-STORE(J)

15 RETURN

16 END

ISN

SOURCE STATEMENT

```
0 $IBFTC ELIMC
1 SUBROUTINE ELIMC(WORK,N,VECTA,VECTB,VECTC)
C
C ELIMC ELIMINATES VECTA,VECTB AND VECTC FROM WORK.
C
2 REAL WORK(16),STORE(16),VECTA(16),VECTB(16),VECTC(16)
3 CONST=0.0
4 CONST2=0.0
5 CONST3=0.0
6 DO 1 J=1,N
7 CONST2=CONST2+WORK(J)*VECTB(J)
10 CONST3=CONST3+WORK(J)*VECTC(J)
11 1 CONST=CONST+WORK(J)*VECTA(J)
13 DO 2 J=1,N
14 STORE(J)=CONST*VECTA(J)+CONST2*VECTB(J) +CONST3*VECTC(J)
15 2 WORK(J)=WORK(J)-STORE(J)
17 RETURN
20 END
```


ISN SOURCE STATEMENT

```

0 $IBFTC ELIMD
1 SUBROUTINE ELIMD(WORK,N,VECTA,VECTB,VECTC,VECTD)
C
C ELIMD ELIMINATES VECTA,VECTB,VECTC AND VECTD FROM WORK.
C
2 REAL WORK(16),STORE(16),VECTA(16),VECTB(16),VECTC(16),VECTD(16)
3 CONST=0.0
4 CONST2=0.0
5 CONST3=0.0
6 CONST4=0.0
7 DO 1 J=1,N
10 CONST2=CONST2+WORK(J)*VECTB(J)
11 CONST3=CONST3+WORK(J)*VECTC(J)
12 CONST4=CONST4+WORK(J)*VECTD(J)
13 1 CONST=CONST+WORK(J)*VECTA(J)
15 DO 2 J=1,N
16 STORE(J)=CONST*VECTA(J)+CONST2*VECTB(J) +CONST3*VECTC(J) +
1CONST4*VECTD(J)
17 2 WORK(J)=WORK(J)-STORE(J)
21 RETURN
22 END

```


PROGRAM IS BEING ENTERED INTO STORAGE.

SOLUTION TO THE INVERSE PROBLEM
NON-HOMOGENEOUS MEDIUM

BOUNDARY CONDITION VECTOR

0.0000 -0.0000 -0.0000 -0.0000 -0.0000

COEFFICIENT MATRIX

-7.222222	4.027778	0.000000	0.000000	0.000000
4.027778	-8.888889	4.861111	0.000000	0.000000
0.000000	4.861111	-10.555555	5.694444	0.000000
0.000000	0.000000	5.694444	-12.222222	6.527778
0.000000	0.000000	0.000000	6.527778	-6.527778

EIGENVALUES

-0.319788 -3.255099 -8.163303 -13.683781 -19.994694

EIGENVECTORS

-0.214814 -0.368128 -0.470943 -0.532264 -0.559682

0.574464 0.565813 0.179765 -0.252547 -0.503737

-0.666445 0.155713 0.575440 0.108818 -0.434317

-0.414723 0.665320 -0.312629 -0.396214 0.361430

0.087755 -0.278280 0.563054 -0.695765 0.337256

PROGRAM IS BEING TESTED INTO STAGE.

RESULTS TO THE FOLLOWING
THE FOLLOWING

PROGRAM EXECUTION ACTION
-0.0000 -0.0000 -0.0000 -0.0000

OFFICIAL DATA
0.00000 0.00000 0.00000 0.00000
0.00000 0.00000 0.00000 0.00000
0.00000 0.00000 0.00000 0.00000
0.00000 0.00000 0.00000 0.00000
0.00000 0.00000 0.00000 0.00000

FINANCIALS
-0.31288 -2.52203 -8.10303 -13.08301 -14.00000

FINANCIALS
-0.31288 -0.30815 -0.40000 -0.52500 -0.60000

0.23444 0.22213 0.13075 -0.52500 -0.60000

-0.00000 0.12213 0.23000 0.10000 -0.40000

-0.41453 0.00000 -0.31500 -0.30000 0.20000

0.00000 -0.20000 0.20000 -0.40000 0.00000

EXPERIMENT NO.= 1

TIME	TEMPERATURES AT GRID POINTS				
	1	2	3	4	5
0.0	1.000000	1.000000	1.000000	1.000000	1.000000
0.0	0.865124	0.987550	0.999054	0.999936	0.999996
0.1	0.767541	0.960562	0.994379	0.999282	0.999906
0.3	0.556436	0.833233	0.943843	0.982203	0.993385
0.6	0.427721	0.696950	0.847394	0.922348	0.951862
1.0	0.347336	0.586030	0.737842	0.824046	0.861355
1.5	0.287776	0.491375	0.626286	0.705889	0.741234
1.9	0.251730	0.430905	0.550621	0.621787	0.653540
2.6	0.200776	0.344022	0.440039	0.497282	0.522870
3.5	0.150515	0.257937	0.329973	0.372935	0.392145
4.5	0.109317	0.187338	0.239659	0.270865	0.284818
5.5	0.079397	0.136064	0.174065	0.196730	0.206864
7.0	0.049145	0.084221	0.107743	0.121772	0.128045
8.0	0.035694	0.061170	0.078254	0.088443	0.092999
10.0	0.018829	0.032268	0.041280	0.046655	0.049059

TIME	LENGTH	NORMALISED TEMPERATURE VECTOR				
0.00	2.2361	0.447214	0.447213	0.447213	0.447214	0.447214
0.05	2.1729	0.398135	0.454476	0.459771	0.460177	0.460204
0.10	2.1211	0.361864	0.452865	0.468809	0.471120	0.471414
0.30	1.9612	0.283723	0.424860	0.481260	0.500820	0.506521
0.60	1.7730	0.241241	0.393091	0.477944	0.520219	0.536865
1.00	1.5587	0.222840	0.375980	0.473378	0.528683	0.552620
1.50	1.3282	0.216661	0.369946	0.471518	0.531449	0.558060
1.90	1.1687	0.215385	0.368690	0.471121	0.532013	0.559181
2.60	0.9343	0.214887	0.368200	0.470966	0.532232	0.559618
3.50	0.7007	0.214819	0.368133	0.470944	0.532262	0.559678
4.50	0.5089	0.214814	0.368128	0.470943	0.532264	0.559682
5.50	0.3696	0.214814	0.368128	0.470943	0.532264	0.559682
7.00	0.2288	0.214814	0.368128	0.470943	0.532264	0.559682
8.00	0.1662	0.214814	0.368128	0.470943	0.532264	0.559682
10.00	0.0877	0.214814	0.368128	0.470943	0.532264	0.559682

EIGENVALUE= -0.32182167

EXPERIMENT NO. 1

TIME	1	2	3	4	5
0.0	1.00000	1.00000	1.00000	1.00000	1.00000
0.0	0.99999	0.99999	0.99999	0.99999	0.99999
0.1	0.99998	0.99998	0.99998	0.99998	0.99998
0.3	0.99996	0.99996	0.99996	0.99996	0.99996
0.6	0.99992	0.99992	0.99992	0.99992	0.99992
1.0	0.99984	0.99984	0.99984	0.99984	0.99984
1.5	0.99972	0.99972	0.99972	0.99972	0.99972
2.0	0.99956	0.99956	0.99956	0.99956	0.99956
2.5	0.99936	0.99936	0.99936	0.99936	0.99936
3.0	0.99912	0.99912	0.99912	0.99912	0.99912
3.5	0.99884	0.99884	0.99884	0.99884	0.99884
4.0	0.99852	0.99852	0.99852	0.99852	0.99852
4.5	0.99816	0.99816	0.99816	0.99816	0.99816
5.0	0.99776	0.99776	0.99776	0.99776	0.99776
5.5	0.99732	0.99732	0.99732	0.99732	0.99732
6.0	0.99684	0.99684	0.99684	0.99684	0.99684
6.5	0.99632	0.99632	0.99632	0.99632	0.99632
7.0	0.99576	0.99576	0.99576	0.99576	0.99576
7.5	0.99516	0.99516	0.99516	0.99516	0.99516
8.0	0.99452	0.99452	0.99452	0.99452	0.99452
8.5	0.99384	0.99384	0.99384	0.99384	0.99384
9.0	0.99312	0.99312	0.99312	0.99312	0.99312
9.5	0.99236	0.99236	0.99236	0.99236	0.99236
10.0	0.99156	0.99156	0.99156	0.99156	0.99156

TIME	LENGTH	1	2	3	4	5
0.00	2.5361	0.99999	0.99999	0.99999	0.99999	0.99999
0.05	2.1759	0.99998	0.99998	0.99998	0.99998	0.99998
0.10	2.1211	0.99996	0.99996	0.99996	0.99996	0.99996
0.30	1.9615	0.99984	0.99984	0.99984	0.99984	0.99984
0.60	1.7730	0.99972	0.99972	0.99972	0.99972	0.99972
1.00	1.5285	0.99956	0.99956	0.99956	0.99956	0.99956
1.50	1.3585	0.99936	0.99936	0.99936	0.99936	0.99936
2.00	1.1685	0.99912	0.99912	0.99912	0.99912	0.99912
2.50	0.9343	0.99884	0.99884	0.99884	0.99884	0.99884
3.00	0.7007	0.99852	0.99852	0.99852	0.99852	0.99852
3.50	0.5059	0.99816	0.99816	0.99816	0.99816	0.99816
4.00	0.3696	0.99776	0.99776	0.99776	0.99776	0.99776
4.50	0.2588	0.99732	0.99732	0.99732	0.99732	0.99732
5.00	0.1665	0.99684	0.99684	0.99684	0.99684	0.99684
5.50	0.0875	0.99632	0.99632	0.99632	0.99632	0.99632
6.00	0.0414	0.99576	0.99576	0.99576	0.99576	0.99576
6.50	0.0215	0.99516	0.99516	0.99516	0.99516	0.99516
7.00	0.0115	0.99452	0.99452	0.99452	0.99452	0.99452
7.50	0.0061	0.99384	0.99384	0.99384	0.99384	0.99384
8.00	0.0032	0.99312	0.99312	0.99312	0.99312	0.99312
8.50	0.0017	0.99236	0.99236	0.99236	0.99236	0.99236
9.00	0.0009	0.99156	0.99156	0.99156	0.99156	0.99156
9.50	0.0005	0.99076	0.99076	0.99076	0.99076	0.99076
10.00	0.0003	0.99000	0.99000	0.99000	0.99000	0.99000

EIGENVALUE = -0.2518154

EXPERIMENT NO.= 2

TIME	TEMPERATURES AT GRID POINTS				
	1	2	3	4	5
0.0	1.000000	1.000000	1.000000	-0.000000	0.000000
0.0	0.863552	0.964255	0.801827	0.190452	0.031642
0.1	0.758801	0.895046	0.701864	0.276814	0.089957
0.2	0.673416	0.820655	0.642587	0.322396	0.149261
0.2	0.601675	0.750840	0.601702	0.350549	0.201874
0.3	0.540526	0.688428	0.570238	0.370070	0.246288
0.3	0.488054	0.633700	0.544344	0.384448	0.282966
0.4	0.403923	0.544696	0.502839	0.403207	0.337041
0.5	0.341179	0.477456	0.470247	0.412712	0.371586
0.6	0.294109	0.426234	0.443611	0.415783	0.392330
0.7	0.258518	0.386688	0.421205	0.414316	0.403333
0.8	0.231318	0.355654	0.401895	0.409676	0.407469
0.9	0.210251	0.330841	0.384893	0.402852	0.406785
1.0	0.193668	0.310596	0.369643	0.394558	0.402743
1.1	0.180372	0.293720	0.355749	0.385307	0.396392

TIME	LENGTH	NORMALISED TEMPERATURE VECTOR				
0.00	1.3745	0.562820	0.445269	0.366439	-0.408100	-0.429122
0.05	1.1314	0.566354	0.514814	0.276993	-0.319609	-0.485114
0.10	0.9552	0.564858	0.543662	0.231544	-0.278982	-0.503908
0.15	0.8106	0.564576	0.556225	0.209127	-0.261885	-0.509460
0.20	0.6886	0.565382	0.561897	0.197700	-0.255078	-0.510358
0.25	0.5851	0.566634	0.564483	0.191514	-0.252569	-0.509721
0.30	0.4972	0.567950	0.565640	0.187913	-0.251785	-0.508702
0.40	0.3590	0.570195	0.566292	0.184144	-0.251747	-0.506858
0.50	0.2593	0.571763	0.566276	0.182305	-0.252030	-0.505632
0.60	0.1872	0.572782	0.566150	0.181284	-0.252238	-0.504883
0.70	0.1352	0.573425	0.566037	0.180685	-0.252364	-0.504432
0.80	0.0976	0.573825	0.565956	0.180325	-0.252437	-0.504160
0.90	0.0705	0.574072	0.565903	0.180107	-0.252481	-0.503995
1.00	0.0509	0.574224	0.565869	0.179974	-0.252507	-0.503894
1.10	0.0368	0.574317	0.565848	0.179893	-0.252522	-0.503833

EIGENVALUE= -3.26797200

EXPERIMENT NO. 2

TEMPERATURES AT 5-MIN. INTERVALS

TIME	1	2	3	4	5
0.0	1.00000	1.00000	1.00000	1.00000	1.00000
0.0	0.99995	0.99995	0.99995	0.99995	0.99995
0.1	0.99981	0.99981	0.99981	0.99981	0.99981
0.2	0.99946	0.99946	0.99946	0.99946	0.99946
0.3	0.99895	0.99895	0.99895	0.99895	0.99895
0.4	0.99824	0.99824	0.99824	0.99824	0.99824
0.5	0.99735	0.99735	0.99735	0.99735	0.99735
0.6	0.99619	0.99619	0.99619	0.99619	0.99619
0.7	0.99478	0.99478	0.99478	0.99478	0.99478
0.8	0.99313	0.99313	0.99313	0.99313	0.99313
0.9	0.99125	0.99125	0.99125	0.99125	0.99125
1.0	0.98916	0.98916	0.98916	0.98916	0.98916
1.1	0.98687	0.98687	0.98687	0.98687	0.98687

TIME	LENGTH	1	2	3	4	5
0.00	1.3742	0.98580	0.98580	0.98580	0.98580	0.98580
0.02	1.1314	0.98338	0.98338	0.98338	0.98338	0.98338
0.10	0.9225	0.97609	0.97609	0.97609	0.97609	0.97609
0.15	0.8106	0.96878	0.96878	0.96878	0.96878	0.96878
0.20	0.6888	0.96147	0.96147	0.96147	0.96147	0.96147
0.25	0.5671	0.95416	0.95416	0.95416	0.95416	0.95416
0.30	0.4453	0.94685	0.94685	0.94685	0.94685	0.94685
0.40	0.3236	0.93954	0.93954	0.93954	0.93954	0.93954
0.50	0.2527	0.93223	0.93223	0.93223	0.93223	0.93223
0.60	0.1818	0.92492	0.92492	0.92492	0.92492	0.92492
0.70	0.1325	0.91761	0.91761	0.91761	0.91761	0.91761
0.80	0.0936	0.91030	0.91030	0.91030	0.91030	0.91030
0.90	0.0702	0.90300	0.90300	0.90300	0.90300	0.90300
1.00	0.0504	0.89569	0.89569	0.89569	0.89569	0.89569
1.10	0.0368	0.88838	0.88838	0.88838	0.88838	0.88838

EIGENVALUE = -8.5034500

EXPERIMENT NO.= 3

TIME	TEMPERATURES AT GRID POINTS				
	1	2	3	4	5
0.0	1.000000	0.000000	0.000000	-0.000000	1.000000
0.0	0.710883	0.139229	0.045083	0.212718	0.757208
0.1	0.525441	0.204232	0.119085	0.298747	0.619943
0.2	0.403820	0.235497	0.183359	0.334740	0.536485
0.2	0.322722	0.250361	0.230467	0.349921	0.482837
0.3	0.267880	0.256717	0.262217	0.355779	0.446827
0.3	0.230281	0.258420	0.282319	0.357066	0.421736
0.4	0.204109	0.257484	0.294169	0.355895	0.403597
0.4	0.185564	0.255030	0.300369	0.353296	0.389957
0.4	0.172140	0.251708	0.302779	0.349823	0.379254
0.5	0.162177	0.247907	0.302681	0.345801	0.370478
0.5	0.154566	0.243864	0.300942	0.341430	0.362970
0.6	0.148569	0.239723	0.298147	0.336842	0.356302
0.6	0.143687	0.235572	0.294681	0.332123	0.350192
0.7	0.139587	0.231461	0.290803	0.327332	0.344458

TIME	LENGTH	NORMALISED TEMPERATURE VECTOR				
0.00	1.1812	0.671487	-0.275146	-0.319530	-0.333935	0.509694
0.05	0.7489	0.684651	-0.234056	-0.433523	-0.237478	0.481801
0.10	0.4902	0.683484	-0.205534	-0.496443	-0.178801	0.460630
0.15	0.3243	0.679483	-0.187433	-0.530678	-0.145691	0.447579
0.20	0.2153	0.675877	-0.176209	-0.549554	-0.127620	0.440277
0.25	0.1431	0.673197	-0.169223	-0.560150	-0.117966	0.436446
0.30	0.0951	0.671309	-0.164807	-0.566204	-0.112921	0.434563
0.35	0.0632	0.669990	-0.161955	-0.569725	-0.110364	0.433724
0.40	0.0420	0.669064	-0.160070	-0.571812	-0.109129	0.433421
0.45	0.0279	0.668405	-0.158795	-0.573073	-0.108582	0.433377
0.50	0.0186	0.667931	-0.157914	-0.573853	-0.108384	0.433447
0.55	0.0124	0.667586	-0.157293	-0.574347	-0.108354	0.433558
0.60	0.0082	0.667332	-0.156848	-0.574668	-0.108398	0.433674
0.65	0.0055	0.667145	-0.156525	-0.574881	-0.108467	0.433780
0.70	0.0036	0.667004	-0.156287	-0.575028	-0.108541	0.433869

EIGENVALUE= -8.21136008

EXPERIMENT NO. 3

TEMPERATURES AT UNIT INTERVALS

TIME	1	2	3	4	5
0.0	1.00000	0.00000	0.00000	0.00000	0.00000
0.0	0.91083	0.13553	0.04203	0.01111	0.00000
0.1	0.82241	0.25233	0.11088	0.04444	0.01111
0.2	0.74030	0.36244	0.18279	0.07778	0.02222
0.2	0.65352	0.46801	0.24678	0.11111	0.03333
0.3	0.56760	0.56814	0.30287	0.14444	0.04444
0.3	0.48351	0.66281	0.35100	0.17778	0.05556
0.4	0.40109	0.75284	0.39129	0.21111	0.06667
0.4	0.32139	0.83730	0.42379	0.24444	0.07778
0.5	0.24517	0.91517	0.44941	0.27778	0.08889
0.5	0.17456	0.98695	0.46825	0.31111	0.09999
0.6	0.14829	1.04223	0.48047	0.34444	0.11111
0.6	0.12387	1.08255	0.48611	0.37778	0.12222
0.7	0.13293	1.10841	0.48603	0.40000	0.12222

TIME	LENGTH	NORMALIZED TEMPERATURE	VELOCITY
0.00	1.1815	0.67147 - 0.25148 - 0.31951 - 0.44753	0.00000
0.02	0.9489	0.68651 - 0.34024 - 0.45274 - 0.54726	0.00000
0.10	0.6905	0.68306 - 0.50224 - 0.45024 - 0.71441	0.00000
0.12	0.8243	0.67983 - 0.41433 - 0.52047 - 0.74021	0.00000
0.20	0.5123	0.67007 - 0.17209 - 0.49959 - 0.75729	0.00000
0.22	0.4141	0.67197 - 0.10252 - 0.49019 - 0.74740	0.00000
0.30	0.0921	0.67130 - 0.16007 - 0.48050 - 0.71721	0.00000
0.32	0.0635	0.66990 - 0.14122 - 0.46979 - 0.71804	0.00000
0.40	0.0450	0.66909 - 0.10070 - 0.45141 - 0.70124	0.00000
0.42	0.0279	0.66602 - 0.12870 - 0.45303 - 0.70082	0.00000
0.50	0.0160	0.667031 - 0.127014 - 0.457623 - 0.704044	0.00000
0.52	0.0154	0.66788 - 0.12723 - 0.45740 - 0.70400	0.00000
0.60	0.0085	0.66735 - 0.12688 - 0.45700 - 0.70390	0.00000
0.62	0.0022	0.66748 - 0.12628 - 0.45681 - 0.70385	0.00000
0.70	0.0038	0.66704 - 0.12527 - 0.45604 - 0.70301	0.00000

EIGENVALUE = -8.511800E

EXPERIMENT NO.= 4

TIME	TEMPERATURES AT GRID POINTS				
	1	2	3	4	5
0.0	0.000000	1.000000	-0.000000	-0.000000	1.000000
0.0	0.068757	0.844000	0.086568	0.113331	0.885028
0.0	0.136979	0.675733	0.183185	0.232191	0.759458
0.1	0.176942	0.562702	0.250362	0.309082	0.673239
0.1	0.203686	0.466922	0.308729	0.370781	0.597646
0.2	0.213369	0.409459	0.343764	0.404453	0.549946
0.2	0.214349	0.374184	0.364181	0.422038	0.518849
0.2	0.211170	0.351749	0.375418	0.430322	0.497727
0.3	0.206242	0.336753	0.380867	0.433188	0.482659
0.3	0.200795	0.326081	0.382649	0.432878	0.471299
0.4	0.195423	0.317940	0.382098	0.430695	0.462235
0.4	0.190379	0.311300	0.380058	0.427401	0.454608
0.4	0.185743	0.305570	0.377068	0.423449	0.447891
0.5	0.182534	0.301659	0.374415	0.420217	0.443250
0.5	0.179540	0.297979	0.371524	0.416840	0.438845

TIME	LENGTH	NORMALISED TEMPERATURE VECTOR				
0.00	1.0287	-0.408343	0.648123	-0.280429	-0.435542	0.380600
0.02	0.7822	-0.409089	0.650254	-0.284364	-0.430967	0.378444
0.05	0.5186	-0.410020	0.652945	-0.289395	-0.425066	0.375653
0.08	0.3439	-0.410767	0.655137	-0.293545	-0.420155	0.373320
0.12	0.1989	-0.411544	0.657446	-0.297974	-0.414869	0.370799
0.16	0.1151	-0.412131	0.659216	-0.301407	-0.410740	0.368822
0.20	0.0666	-0.412577	0.660575	-0.304071	-0.407517	0.367276
0.24	0.0385	-0.412917	0.661624	-0.306141	-0.405002	0.366067
0.28	0.0223	-0.413177	0.662435	-0.307753	-0.403037	0.365122
0.32	0.0129	-0.413376	0.663064	-0.309011	-0.401501	0.364384
0.36	0.0075	-0.413526	0.663553	-0.310000	-0.400297	0.363807
0.40	0.0043	-0.413635	0.663936	-0.310784	-0.399349	0.363358
0.44	0.0025	-0.413708	0.664237	-0.311415	-0.398599	0.363006
0.47	0.0017	-0.413738	0.664417	-0.311818	-0.398139	0.362803
0.50	0.0011	-0.413743	0.664577	-0.312173	-0.397743	0.362632

EIGENVALUE= -13.67893520

EXPERIMENT 11

TEMPERATURE AT 25°C

TIME	1	2	3	4	5
0.0	0.00000	0.00000	0.00000	0.00000	0.00000
0.0	0.00000	0.00000	0.00000	0.00000	0.00000
0.0	0.00000	0.00000	0.00000	0.00000	0.00000
0.1	0.00000	0.00000	0.00000	0.00000	0.00000
0.1	0.00000	0.00000	0.00000	0.00000	0.00000
0.2	0.00000	0.00000	0.00000	0.00000	0.00000
0.2	0.00000	0.00000	0.00000	0.00000	0.00000
0.3	0.00000	0.00000	0.00000	0.00000	0.00000
0.3	0.00000	0.00000	0.00000	0.00000	0.00000
0.4	0.00000	0.00000	0.00000	0.00000	0.00000
0.4	0.00000	0.00000	0.00000	0.00000	0.00000
0.5	0.00000	0.00000	0.00000	0.00000	0.00000
0.5	0.00000	0.00000	0.00000	0.00000	0.00000
0.6	0.00000	0.00000	0.00000	0.00000	0.00000
0.6	0.00000	0.00000	0.00000	0.00000	0.00000
0.7	0.00000	0.00000	0.00000	0.00000	0.00000
0.7	0.00000	0.00000	0.00000	0.00000	0.00000
0.8	0.00000	0.00000	0.00000	0.00000	0.00000
0.8	0.00000	0.00000	0.00000	0.00000	0.00000
0.9	0.00000	0.00000	0.00000	0.00000	0.00000
0.9	0.00000	0.00000	0.00000	0.00000	0.00000

TIME	LENGTH	TEMPERATURE	TEMPERATURE
0.00	1.0587	0.00000	0.00000
0.05	0.7853	0.00000	0.00000
0.05	0.7853	0.00000	0.00000
0.08	0.3439	0.00000	0.00000
0.15	0.1989	0.00000	0.00000
0.16	0.1121	0.00000	0.00000
0.20	0.0666	0.00000	0.00000
0.24	0.0385	0.00000	0.00000
0.28	0.0253	0.00000	0.00000
0.32	0.0159	0.00000	0.00000
0.36	0.0092	0.00000	0.00000
0.40	0.0043	0.00000	0.00000
0.44	0.0025	0.00000	0.00000
0.47	0.0017	0.00000	0.00000
0.50	0.0011	0.00000	0.00000

EIGENVALUE = -13.0702250

EXPERIMENT NO.= 5

TIME	TEMPERATURES AT GRID POINTS				
	1	2	3	4	5
0.0	0.000000	-0.000000	1.000000	-0.000000	1.000000
0.0	0.003290	0.080630	0.824910	0.200130	0.890966
0.0	0.011140	0.135689	0.705650	0.331794	0.815211
0.1	0.021364	0.173672	0.624013	0.417615	0.761789
0.1	0.032596	0.200235	0.567767	0.472768	0.723366
0.1	0.043996	0.219137	0.528687	0.507433	0.695032
0.1	0.055069	0.232878	0.501239	0.528435	0.673497
0.1	0.065542	0.243117	0.481692	0.540350	0.656562
0.2	0.075275	0.250954	0.467529	0.546245	0.642756
0.2	0.084220	0.257121	0.457048	0.548174	0.631097
0.2	0.092377	0.262103	0.449097	0.547511	0.620927
0.2	0.099775	0.266223	0.442891	0.545174	0.611809
0.2	0.106461	0.269696	0.437897	0.541777	0.603450
0.3	0.112489	0.272668	0.433747	0.537725	0.595653
0.3	0.117913	0.275237	0.430191	0.533290	0.588289

TIME	LENGTH	NORMALISED TEMPERATURE VECTOR				
0.00	0.9002	0.088514	-0.279594	0.563739	-0.694959	0.336485
0.02	0.6035	0.088512	-0.279592	0.563739	-0.694960	0.336486
0.04	0.4046	0.088512	-0.279591	0.563738	-0.694961	0.336486
0.06	0.2712	0.088512	-0.279590	0.563738	-0.694961	0.336486
0.08	0.1818	0.088513	-0.279592	0.563738	-0.694961	0.336485
0.10	0.1219	0.088517	-0.279596	0.563739	-0.694959	0.336483
0.12	0.0817	0.088525	-0.279605	0.563742	-0.694954	0.336479
0.14	0.0548	0.088540	-0.279623	0.563748	-0.694944	0.336471
0.16	0.0367	0.088564	-0.279654	0.563758	-0.694927	0.336457
0.18	0.0246	0.088605	-0.279707	0.563776	-0.694897	0.336433
0.20	0.0165	0.088670	-0.279793	0.563807	-0.694849	0.336392
0.22	0.0111	0.088774	-0.279930	0.563855	-0.694772	0.336329
0.24	0.0074	0.088933	-0.280148	0.563933	-0.694650	0.336227
0.26	0.0050	0.089180	-0.280487	0.564059	-0.694458	0.336066
0.28	0.0033	0.089559	-0.281008	0.564254	-0.694160	0.335817

EIGENVALUE= -19.99522528

[illegible]

TIME	LENGTH	WAVELENGTH (nm)	WAVELENGTH (nm)
0.00	0.0000	0.0000	0.0000
0.05	0.0005	0.0005	0.0005
0.10	0.0010	0.0010	0.0010
0.15	0.0015	0.0015	0.0015
0.20	0.0020	0.0020	0.0020
0.25	0.0025	0.0025	0.0025
0.30	0.0030	0.0030	0.0030
0.35	0.0035	0.0035	0.0035
0.40	0.0040	0.0040	0.0040
0.45	0.0045	0.0045	0.0045
0.50	0.0050	0.0050	0.0050
0.55	0.0055	0.0055	0.0055
0.60	0.0060	0.0060	0.0060
0.65	0.0065	0.0065	0.0065
0.70	0.0070	0.0070	0.0070
0.75	0.0075	0.0075	0.0075
0.80	0.0080	0.0080	0.0080
0.85	0.0085	0.0085	0.0085
0.90	0.0090	0.0090	0.0090
0.95	0.0095	0.0095	0.0095
1.00	0.0100	0.0100	0.0100

EIGENARTIGE = -19.088755

REPRODUCTION OF THE ORIGINAL MATRIX

-7.247924	4.032952	0.002045	0.023639	-0.018400
4.032952	-8.910943	4.881922	-0.019982	0.012474
0.002045	4.881922	-10.591453	5.688623	0.019685
0.023639	-0.019982	5.688623	-12.195157	6.509119
-0.018400	0.012474	0.019685	6.509119	-6.529837

ERROR MATRIX

0.025702	-0.005174	-0.002045	-0.023639	0.018400
-0.005174	0.022054	-0.020811	0.019982	-0.012474
-0.002045	-0.020811	0.035898	0.005821	-0.019685
-0.023639	0.019982	0.005821	-0.027065	0.018659
0.018400	-0.012474	-0.019685	0.018659	0.002060

REPRODUCTION OF THE INITIAL MATRIX

-0.018400	0.01544	0.01688	0.0111	-0.018400
0.03333	-0.01688	0.03333	-0.01688	0.03333
0.00505	0.01688	-0.01688	0.01688	0.00505
0.03333	-0.01688	0.03333	-0.01688	0.03333
-0.018400	0.01544	0.01688	0.0111	-0.018400

ERROR MATRIX

0.018400	-0.01544	-0.01688	-0.0111	0.018400
-0.03333	0.01688	-0.03333	0.01688	-0.03333
-0.00505	-0.01688	0.00505	-0.01688	-0.00505
-0.03333	0.01688	-0.03333	0.01688	-0.03333
0.018400	-0.01544	-0.01688	-0.0111	0.018400

APPENDIX J.Listings of the Various Subroutines

The subroutines listed in this appendix are:

1. EIG 1
2. JACOBI
3. CHECK
4. INTERP
5. LINEAR
6. MAIN
7. MODIFY
8. PLOT
9. ANALYT
10. LESQFT
11. LINECT

"Comment cards" added to these various subroutines explain the purpose of these programs and the way to use them.

APPENDIX

TABLE I. SUMMARY OF DATA

The following table gives a summary of the data used in the present study.

1. Age	10
2. Sex	10
3. Race	10
4. Education	10
5. Income	10
6. Religion	10
7. Marital Status	10
8. Occupation	10
9. Health	10
10. Family Size	10

Source: Data collected from the 1960 Census of the United States.

Note: The figures in parentheses indicate the percentage of the total sample.

Continued

SUBROUTINE EIG 1

Calling Sequence: CALL EIG 1 (N, MAXN, M, S, R, V, A, B, W1, W2)

N Order of the input square symmetric matrix S. Range $1 \leq N \leq \text{MAXN}$.

MAXN Maximum order N may attain, i.e., the value in the dimension statement of the main program for the matrix S. Therefore the upper bound on solution size is determined by space requirements in the main program.

M The absolute value of M indicates the number of eigenvectors to be calculated, where $0 \leq M \leq N$. The ordering of the eigenroots, to which the calculated eigenvectors will correspond, is determined by the sign of M as follows:

$M \geq +0$: Eigenroots arranged in order to descending absolute value.

$M \leq -0$: Eigenroots arranged in order of ascending absolute value.

Input

S S(MAXN, MAXN) is the input symmetric matrix. Although partially used for temporary storage, S is restored to its original value before leaving.

Output

R Column vector, length at least N, for calculated eigenroots.

V V(MAXN, L), where L = M, is the matrix for calculated eigenvectors.

- A Column vector, length at least N , for calculated diagonal of tri-diagonal matrix.
- B Column vector, length at least N , where $B(2)$ through $B(N)$ will contain the calculated off diagonal of the tri-diagonal matrix.

Working Space

- W1) (Column vectors, each of length at least N , for
- W2) (temporary storage.


```

0 $IBFTC EIG1
1 SUBROUTINE EIG1(LP,NM,M,R,E,V,A,B,W1,W2)
2 DIMENSION R(20),E(10),V(10),A(10),B(10),W1(10),W2(10)
3 IF(LP-1)20,14,4
4 C TRI-DIAGONALIZE MATRIX
5 4 CALL TRIDI(LP,NM,R,A,B,W1,W2)
6 C FIND EIGENVALUES
7 CALL EIGVAL(LP,NM,M,E,A,B,W1,W2)
8 IF(M)5,9,5
9 C FIND EIGENVECTORS
10 5 K=IABS(M)
11 J=1
12 DO 7 I=1,K
13 CALL EIGVEC(LP,NM,R,A,B,E(I),V(J),W1,W2)
14 7 J=J+NM
15 C RESTORE INPUT MATRIX
16 9 NM1=NM+1
17 JJ=NM1
18 LP2=LP*NM
19 DO 12 I=2,LP2,NM1
20 K=I
21 DO 10 J=JJ,LP2,NM
22 R(K)=R(J)
23 10 K=K+1
24 12 JJ=JJ+NM1
25 GO TO 20
26 C SOLUTION FOR ORDER 1
27 14 E(1)=R(1)
28 V(1)=1.0
29 A(1)=R(1)
30 B(1)=0.0
31 20 RETURN
32 END

```


SN SOURCE STATEMENT

```
0 $IBFTC TRIDI DECK
C TRIDIAGONALIZATION SUBROUTINE
C RETURN ORIGINAL R IN UPPER TRIAN HALF INCLUDING DIAGONAL
C RETURN MODIFIED W MATRICES IN LOWER HALF OF R MATRIX
C RETURN NEW DIAGONAL IN A
C RETURN NEW FIRST OFF DIAGONAL IN B
1 SUBROUTINE TRIDI(LP,NM,R,A,B,W,Q)
2 DIMENSION R(10),A(10),B(10),Q(10),W(10)
3 LP1=LP-1
4 LP2=LP1*NM+LP
5 LPP=LP2-NM
6 NM1=NM+1
C STORE ORIGINAL DIAGONAL
7 L=0
10 DO 10 I=1,LP2,NM1
11 L=L+1
12 10 A(L)=R(I)
14 B(1)=0.
15 IF(LP-2)9,65,15
16 15 KK=0
17 DO 50 K=2,LP1
20 KL=KK+K
21 KU=KK+LP
22 KJ=K+1
C CALCULATE AND STORE MODIFIED COLUMN MATRIX W
23 SUM=0.0
24 DO 20 J=KL,KU
25 20 SUM=SUM+R(J)**2
27 S=SQRT(SUM)
30 RR=-R(KL)
31 B(K)=SIGN (S,RR)
32 S=1./S
33 W(K)=SQRT(ABS(R(KL))*S+1.0)
34 X=SIGN (S/W(K),R(KL))
35 R(KL)=W(K)
36 DO 30 I=KJ,LP
37 JJ=I+KK
40 W(I)=X*R(JJ)
41 30 R(JJ)=W(I)
C MOVE NEW R MATRIX WITH ROW K-U NOW HAVE 3 OFF 2ND DIAG
43 DO 35 J=K,LP
44 JJ=J+1
45 Q(J)=0.0
46 L=KK+J
47 DO 33 I=K,J
50 L=L+NM
51 33 Q(J)=Q(J)+R(L)*W(I)
53 IF(JJ-LP)34,34,36
54 34 DO 35 I=JJ,LP
55 L=L+1
56 35 Q(J)=Q(J)+R(L)*W(I)
61 36 X=0.0
62 DO 40 J=K,LP
63 40 X=X+W(J)*Q(J)
65 X=.5*X
```



```

66      DO 45 I=K,LP
67      45 Q(I)=X*W(I)-Q(I)
71      LL=KK
72      KK=KK+NM
73      DO 50 I=K,LP
74      LL=LL+NM
75      DO 50 J=I,LP
76      L=LL+J
77      50 R(L)=R(L)+Q(I)*W(J)+Q(J)*W(I)
C      SORT OUTPUT
03      L=1
04      DO 60 I=1,LP
05      X=A(I)
06      A(I)=R(L)
07      R(L)=X
10      60 L=L+NM1
12      65 B(LP)=R(LPP)
13      99 RETURN
14      END

```


SN SOURCE STATEMENT

```

0 $IBFTC EIGVAL DECK
C EIGENVALUE SUB FOR TRI-DIAGONAL MATRICES
1 SUBROUTINE EIGVAL(LP,NM,M,E,A,B,F,W)
2 DIMENSION E(10),A(10),B(10),F(10),W(10)
3 EQUIVALENCE (S1,IS1),(S2,IS2)
C FIND UPPER AND LOWER BOUNDS AND NORMALIZE INPUT
4 BD=ABS(A(1))
5 DO 5 I=2,LP
6 5 BD=AMAX1(BD,ABS(A(I))+B(I)**2)
10 BD=BD+1.
11 DO 6 I=1,LP
12 A(I)=A(I)/BD
13 B(I)=B(I)/BD
14 W(I)=1.
15 6 E(I)=-1.
17 DO 50 K=1,LP
20 8 IF((W(K)-E(K))/AMAX1(ABS(W(K)),ABS(E(K)),1.0E-9)-1.0E-7)50,50,10
21 10 X=(W(K)+E(K))*0.5
C FIND NUMBER OF EIGENVALUES,N, GTR THAN OR EQUAL TO X
22 IS2=1
23 F(1)=A(1)-X
24 IF(F(1))102,104,104
25 102 IS1=-1
26 N=0
27 GO TO 105
30 104 IS1=1
31 N=1
32 105 DO 120 I=2,LP
33 IF(B(I))106,113,106
34 106 IF(B(I-1))107,114,107
35 107 IF(ABS(F(I-1))+ABS(F(I-2))-1.0E-15)110,112,112
36 110 F(I-1)=F(I-1)*1.0E15
37 F(I-2)=F(I-2)*1.0E15
40 112 F(I)=(A(I)-X)*F(I-1)-B(I)**2*F(I-2)
41 GO TO 115
42 113 F(I)=(A(I)-X)*SIGN(1.,S1)
43 GO TO 115
44 114 F(I)=(A(I)-X)*F(I-1)-SIGN(B(I)**2,S2)
45 115 S2=S1
46 IF(F(I))116,117,116
47 116 S1=SIGN(S1,F(I))
50 IF(IS2+IS1)117,120,117
51 117 N=N+1
52 120 CONTINUE
C TRAP EIGENVALUES IN SMALLER AND SMALLER BOUNDS
54 N=LP-N
55 IF(N-K)20,12,12
56 12 DO 15 J=K,N
57 15 W(J)=X
61 20 N=N+1
62 IF(LP-N)8,24,24
63 24 DO 26 J=N,LP
64 IF(X-E(J))8,8,26
65 26 E(J)=X
67 GO TO 8

```



```
70      50 CONTINUE  
C      RESTORE INPUT AND ORDER EIGENVALUES  
72      DO 60 I=1,LP  
73      A(I)=A(I)*BD  
74      B(I)=B(I)*BD  
75      60 F(I)=(W(I)+E(I))*BD*.5  
77      J=LP  
00      K=1  
01      DO 70 I=1,LP  
02      IF(ABS (F(K))-ABS (F(J)))63,63,65  
03      63 E(I)=F(J)  
04      J=J-1  
05      GO TO 70  
06      65 E(I)=F(K)  
07      K=K+1  
10      70 CONTINUE  
12      IF(ISIGN (1,M))75,80,80  
13      75 DO 77 I=1,LP  
14      77 F(I)=E(I)  
16      J=LP  
17      DO 79 I=1,LP  
20      E(I)=F(J)  
21      79 J=J-1  
23      80 CONTINUE  
24      RETURN  
25      END
```


LSN SOURCE STATEMENT

```
0 $IBFTC EIGVEC DECK
1 SUBROUTINE EIGVEC (LP,NM,R,A,B,E,V,P,Q)
2 DIMENSION R(10),A(10),B(10),V(10),P(10),Q(10)
C SETUP SIMULTANEOUS EQN FOR EGENVECTOR WITH EIGNVALUE E
3 X=A(1)-E
4 Y=B(2)
5 LP1=LP-1
6 DO 10 I=1, LP1
7 IF (ABS (X)-ABS (B(I+1)))4,6,8
10 4 P(I)=B(I+1)
11 Q(I)=A(I+1)-E
12 V(I)=B(I+2)
13 Z=-X/P(I)
14 X=Z*Q(I)+Y
15 IF(LP1-I)5,10,5
16 5 Y=Z*V(I)
17 GO TO 10
20 6 IF(X)8,7,8
21 7 X=1.0E-10
22 8 P(I)=X
23 Q(I)=Y
24 V(I)=0.
25 X=A(I+1)-(B(I+1)/X*Y+E)
26 Y=B(I+2)
27 10 CONTINUE
C SOLVE SIMULT EQN FOR EGENVECTOR OF TRI-DIAG MATRIX
31 20 IF(X)21,28,21
32 21 V(LP)=1./X
33 22 I=LP1
34 V(I)=(1.-Q(I)*V(LP))/P(I)
35 X=V(LP)**2+V(I)**2
36 25 I=I-1
37 IF(I)26,30,26
40 26 V(I)=(1.-(Q(I)*V(I+1)+V(I)*V(I+2)))/P(I)
41 X=X+V(I)**2
42 GO TO 25
43 28 V(LP)=1.0E10
44 GO TO 22
45 30 X=SQRT (X)
46 DO 31 I=1,LP
47 31 V(I)=V(I)/X
C TRANSFORM EIGENVECTOR TO SOLN OF ORIGNAL MATRIX
51 J=LP1*NM-NM
52 K=LP
53 GO TO 42
54 32 K=K-1
55 J=J-NM
56 Y=0.0
57 DO 35 I=K,LP
60 L=J+I
61 35 Y=Y+V(I)*R(L)
63 DO 40 I=K,LP
64 L=J+I
65 40 V(I)=V(I)-Y*R(L)
67 42 IF(J)32,44,32
```


70 44 RETURN
71 END


```
0 $IBFTC JACOBI
1 SUBROUTINE JACOBI(N,MATRIX,VECTOR,TOLERC,NORM)
C
C
C JACOBI EVALUATES THE EIGENVALUES AND EIGENVECTORS OF A
C SYMMETRIC MATRIX. THIS PROGRAM WAS WRITTEN BY MR.G.R.JACKSON,
C DEPT.OF COMPUTING SCIENCE,U.OF A,EDMONTON.
C
C INPUT DATA
C
C N=ORDER OF THE MATRIX.
C MATRIX=THE MATRIX ITSELF
C VECTOR=MATRIX OF EIGENVECTORS
C TOLERC=TOLERANCE LIMIT,INTERNALLY SET IF NOT SPECIFIED.
C NORM=SPECIFIES THE TYPE OF NORMALISATION OF THE EIGENVECTORS.
C IF THIS IS 2,ORTHONORMAL EIGENVECTORS ARE OBTAINED.
C
2 DOUBLE PRECISION SINE,COSINE,TS1,TS2,LAMDA,MU,OMEGA,TOLERC,
1 MATRIX(25,25),VECTOR(25,25),TS3
3 INTEGER I,J,P,Q,N,NM1,PP1,DONE,NORM
4 IF(TOLERC .EQ.0.000000000000) TOLERC =0.0000000000001
7 NM1=N-1
10 DO 1 I=1,N
11 DO 1 J=1,N
12 VECTOR(I,J)=0.0000000000000000
13 1 IF(I.EQ.J) VECTOR(I,I)=1.0000000000000000
20 2 DONE=0
21 DO 5 P=1,NM1
22 PP1=P+1
23 DO 5 Q=PP1,N
24 IF(DABS(MATRIX(P,Q)).LE.TOLERC) GO TO 5
27 DONE=1
30 LAMDA=-MATRIX(P,Q)
31 MU=.5000000000000000*(MATRIX(P,P)-MATRIX(Q,Q))
32 OMEGA=LAMDA/DSQRT(LAMDA*LAMDA+MU*MU)
33 IF(MU.LT.0.0000000000000000) OMEGA=-OMEGA
36 SINE=OMEGA/DSQRT(2.0000000000000000+2.0000000000000000*
1 DSQRT(1.0000000000000000-OMEGA*OMEGA))
37 COSINE=DSQRT(1.0000000000000000-SINE*SINE)
40 DO 4 I=1,N
41 IF(I.EQ.P.OR.I.EQ.Q) GO TO 3
44 TS1=COSINE*MATRIX(P,I)-SINE*MATRIX(Q,I)
45 TS2=SINE*MATRIX(P,I)+COSINE*MATRIX(Q,I)
46 MATRIX(P,I)=TS1
47 MATRIX(Q,I)=TS2
50 MATRIX(I,P)=TS1
51 MATRIX(I,Q)=TS2
52 3 TS1=VECTOR(I,P)*COSINE-VECTOR(I,Q)*SINE
53 TS2=VECTOR(I,P)*SINE+VECTOR(I,Q)*COSINE
54 VECTOR(I,P)=TS1
55 4 VECTOR(I,Q)=TS2
57 TS1=MATRIX(P,P)*COSINE*COSINE-2.0000000000000000*MATRIX(P,Q)*
1 COSINE*SINE + MATRIX(Q,Q)*SINE*SINE
60 TS2=MATRIX(P,P)*SINE*SINE + 2.0000000000000000*MATRIX(P,Q)*
```


ISN SOURCE STATEMENT

```
1      COSINE*SINE + MATRIX(Q,Q)*COSINE*COSINE
61     TS3=(MATRIX(P,P)-MATRIX(Q,Q))*SINE*COSINE + MATRIX(P,Q)*
1      (COSINE*COSINE-SINE*SINE)
62     MATRIX(P,P)=TS1
63     MATRIX(Q,Q)=TS2
64     MATRIX(P,Q)=TS3
65     MATRIX(Q,P)=TS3
66     5  CONTINUE
71     IF(DONE.NE.0) GO TO 2
74     IF(NORM.NE.1) GO TO 8
77     DO 7 J=1,N
00     TS1=VECTOR(1,J)
01     DO 6 I=2,N
02     6  IF(DABS(TS1).LT.DABS(VECTOR(I,J))) TS1 = VECTOR(I,J)
06     DO 7 I=1,N
07     7  VECTOR(I,J) = VECTOR(I,J)/TS1
12     RETURN
13     8  DO 11 J=1,N
14     TS1 = 0.0000000000000000
15     DO 9 I=1,N
16     9  TS1 = TS1 + VECTOR(I,J)*VECTOR(I,J)
20     TS1 = 1.0000000000000000/DSQRT(TS1)
21     DO 10 I=1,N
22     10 VECTOR(I,J) = VECTOR(I,J)*TS1
24     11 CONTINUE
26     RETURN
27     END
```



```
0 $IBFTC CHECK
1 SUBROUTINE CHECK(V,R,N,W)
C
C THIS SUBROUTINE MAKES A SIMILARITY TRANSFORMATION.
C
2 REAL V(16,16),R(16),W(16,16)
3 DO 400 J=1,N
4 DO 400 I=1,N
5 W(I,J)=0.0
6 DO 400 K=1,N
7 W(I,J)=W(I,J)+V(I,K)*R(K)*V(J,K)
10 400 CONTINUE
14 RETURN
15 END
```



```
0 $IBFTC INTERP
1 SUBROUTINE INTERP(NP,GRIDX,X,Y,KA,RR)
C
C INTERP USES THIRD ORDER LAGRANGIAN INTERPOLATION FORMULA
C AND HAS BUILT-IN SAFETY DEVICES.THE INTERPOLATED VALUE IS RR.
C
2 REAL GRIDX(10),X(75),Y(75)
3 DO 202 KB=1,NP
4 IF(GRIDX(KA).GT.X(KB)) GO TO 202
7 GO TO 203
10 202 CONTINUE
12 203 LA=KB-2
13 IF(LA.EQ.0) LA=1
16 LB=KB+1
17 IF(KB.EQ.NP) LB=NP
22 RR=0.0
23 DO 207 KC=LA,LB
24 RP=1.0
25 DO 206 KD=LA,LB
26 IF(KD.EQ.KC) GO TO 205
31 RS=(GRIDX(KA)-X(KD))/(X(KC)-X(KD))
32 RP=RP*RS
33 GO TO 206
34 205 RS=1.0
35 206 CONTINUE
37 RP=RP*Y(KC)
40 207 RR=RR+RP
42 RETURN
43 END
```



```
0 $IBFTC LINEAR
1 SUBROUTINE LINEAR(NP,GRIDX,X,Y,KA,RR)
C
C LINEAR IS A LINEAR INTERPOLATION FORMULA.
C
2 REAL GRIDX(10),X(75),Y(75)
3 DO 202 KB=1,NP
4 IF(GRIDX(KA).GT.X(KB)) GO TO 202
7 GO TO 203
10 202 CONTINUE
12 203 RR=Y(KB-1)+(Y(KB)-Y(KB-1))*(GRIDX(KA)-X(KB-1))/(X(KB)-X(KB-1))
13 RETURN
14 END
```



```
0 $IBFTC MAIN
1 SUBROUTINE MAIN(N,WORK,V,C1,PZ,R,TIME1,NT,P)
C
C THIS SUBROUTINE USES THE SEMI-ANALYTICAL SOLUTION TO EVA-
C LUATE THE TEMPERATURE PROFILES AT VARIOUS ELAPSED TIMES.
C THIS PROGRAM DOES NOT TAKE THE VARIATION OF HEAT CAPACITY
C INTO ACCOUNT. THE TEMPERATURE PROFILES COME OUT AT P.
C
2 REAL WORK(16),V(16,16),C1(16),PZ(16),R(16),TIME1(15),P(15,16)
3 DO 8 J=1,N
4 WORK(J)=0.0
5 DO 8 K=1,N
6 8 WORK(J)=WORK(J)+V(K,J)*C1(K)
11 DO 9 J=1,N
12 9 C1(J)=WORK(J)
14 DO 10 J=1,N
15 WORK(J)=0.0
16 DO 10 K=1,N
17 10 WORK(J)=WORK(J)+V(K,J)*PZ(K)
22 DO 11 J=1,N
23 11 PZ(J)=WORK(J)
25 DO 18 J=1,NT
26 DO 15 K=1,N
27 UAR=R(K)*TIME1(J)
30 15 WORK(K)=((R(K)*PZ(K)+C1(K))* EXP(UAR)-C1(K))/R(K)
32 DO 17 L1=1,N
33 P(J,L1)=0.0
34 DO 16 L2=1,N
35 16 P(J,L1)=P(J,L1)+V(L1,L2)*WORK(L2)
37 17 CONTINUE
41 18 CONTINUE
43 RETURN
44 END
```



```
0 $IBFTC MODIFY
1 SUBROUTINE MODIFY(WORK,V,C3,CZ,Z,C,N,NT)
C
C MODIFY CALCULATES THE MODIFIED VARIABLES.
C
2 REAL WORK(10),V(16,16),C3(10),CZ(10),Z(25,10),C(25,10)
3 DO 65 J=1,N
4 WORK(J)=0.0
5 DO 65 K=1,N
6 65 WORK(J)=WORK(J)+V(K,J)*C3(K)
11 DO 66 J=1,N
12 66 C3(J)=WORK(J)
14 DO 67 J=1,N
15 WORK(J)=0.0
16 DO 67 K=1,N
17 67 WORK(J)=WORK(J)+V(K,J)*CZ(K)
22 DO 68 J=1,N
23 68 CZ(J)=WORK(J)
25 DO 75 K=1,NT
26 DO 75 J=1,N
27 Z(K,J)=0.0
30 DO 74 L=1,N
31 74 Z(K,J)=Z(K,J)+V(L,J)*C(K,L)
33 75 CONTINUE
36 RETURN
37 END
```



```
0 $IBFTC PLOT
1 SUBROUTINE PLOT(C,R,Z,C3,CZ,N,NT)
C
C PLOT CALCULATES VALUES FOR PLOTTING,REQUIRED BY THE PROGRAM
C IH-20 LOGFIT.
C
2 REAL R(16),Z(25,10),C3(10), CZ(10),C(25,10)
3 DO 87 J=1,NT
4 DO 85 K=1,N
5 85 C(J,K)=((R(K)*Z(J,K)+C3(K))/(R(K)*CZ(K)+C3(K)))
7 87 CONTINUE
11 RETURN
12 END
```



```
0 $IBFTC ANALYT NODECK
1 SUBROUTINE ANALYT(ALPHA,RM,TBOUND,TINC,N,NT,X,TIME,TANAL)
C
C
C ANALYT USES AN ANALYTICAL SOLUTION(REF.MCADAMS,P.34) TO
C CALCULATE TEMPERATURE PROFILES AT THE SPECIFIED TIMES.
C
C
2 REAL X(10),TIME(25),TANAL(25,10)
3 PI=3.1415927
4 DO 10 K=1,N
5 DO 10 J=1,NT
6 TANAL(J,K)=0.0
7 DO 9 L=1,20
10 AL=L
11 AA=(2.*AL-1.)*PI*X(K)
12 AB=-(2.*AL-1.)**2*(PI/2.)**2*ALPHA*TIME(J)/RM
13 9 TANAL(J,K)=TANAL(J,K)+SIN(AA)*EXP(AB)/(2.*AL-1.)
15 10 TANAL(J,K)=TBOUND+4.*TANAL(J,K)*(TINC-TBOUND)/PI
20 RETURN
21 END
```


SUBROUTINE LISTINGS

FORTRAN SOURCE LIST

ISN

SOURCE STATEMENT

```
0 $IBFTC LESQFT
1      SUBROUTINE LESQFT(AX,AY,SLOPE)
   C
   C      LESQFT FITS AX AND AY TO A STRAIGHT LINE AND EVALUATES THE
   C      SLOPE OF THE LINE.
   C
2      REAL AX(3),AY(3)
3      NC=3
4      SUM1=0.0
5      DO 100 J=1,NC
6      100 SUM1=SUM1+AX(J)
10     SUM2=0.0
11     DO 101 J=1,NC
12     101 SUM2=SUM2+AX(J)**2
14     SUM3=0.0
15     DO 102 J=1,NC
16     102 SUM3=SUM3+AY(J)
20     SUM4=0.0
21     DO 103 J=1,NC
22     103 SUM4=SUM4+AX(J)*AY(J)
24     AN=NC
25     SLOPE=(SUM3*SUM1-SUM4*AN)/(SUM1**2-AN*SUM2)
26     RETURN
27     END
```



```

0 $IBFTC LINECT
1 SUBROUTINE LINECT(LINES,N,REFER)
  C
  C LINECT KEEPS TRACK OF NUMBER OF LINES PRINTED AND SKIPS
  C TO A NEW PAGE LEAVING SUFFICIENT MARGIN.
  C
2 INTEGER LINES,N,REFER
3 1 FORMAT(1H2,10X,8H ..CONTD)
4 2 FORMAT(1H ,10X)
5 LINES=LINES+N
6 IF(LINES.GT.61) GO TO 5
11 RETURN
12 5 WRITE(6,1)
13 LINES=9
14 IF(REFER.EQ.1) GO TO 6
17 RETURN
20 6 WRITE(6,2)
21 LINES=10
22 RETURN
23 END
  
```




Date Due

[illegible]

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